Blue Pelican Algebra I

First Semester

Absent-student Version 1.01

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Alg1 Syllabus (First Semester)

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Semester summary

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Enrichment Topics

Topic A: Commutative, distributive, and associative properties
Topic B: Inequality conjunctions and disjunctions
Topic C: Two dimensional inequalities
Topic D: Combining direct and indirect variations
Topic E: Scientific notation
Topic F: Greatest common factor (GCF) and least common multiple (LCM)
Topic G: Derivation of the Quadratic Formula
Topic H: Completing the square
Topic I: Statistics
Topic J: Real-world applications of parabolas and the other three conic sections

Alg 1, Unit 1

Basic Operations

Unit 1: Lesson 01 **Order of operations (PEMDAS)**

In arithmetic expressions it is important to know the **order** in which to do the operations. The correct order is given by **PEMDAS**:

- PEMDAS is a memory aid for the correct order: **parentheses, exponents, multiplication, division, addition, and subtraction**.
- Even though multiplication is listed before division, they are actually of the **same** priority.
- Even though addition is listed before subtraction, they are actually of the **same** priority.
- When deciding which of two operations of the same priority to do first, do them in a **left-to-right order**.

In the following examples, perform the arithmetic operations in the correct order to produce a final value for the expression.

Example 1: $2 \cdot 8 + 5 - 6 + 1 \cdot 3$

$$= 16 + 5 - 6 + 1 \cdot 3$$
$$= 16 + 5 - 6 + 3$$
$$= 21 - 6 + 3$$
$$= 15 + 3 = \boxed{18}$$

Example 2: $17 + 6 \cdot 3 \div 2$

$$= 17 + 18 \div 2$$
$$= 17 + 9$$
$$= \boxed{26}$$

Example 3: $2 \cdot (7 + 2) + 1 - 8/2$

$$= 2(9) + 1 - 8/2$$
$$= 18 + 1 - 8/2$$
$$= 18 + 1 - 4$$
$$= 19 - 4 = \boxed{15}$$

Example 4: $2 \cdot 3^2 - 15/3$

$$= 2 \cdot 9 - 15/3$$
$$= 18 - 15/3$$
$$= 18 - 5 = \boxed{13}$$

Example 5: $24 \div 2^2 \cdot 10 - 2(3 \cdot 5)$

$= 24 \div 2^2 \cdot 10 - 2(15)$
$= 24 \div 4 \cdot 10 - 30$
$= 6 \cdot 10 - 30$
$= 60 - 30$
$= \boxed{30}$

Example 6: $(18 - (12/2) + 3)/(4 + 1)$

$= (18 - 6 + 3)/(4+1)$
$= (12+3)/(5)$
$= 15/5$
$= \boxed{3}$

As a special case of parentheses, consider a fraction written in this form:

$$\frac{a + b}{c + d}$$

Rewrite with parentheses in this form $(a + b)/(c + d)$ and simplify in the parentheses first.

Example 7: $\dfrac{3 \cdot 2 + 6 \cdot 5}{28 - 25}$

$= (3 \cdot 2 + 6 \cdot 5)/(28 - 25)$
$= (6 + 6 \cdot 5)/3$
$= (6 + 30)/3$
$= 36/3 = \boxed{12}$

Assignment: In the following examples, perform the arithmetic operations in the correct order to produce a final value for the expression.

1. $8 + 4(7 - 2)$

2. $3(4 + 1) - 12 \div 2^2$

3. $11 - 22/11 + 2^3 \cdot 6$

4. $40 - 25 \div 5$

5. $(6 \cdot 5)/(11 - 8)$

6. $\dfrac{4 \cdot 3^2}{18 - 2 \cdot 3}$

7. $11 + 1 \cdot 2 - 4 \cdot 1 + 36 \div 3$

8. $200/2/2 \cdot 3 + 1$

9. $\dfrac{10 \cdot 2 + 1 \cdot 12}{1 + 2 \cdot 3 - 3}$

10. $8 \cdot 5 - 2(22 \div 2) + 3(5 - 2)$

11. $3(36 \div 9) + 2(80 - 60) - 3 \cdot 4$

12. $\dfrac{5 \cdot 2 + 48 \div 12}{9 - 2 - 5}$

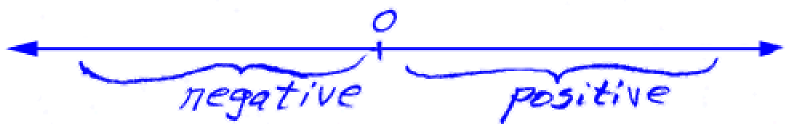
*13. $\{\,72 - 4[11 - 3(12/4)]\ \}/2$

*14. $\dfrac{15[\,5 + 3(8 \div 4 + 2)\,] + 15}{7 - 45 \div [5 + 2(6 \div 3)]}$

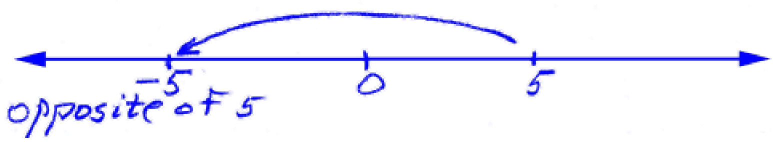
Unit 1: Lesson 02

Negative numbers, opposites, absolute value
Inequalities

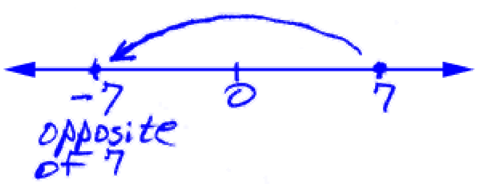
Negative numbers are to the left of the origin (0) while positive numbers are to the right.



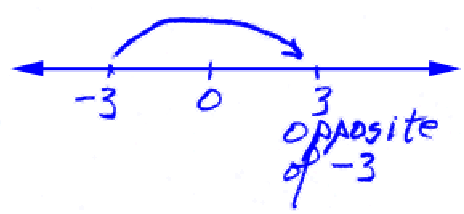
Opposite numbers are mirror images of each other across the origin.



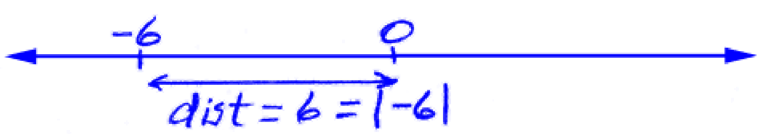
Example 1: Locate 7 on a number line and then locate its opposite.



Example 2: Locate –3 on a number line and then locate its opposite.

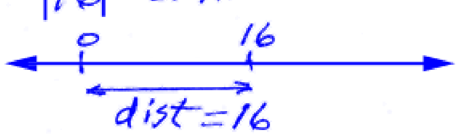


The **absolute value** of a number (indicated with vertical bars, |4|) is the distance of a number from the origin. The absolute value of a number is **always positive**.



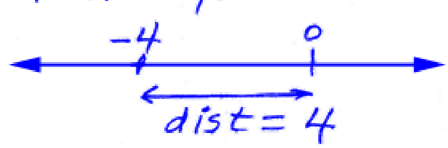
Example 3: $|\mathbf{16}| = ?$

$|16| = 16$



Example 4: $|-\mathbf{4}| = ?$

$|-4| = 4$



When an expression is inside an absolute value,

- simplify the expression with PEMDAS (down to a **single number**),
- and then take the absolute value of that number.

Example 5: $|9 - 2 \cdot 3|$

$$|9-2\cdot 3| = |9-6| = |3| = 3$$

Example 6: In the following table, fill in the blank areas with the appropriate integer that best describes the phrase, its opposite, and its absolute value.

Description	Integer	Opposite	Absolute value
A price **increase** of $4	**4**	**– 4**	**4**
Ten degrees **below** freezing	**– 10**	**10**	**10**
A bank **deposit** of $40	**40**	**–40**	**40**
3 points **off** on a test question	**–3**	**3**	**3**
A five point **bonus** on a test	**5**	**–5**	**5**

Any number, *a*, that lies to the **left** on a number line of another number, *b*, is said to be **less** than *b*:

$a < b$ (read this as, "*a* is less than *b*.")

Any number, *c*, that lies to the **right** on a number line of another number, *d*, is said to be **greater** than *d*:

$c > d$ (read this as, "*c* is greater than *d*.")

An easy way to remember these **inequality** relationships is, "The alligator eats the big one."

Use the number line above to fill in the appropriate symbol (< , >, or =) in the blanks in the examples below. Give the reasons for your choices.

Example 7: -4 <u> < </u> -2 *because -4 lies to the left of -2*

Example 8: 1 <u> > </u> -2 *because 1 lies to the right of -2*

Example 9: b <u> > </u> a *because b lies to the right of a*

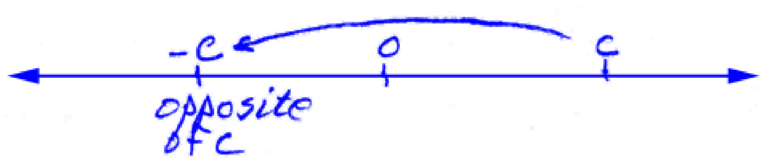
Example 10: a <u> < </u> c *because a lies to the left of c*

Example 11: $|-2|$ <u> = </u> 2 *because absolute value is always positive*

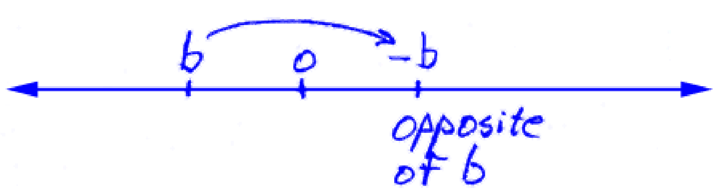
Consider -2 on a number line as seen at the top of this page. It is represented to the **left** of the origin since it is a negative number. The point *b* is also to the left of the origin, so what would be the meaning of $-b$?

The meaning of the **negative of a variable** is that it is the **opposite** of that variable.

Example 12: Redraw the number line at the top of this page and locate $-c$.



Example 13: Redraw the number line at the top of this page and locate $-b$.



Assignment:

1. Locate –8 on a number line and then locate its opposite.	2. Locate 6 on a number line and then locate its opposite.
3. Locate –4 on a number line and then locate its absolute value.	4. Locate 2 on a number line and then locate its absolute value.
5. How far from the origin is $\lvert -10 \rvert$?	6. What is the value of $7 - \lvert -7 \rvert$?
7. Simplify $\lvert 17 - 6 - 1 \rvert$.	8. Simplify $\lvert (17 - 6 - 1)/2 \rvert$.

9. Simplify |–2| + 6 – 7

10. Simplify (5 + |–17|) –3^2

11. In the following table, fill in the blank areas with the appropriate integer that best describes the phrase, its opposite, and its absolute value.

Description	Integer	Opposite	Absolute value
A 15 yard penalty			
An 11 yard gain			
A bank **withdrawal** of $36			
8 points **off** on a test question			
Thrown for a loss of 3 yards			
4 points above average			

Use the number line above to fill in the appropriate symbol (< , >, or =) in the blanks in the examples below. Give the reasons for your choices.

12.	5 _____ –15	
13.	–15 _____ –10	
14.	x ____ y	
15.	z ____ 0	
16.	|–10| ____ –10	
17.	0 ____ x	
18.	|y| ____ 5	
*19.	–x ____ y	

20. Redraw the number line given on the previous page and locate – y.

21. Redraw the number line given on the previous page and locate – z.

Unit 1: Lesson 03

Review of sign rules for arithmetic operations
Unit multipliers

Rules for addition and subtraction:

If signs are alike: Add the two numbers and apply their sign.

Example group 1:

3 + (+4) = ***+7***

(-5) – 4 = ***-9***

5 + 8 = ***+13***

-4 + (-6) = ***-10***

-9 –2 = ***-11***

If signs are different: Subtract and give the answer the sign of the largest number.

Examples group 2:

3 + (-7) = ***-4***

14 – 8 = ***6***

9 – 11 = ***-2***

22 + (-1) = ***21***

Rules for multiplication:

If signs are alike: Multiply and give the answer a positive sign.

Example group 3:

3(4) = ***12***

-3(-12) = ***36***

(-5)(-3) = ***15***

If signs are different: Multiply and give the answer a negative sign.

Example group 4:

(-3)4 = ***-12***

5(-2) = ***-10***

Rules for division (same as for multiplication):

If signs are alike: Divide and give the answer a positive sign.

Example group 5:

12 / (4) = *3*   -12 / (-3) = *4*

6 / 2 = *3*   (-15) / (-3) = *5*

If signs are different: Divide and give the answer a negative sign.

Example group 6:

(-30) / 5 = *-6*   -8 / 2 = *-4*

16 / (-2) = *-8*

Unit multipliers:

Now consider the various ways in which we could express 1 as any number over itself. For example:

$$\frac{189}{189} = 1, \quad \frac{\pi}{\pi} = 1, \text{ etc.}$$

Consider an unusual way in which we could multiply by 1. Since 12 inches = 1 foot, when we "stack" them as follows, the quotient is exactly 1:

$$\frac{12\text{ in}}{1\text{ ft}} = 1 \quad \text{or} \quad \frac{1\text{ ft}}{12\text{ in}} = 1$$

Some other ways to "build *1*" are:

$$\frac{2\text{ pints}}{1\text{ quart}}, \quad \frac{1\text{ yd}}{36''}, \quad \frac{100\text{ cm}}{1\text{ meter}}$$

These quantities that are equivalent to *1* are known as **unit multipliers**. They are useful in converting a number expressed in one type of units to an **equivalent number of different types of units**. . .for example, from inches to yards.

Example 7: Convert 108.19 inches to yards.

$$\frac{108.19\,\cancel{in}}{1}\;\frac{1\,yd}{36\,\cancel{in}} = \frac{108.19}{36}\,yd$$
$$= \boxed{3.0052\bar{7}\,yd}$$

Example 8: Convert 22.8 feet into inches.

$$\frac{22.8\,\cancel{ft}}{1}\;\frac{12\,in}{1\,\cancel{ft}} = 22.8(12)\,in$$
$$= \boxed{273.6\,in}$$

Example 9: Convert 450 cm into meters.

$$\frac{450\,\cancel{cm}}{1}\;\frac{1\,m}{100\,\cancel{cm}} = \frac{450}{100}\,m$$
$$= \boxed{4.5\,m}$$

Example 10: Use the fact that 1 inch = 2.54 cm to convert 19 cm into inches.

$$\frac{19\,\cancel{cm}}{1}\;\frac{1\,in}{2.54\,\cancel{cm}} = \frac{19}{2.54}\,in$$
$$= \boxed{7.4803\,in}$$

Multiple applications of unit multipliers:

It is possible to apply **more than one unit multiplier in succession** in order to achieve the desired conversion.

***Example 11:** Convert 150 meters into inches.

$$\frac{150\,\cancel{m}}{1}\ \frac{100\,cm}{1\,\cancel{m}} = 15{,}000\,\cancel{cm}\ \frac{1\,in}{2.54\,\cancel{cm}}$$

$$= \frac{15{,}000}{2.54}\ in$$

$$= \boxed{5{,}905.5118\ in}$$

Assignment:

1. 5(-3) =	**2.** 8(5) =	**3.** -9/(-3) =
4. -2(-6) **=**	**5.** 22(-1) =	**6.** -12(-2) =
7. 3 + (-8) =	**8.** (-50)/10 =	**9.** 2 + (19) =
10. 16(2) =	**11.** 23 + (-2) =	**12.** -8/4 =
13. 15 – 6 =	**14.** 16/(-2) =	**15.** 36/4 =
16. (-3)(-8) =	**17.** 5(-4) =	**18.** -3(-22) =
19. 9 – 12 =	**20.** 5 + (8) =	**21.** -6 + (-7) =
22. 8 + (-11) =	**23.** (-2) – 4 =	**24.** -19(-2) =
***25.** (400 – 20)/(-10) =	***26.** -4 + (-2)(-6) =	***27.** (-5)(-4)(-3) =

28. Use a unit multiplier to convert 24.1 quarts to pints (1 quart = 2 pints).

29. Use a unit multiplier to convert 80.9 millimeters to meters (1000 mm = 1 m).

30. Use a unit multiplier to convert 11.28 inches to centimeters (2.54 cm = 1 in).

31. Use a unit multiplier to convert 102 centimeters to inches.

32. Use a unit multiplier to convert 82,000 feet to miles (5280 ft = 1 mi).

***33.** Use multiple unit multipliers to convert 82,000 inches to meters.

Unit 1: Lesson 04

Evaluating algebraic expressions
Combining like terms

Example 1: Evaluate x + y – 2 if x = 3 and y = 11.

$$x+y-2=3+11-2$$
$$=14-2=\boxed{12}$$

Example 2: Evaluate $\frac{a\,b\,c}{a-c}$ if a = –10, b = 2, and c = 5.

$$\frac{abc}{a-c}=\frac{-10\cdot 2\cdot 5}{-10-5}=\frac{-100}{-15}$$
$$=\frac{100}{15}=\boxed{\frac{20}{3}}$$

Example 3: Evaluate |z – x/2 + y| if x = 6, y = 10, z = 15.

$$\left|z-\frac{x}{2}+y\right|=\left|15-\frac{6}{2}+10\right|$$
$$=|15-3+10|=|12+10|=|22|$$
$$=\boxed{22}$$

Like terms are those that contain exactly the same variables and with corresponding variables having the **same** exponent.

Example 4: (like terms)

$3x, -7x$ like

$5ax^2, 12ax^2$ like

Example 5: (unlike terms)

$4x, 4y$
unlike

$8z^2, -3z^3$
unlike

Simplify algebraic expressions by adding or subtracting the coefficients of **like terms** according to the rules of addition and subtraction given in Lesson 3.

Example 6: Simplify 4x – 3z – 8x + 12z

$$4x - 3z - 8x + 12z = \boxed{-4x + 9z}$$

Example 7: Simplify $3a^2 - 5a + 6a^2 + a - 2a$

$$3a^2 - 5a + 6a^2 + a - 2a$$

$$= 9a^2 - 4a - 2a = \boxed{9a^2 - 6a}$$

Example 8: Combine like terms and then evaluate 6ap –11q + 4q – 3ap at a = 1, p = 2 and q = 15.

$$6ap - 11q + 4q - 3ap = 3ap - 7q$$

$$= 3 \cdot 1 \cdot 2 - 7 \cdot 15$$

$$= 6 - 105 = \boxed{-99}$$

Assignment:

1. Evaluate x – y – z if x = 8, y = 3, and z = 1.	2. Evaluate 3x/y at x = 12 and y = 2.
3. Evaluate |–4a – 2b| where a = 10 and b = –8.	4. Evaluate $\frac{4x + y - z}{x}$ where x = 7, y = 2, and z = 1.
5. Simplify 8m – 6 + 9m + 5 + m	6. Simplify a + 2b – 22a + 17b – 1

7. Simplify $6x - 2y + z - 3z + x + 13y$

8. Simplify $5z^2 - 6y^3 + 20z^2 + y^3 + 14$

9. Simplify $|-5|(x - 5x) + 2x$

10. Evaluate $-2(x - m)(x + m)$ if $x = 8$ and $m = 9$.

11. Simplify $-5x + 2y + 4 + 6x - y + 11$and then evaluate at $x = 4$ and $y = -9$.

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*12. Combine like terms in $3^2z + 2^3 + 7z - |18a|$ and then evaluate at $a = -2$ and $z = -1$.

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*13. Simplify $26xz^2 - 22x^2z + 4xz^2 + 3x^2z$

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14. Evaluate $|1 - x/3 + j|$ if $x = 12$ and $j = 2$.

Unit 1: Lesson 05

Evaluating expressions that distribute negative numbers
Nested groups

Using the **distributive property**, we can write:

$$a(b - c + d) = ab - ac + ad$$

Be especially careful when *a* is negative as in some of the following examples.

Example 1: Simplify 2p – 6(5 – 4p)

$2p - 6(5 - 4p)$

$= 2p - 30 + 24p$

$= \boxed{26p - 30}$

Example 2: Simplify 3(5y – 1) – 2(4 + y)

$3(5y - 1) - 2(4 + y)$

$= 15y - 3 - 8 - 2y$

$= \boxed{13y - 11}$

A lone negative sign in front of a parenthesis means to **distribute in –1**.

$$-(a - b) = -a + b$$

Example 3: Simplify 7x – (4 – 3x) + 1

$7x - 1(4 - 3x) + 1 = 7x - 4 + 3x + 1$

$= \boxed{10x - 3}$

Example 4: Simplify 11m – (– m + n) – 12n and then evaluate at m = 2 and n = 7.

$11m - 1(-m + n) - 12n$

$= 11m + m - n - 12n = 12m - 13n$

$= 12 \cdot 2 - 13 \cdot 7 = 24 - 91 = \boxed{-67}$

Grouping can be indicated with:

{. . .}, [. . .], (. . .), or |. . .| .

Nested grouping occurs when a group appears inside another group. For example:

{ [. . .] . . .] , [. . .(. . .)] , etc.

For such expression, simplify **the innermost group** first and work your way out.

Example 5: Simplify –x[–x(y – b) + xb]

$$-x[-x(y-b)+xb]$$
$$=-x[-xy+xb+xb]$$
$$=-x[-xy+2xb] = \boxed{x^2y-2x^2b}$$

Do not distribute into an "absolute value" group.

If there is only a "+" in front of a parenthesis, simply drop the parenthesis pair (or any other grouping symbol pair except absolute value).

Example 6: Simplify –2x + (5x + 6) + 2|4 – 7|

$$-2x+(5x+6)+2|4-7|$$
$$=-2x+5x+6+2|-3|$$
$$=3x+6+2\cdot 3=3x+6+6=\boxed{3x+12}$$

See **Calculator Appendix A** (and an associated video) for how to nest groups on the graphing calculator.

Assignment:

1. Simplify $10 - (6x + 7)$

2. Simplify $-4(3z - 4) - (-10 + 5z)$

3. Simplify $2 - 8(5p - 3) - 9p$ and evaluate at $p = -1$.

4. Simplify $1 - 2(2 - 5x) - (3x - 14)$ and evaluate if $x = 2$.

5. After simplifying $-8y - (4y + 6) + 12y$, evaluate at $y = -1$.

6. Simplify $b[(-x - y) - (x - y)]$

7. Simplify $-5 - (-3) - \{ -[-6 + 1] \}$

8. Simplify $-2 - |-4 - 9| + (-4)(-4 - 2)$

9. Simplify $-7 - 2[(6x - 3)2 - (5x - 7)]$

10. Simplify { x – 3[2(x + 4) – 1] }

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11. Simplify –8z + (2z + 10) + 2|5 – 8|

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12. Simplify $\frac{3(-x+4)}{-(-x-4)}$

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13. Simplify $-2 - |-4 - 6| + (-5)(-1 - 3)$

14. Simplify $-(g + 4) + (9 - g)$ and then evaluate if g = 10.

15. Simplify $7x - 2(6x - 7) + 1$

16. Simplify $-5c - (8 - c) - 11$

17. Simplify -4x + (5x – 6) – 2|3 – 8|

Unit 1: Lesson 06 *Putting it all together with fractions

When **adding or subtracting** fractions, find a **common denominator**.

Example 1: Simplify $3\left(\frac{3x}{4} - \frac{x}{3}\right)$

$$3\left(\frac{3x}{4}\cdot\frac{3}{3} - \frac{x}{3}\cdot\frac{4}{4}\right) = 3\left(\frac{9x}{12} - \frac{4x}{12}\right) = 3\left(\frac{5x}{12}\right)$$

$$= \frac{15x}{12} = \boxed{\frac{5x}{4}}$$

When **multiplying** fractions, **multiply numerators** to produce the new numerator. **Multiply denominators** to produce the new denominator.

$$\frac{a}{b}\,\frac{c}{d} = \frac{ac}{bd}$$

Example 2: $-\frac{4}{5}\left(\frac{3}{8}x - \frac{5}{6}y\right)$

$$-\frac{4}{5}\left(\frac{3}{8}\,\frac{x}{1} - \frac{5}{6}\,\frac{y}{1}\right) = \frac{-12x}{40} + \frac{20y}{30} = \boxed{\frac{-3x}{10} + \frac{2y}{3}}$$

When **dividing** by a fraction, multiply the numerator by the **reciprocal** of that fraction.

Example 3: Simplify $\frac{3x/(5y)}{4a/(20b)}$

$$\frac{\frac{3x}{5y}}{\frac{4a}{20b}} = \frac{3x}{5y}\,\frac{20b}{4a} = \frac{60xb}{20ay} = \boxed{\frac{3xb}{ay}}$$

***Example 4:** Combine like terms in 4[(3/4)x + (2/5)x – 2] and evaluate at x = 3.

$$4\left[\frac{3}{4}\frac{x}{1}+\frac{2}{5}\frac{x}{1}-2\right]=4\left[\frac{3x}{4}+\frac{2x}{5}-2\right]$$

$$=4\left[\frac{3x}{4}\frac{5}{5}+\frac{2x}{5}\frac{4}{4}-2\right]$$

$$=4\left[\frac{15x}{20}+\frac{8x}{20}-2\right]=\frac{4}{1}\left[\frac{23x}{20}-2\right]$$

$$=\frac{92x}{20}-8=\frac{23x}{5}-8=\frac{23\cdot 3}{5}-\frac{8}{1}\frac{5}{5}=\frac{69-40}{5}=\boxed{\frac{29}{5}}$$

Example 5: Simplify (11x – (5/4)x)/(2/3)

$$\left(\frac{11x}{1}-\frac{5}{4}\frac{x}{1}\right)\frac{3}{2}=\left(\frac{11x}{1}\frac{4}{4}-\frac{5x}{4}\right)\frac{3}{2}$$

$$=\left(\frac{44x}{4}-\frac{5x}{4}\right)\frac{3}{2}=\frac{39x}{4}\frac{3}{2}=\boxed{\frac{117x}{8}}$$

See **Calculator Appendix B** (and an associated video) for how to handle the grouping of numerators and denominators on a graphing calculator. Common pitfalls are discussed.

Assignment:

1. Simplify $\frac{7}{8} + \frac{2}{3}$

2. Simplify $\frac{2}{7}\,\frac{3}{4} \div \frac{2}{3}$

3. Simplify $-\frac{5}{3}\left(\frac{1}{7}\mathrm{m} - \frac{2}{3}\mathrm{n}\right)$

4. Simplify $\left(\frac{2\mathrm{x}}{5} - \frac{\mathrm{x}}{4}\right)$

5. Simplify $-\left(\frac{2\mathrm{x}}{5} - \frac{\mathrm{x}}{3}\right) + 4\mathrm{x}$

6. Combine like terms in 5[(3/4)y + (5/3)y – 1] and evaluate at y = –3.

7. Simplify (11q – (7/3)q)/(-8)

8. Simplify $\frac{3x}{7} - \frac{1}{5} + \frac{2x}{3}$ and evaluate when x = –1.

*9. Simplify (2/3) { –[1/5 – 1/2] + 2|1/3 + 2| }

---

*10. Combine like terms in $\frac{-4}{5x} - \frac{3}{2x}$ + 1 and then evaluate at x = 2.

Unit 01: Review Calculators are not permitted on this review.

1. Simplify $6 \cdot 3/(11-2)$

2. Simplify $2(48 \div 4) + 2(5-2) - 1$

3. Locate the opposite of –7 on a number line.

4. Simplify |4 – 6 + 1|

5. Locate |–5| on a number line.

6. Simplify 18(–2)

7. Simplify –5(–6)

8. Simplify 24/(–8)

9. Simplify –50/(–10)

10. Simplify –12 + 5

11. Simplify –79 – 2

12. Using the fact that 1 inch = 2.54 centimeters, use a unit multiplier to convert 8 inches into centimeters.

13. Using the fact that 2 nerds = 32 twerps, use a unit multiplier to convert 10 nerds to twerps.

14. Using the fact that 1 centimeter = 10 millimeters, use a unit multiplier to convert 82 millimeters to centimeters.

15. Simplify $3x - 7y + 2x - 2y$ by combining like terms and then evaluate at x = 7 and y = – 6.

16. Simplify 11x – 6 – 23x + 1

17. Evaluate |4b – 3c – 9| if b =3 and c = 2.

18. Simplify 3/4 – 1/6 + 2

19. Simplify $\left(\frac{1}{5}\mathrm{x} - \frac{7}{4}\mathrm{x}\right) \div \frac{1}{2}$

20. Simplify 1 – 6(2x – 3) – 2(2 – x) and then evaluate at x = –5.

Alg 1, Unit 2

Solving Linear Equations

Unit 2: Lesson 01 Solving one-step linear equations

The **solution** to an equation is the value of the variable that makes the equation true.

To **prove** that a number is the solution to an equation, substitute the number into the equation for each occurrence of the variable and show that the new equation is true (both sides equal each other).

Example 1: Show that x = 5 is a solution to 3x – 1 = 2x + 4

$$3x - 1 = 2x + 4$$
$$3\cdot 5 - 1 = 2\cdot 5 + 4$$
$$15 - 1 = 10 + 4$$
$$14 \overset{\checkmark}{=} 14$$

Solving an equation means, "getting *x* by itself."

To do this it is sometimes necessary to add a number (either negative or positive) to both sides of an equation. The result is a new equation that is still true.

Example 2: Solve x + 5 = 2

$$x + \cancel{5} = 2$$
$$\underline{-\cancel{5}} \quad \underline{-5}$$
$$x = \boxed{-3}$$

Example 3: Find the solution to x – 4 = 1.

$$x - \cancel{4} = 1$$
$$\underline{\cancel{4}} \quad \underline{4}$$
$$x = \boxed{5}$$

When finding the solution to some equations, it is necessary to multiply (or divide) both sides by a number in order to "get *x* by itself."

In the following examples, multiply both sides by the **reciprocal of the coefficient** of *x* to "get *x* by itself."

Recall that **a number times its reciprocal is 1**.

Example 4: Solve 18 = 6x

$$18 = 6x$$
$$\frac{1}{6}\cdot\frac{18}{1} = \left(\cancel{\frac{1}{6}}\right)\cancel{6}x$$
$$\frac{18}{6} = x$$
$$\boxed{3} = x$$

Example 5: Find the solution to $-\frac{3}{4}$x = 15.

$$-\frac{3}{4}x = 15$$
$$\cancel{-\frac{3}{4}}\left(\cancel{-\frac{4}{3}}\right)x = \frac{15}{1}\left(-\frac{4}{3}\right)$$
$$x = \frac{-60}{3}$$
$$x = \boxed{-20}$$

Assignment:

1. Show that $x = 3$ is a solution of $x + 2x + 1 = 10$.

2. Prove that $y = -1$ is a solution of $-6y - 14 = -8$.

In the following problems, solve each equation.

3. $m + 12 = -3$

4. $-6 + x = 8$

5. $x - 11 = -18$

6. $-36 = -17 + p$

7. $j + (-1) = 11$

8. $x + 15 = 8$

9. $-3x = 21$

10. $6x = -24$

11. $-115 = -5k$

12. $36.3 = 12.1z$

13. $(2/5)x = 20$

14. $(-1/3)p = -9$

15. $10h = -2/7$

16. $(9/10)x = -99$

*17. $12 + 2 = 2x$

*18. $4t = 143 + 1$

19. $5 = 2.5 + x$

20. $0 = y + (-3.1)$

*21. Show that $h = -6$ is a solution of $(-1/2)h + 4 - h = 13$.

Unit 2: Lesson 02 Solving two-step linear equations

In the process of solving some equations it is necessary to **both** add a number to both sides **and** then divide (or multiply) both sides by another number in order to "get *x* by itself."

Example 1: Solve 5x – 7 = 3

$$5x\cancel{-7}=3$$
$$\underline{\cancel{7}}\quad\underline{7}$$
$$5x=10$$
$$\frac{\cancel{5}x}{\cancel{5}}=\frac{10}{5}$$
$$x=\boxed{2}$$

Example 2: Solve y/2 + 12 = 30

$$\frac{y}{2}+\cancel{12}=30$$
$$\underline{\cancel{-12}}\quad\underline{-12}$$
$$\frac{y}{2}=18$$
$$\frac{y}{\cancel{2}}\cdot\frac{\cancel{2}}{1}=18\cdot 2$$
$$y=\boxed{36}$$

Example 3: Solve 8 = –3m – 10

$$8=-3m\cancel{-10}$$
$$\underline{10}\qquad\underline{\cancel{10}}$$
$$18=-3m$$
$$\frac{18}{1}\left(-\frac{1}{3}\right)=\cancel{-\frac{3}{1}\left(-\frac{1}{3}\right)}m$$
$$-\frac{18}{3}=m$$
$$\boxed{-6}=m$$

Example 4: Solve 6.4z – 13.2 = 38

$$6.4z\cancel{-13.2}=38$$
$$\underline{\cancel{13.2}}\quad\underline{13.2}$$
$$6.4z=51.2$$
$$\frac{\cancel{6.4}z}{\cancel{6.4}}=\frac{51.2}{6.4}$$
$$z=\boxed{8}$$

Example 5: Solve $4 = -11 + \frac{p}{-5}$

$$4 = \cancel{-11} + \frac{p}{-5}$$
$$\underline{11} \qquad \underline{\cancel{+11}}$$
$$15 = \frac{p}{-5}$$
$$15(-5) = \frac{p}{\cancel{-5}}\left(\frac{\cancel{-5}}{1}\right)$$
$$\boxed{-75} = p$$

Example 6: Solve $20 - (1/7)c = -9$

$$\cancel{20} - \frac{1}{7}c = -9$$
$$\underline{\cancel{-20}} \qquad \underline{-20}$$
$$-\frac{1}{7}c = -29$$
$$\cancel{-}\frac{1}{\cancel{7}}\frac{\cancel{7}}{1}c = \frac{-29}{1}\,\frac{-7}{1}$$
$$c = \boxed{203}$$

Assignment: Solve for the indicated variable in the following problems.

1. $11x + 2 = 35$

2. $8 = 2b - 22$

3. $100 = 5x - 35$

4. $11 - 6v = -49$

5. $6 - 4h = -22$

6. $-32 = 4t - 16$

7. $.5x - 6 = -1$

8. $2.7d + 11.6 = 19.7$

9. $(4/5)n - 7 = -4$

10. $\frac{x}{-6} + 4 = 12$

11. $10 = \frac{v}{-2} - 4$

12. $2/3 = (-1/6)g - 1/3$

13. $(4/5)x - 9 = 8$

14. $-20 = 10 + (2/3)h$

15. $-19 = 11 - \frac{1}{6}x$

16. $7 - (3/8)x = -1$

17. $8p - 11 = 5$

18. $4/5 = 9 - (1/2)x$

Unit 2: Lesson 03

Solving linear equations by combining like terms
Solving multi-step linear equations

If an equation has several terms of the same type, **combine** those terms before proceeding to solve the equation.

Example 1: Solve x – 5 + 4x = 10

$$x - 5 + 4x = 10$$
$$5x \cancel{-5} = 10$$
$$\underline{\cancel{+5}} \qquad \underline{+5}$$
$$5x = 15$$
$$\frac{\cancel{5}x}{\cancel{5}} = \frac{15}{5}$$
$$x = \boxed{3}$$

Example 2: Solve –x + 8 – 9x = 11

$$-x + 8 - 9x = 11$$
$$-10x \cancel{+8} = 11$$
$$\underline{\cancel{-8}} \qquad \underline{-8}$$
$$-10x = 3$$
$$\frac{\cancel{-10}x}{\cancel{-10}} = \frac{3}{-10}$$
$$x = \boxed{-\frac{3}{10}}$$

Example 3: Find the solution to this equation: 14p – 9 + 6p + 1 = 32

$$14p - 9 + 6p + 1 = 32$$
$$20p \cancel{-8} = 32$$
$$\underline{\cancel{+8}} \qquad \underline{+8}$$
$$20p = 40$$
$$\frac{\cancel{20}p}{\cancel{20}} = \frac{40}{20}$$
$$p = \boxed{2}$$

The solution of some equations requires multiple steps.

If an equation has a multiplier in front of a parentheses (or any other similar group), distribute the multiplier.

Example 4: Solve 3(y – 4) + 12 = – 6

$3(y-4)+12=-6$

$3y\cancel{-12}\cancel{+12}=-6$

$3y=-6$

$\frac{\cancel{3}y}{\cancel{3}}=\frac{-6}{3}$ $\quad y=\boxed{-2}$

Example 5: Solve –9 –3(4t –1) = 30

$-9-3(4t-1)=30$

$-9-12t+3=30$

$\cancel{-6}-12t=30$

$\underline{\cancel{6}}\qquad\quad \underline{6}$

$-12t=36$

$\frac{\cancel{-12}t}{\cancel{-12}}=\frac{36}{-12}$

$t=\boxed{-3}$

***Example 6:** Find the solution to this equation: 5(k – 2) + 2[k – 3(k + 2)] = 0

$5(k-2)+2[k-3(k+2)]=0$

$5k-10+2[k-3k-6]=0$

$5k-10+2[-2k-6]=0$

$5k-10-4k-12=0$

$k\cancel{-22}=0$

$\underline{\cancel{22}}\qquad \underline{22}$ $\longrightarrow$ $k=\boxed{22}$

Assignment: Solve the following equations.

1. $6x + 2x = -48$

2. $-11z + 9 - 4z = 2$

3. $3(x - 5) = 30$

4. $14 = 7r - 4 + 2r$

5. $2(v + 10) - 6 = 2$

6. $11 = 7(f - 3) + 21$

7. $b + 9(b + 4) = -3$

8. $-22 + 2(4n + 10) = 10$

9. $-8 = 7[\, w - (-1)\,]$

10. $(6 - t) + (7 - t) - (4 - t) = 0$

*11. $2a + 3[\,4(2 - a) - 6(1 + a)\,] = 5$

*12. $(x + 4) - x - (5 - 6x) = 1$

Unit 2: Lesson 04 Solving linear equations with variables on both sides

To solve an equation with **variables on both sides**, **eliminate** the variable on one side, thus collecting all of the variables on the other side.

It is not a requirement, but is suggested that variables be collected on the **left side** of the equation.

Example 1: Solve 4x – 6 = 7x

$$
\begin{aligned}
4x - 6 &= \cancel{7x} \\
-7x \quad & \quad \cancel{-7x} \\
-3x \cancel{-6} &= 0 \\
\cancel{6} \quad & \quad 6 \\
-3x &= 6
\end{aligned}
$$

$$\frac{\cancel{-3}x}{\cancel{-3}} = \frac{6}{-3}$$

$$x = \boxed{-2}$$

At this point we want to increase our level of sophistication in how to add (or subtract) numbers or terms from each side of an equation.

In the following example (which is the same problem in Example 1), notice how we still add –7x and +6 to both side, but in a new way.

Example 2: Solve 4x – 6 = 7x

$$
\begin{aligned}
4x - 6 &= 7x \\
4x - 6 - 7x &= \cancel{7x - 7x} \\
-3x - 6 &= 0 \\
-3x \cancel{-6 + 6} &= +6 \\
-3x &= 6
\end{aligned}
$$

$$\frac{\cancel{-3}x}{\cancel{-3}} = \frac{6}{-3}$$

$$x = \boxed{-2}$$

Example 3: Solve 4(2 + x) – 5x = x + 12

$$4(2+x)-5x=x+12$$
$$8+\underbrace{4x-5x}=x+12$$
$$8-x=x+12$$
$$8-\underbrace{x-x}=\underbrace{\cancel{x}+12\cancel{-x}}$$
$$8-2x=12$$
$$\underbrace{\cancel{8}-2x\cancel{-8}}=12-8$$
$$-2x=4$$
$$\frac{\cancel{-2}x}{\cancel{-2}}=\frac{4}{-2}$$
$$x=\boxed{-2}$$

Example 4: Solve 2(y – 3) + 4 = 6(7 – y)

$$2(y-3)+4=6(7-y)$$
$$2y-\underbrace{6+4}=42-6y$$
$$\underbrace{2y-2+6y}=42\cancel{-6y}\cancel{+6y}$$
$$8y-2=42$$
$$8y\cancel{-2}\cancel{+2}=42+2$$
$$8y=44$$
$$\frac{\cancel{8}y}{\cancel{8}}=\frac{44}{8}$$
$$y=\frac{44}{8}=\boxed{\frac{11}{2}}$$

Example 5: Solve 2(f –3) = 2(f – 2) – 5

$$2(f-3) = 2(f-2) - 5$$
$$2f - 6 = 2f - \underbrace{4 - 5}$$
$$\underbrace{\cancel{2f} - 6 - \cancel{2f}} = \underbrace{\cancel{2f} - 9 - \cancel{2f}}$$
$$-6 \neq -9 \quad \text{No solution}$$

Sometimes (as in the example above) a statement is produced that is not true. This means there is **no solution** to the equation.

Example 6: Solve 4(z + 5) – 8 = 4(z + 3)

$$4(z+5) - 8 = 4(z+3)$$
$$4z + \underbrace{20 - 8} = 4z + 12$$
$$\underbrace{\cancel{4z} + 12 - \cancel{4z}} = \underbrace{\cancel{4z} + 12 - \cancel{4z}}$$
$$12 = 12 \quad \text{All real } x$$
$$\text{ARX}$$

Sometimes (as in the example above) a statement is produced that is true; however, the variables are no longer present (they all canceled out). This means there are an **infinite number of solutions** (all real numbers).

Assignment: Solve the following equations.

1. $3(x + 6) = 5(x + 2)$

2. $v + 8 = 5v + 5(1 - v)$

3. $6x - 2 + x = 7x - 13$

4. $-11d - 2d - 1 = 27 + d$

5. $-3(p + 5) + 6 = 3(-p - 3)$

6. $-4m - 9 + 5m = 51 - 5m$

7. $w - 4(w + 2) = 7 - 2w$

*8. $(3/2)x + 1/2 = (7/3)x + 4$

9. $3q - (2 - q) = 2(2q - 1)$

10. $5r + 22r - 7 = 2 - r$

11. $8(j + 2) + 9(-j - 1) = -j + 2$

*12. $2[\, x - 3(-x - 5) + 1\,] = 2(\, x + 11) - (-4 + x)$

Unit 2: Review Solve the following equations for the indicated variables.

1. $m + 11 = -4$	2. $-24 = -18 + p$
3. $5x - 8 = 2$	4. $10 = -3m - 14$
5. $2x - 5 + 4x = 31$	6. $3(y - 2) + 12 = -6$

7. $11(k - 2) + 2[\ k - 3(k + 1)\ ] = 0$

8. $(x + 8) - x - (5 - 6x) = 15$

9. $-3(p + 1) + 2 = 3(-p - 3)$

10. $10(f - 3) = 2(f - 20) - 50$

11. $5(x + 6) = 5(x + 2)$

12. $-4m + 10 + 5m = 14 - 4m$

13. $3(-x - 3) = -3(x + 5) + 6$

*14. $(1/2)x + 1 - x = (3/5)(x - 7/2) - 1$

Alg 1, Unit 3

Inequality basics
Solving linear, single-variable inequalities

Unit 3: Lesson 01 Inequality statements

Read the symbol, > , “greater than.”

Read the symbol, < , “less than.”

If *a* lies to the left of *b* on a number line, then we can make the statement, **a < b.** (**Read this, “*a* is less than *b*.**”)

If *x* lies to the right of *y* on a number line, then we can make the statement, **x > y.** (Read this, “*x* is **greater than** *y*.”)

Just remember, “The alligator eats the big one.”

Example 1: Express “x + 3 is greater than 2y” in symbols.

$x + 3 > 2y$

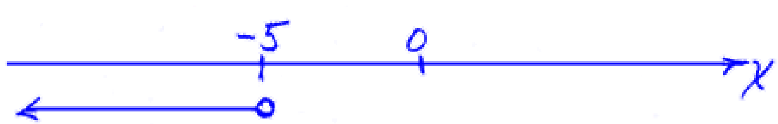
Example 2: The number represented by *m* lies to the right of the number represented by *n* on a number line. Express the inequality relationship between *m* and *n* using “<”.

$n < m$

When graphing x > a or x < a on a number line just remember that the “inequality arrow” is in the **same direction** as the “graph arrow” (only true when the **variable is on the left side)**.

Graph with an **open circle** as illustrated in the example below.

Example 3: Sketch the graph of x < –5 on a number line.

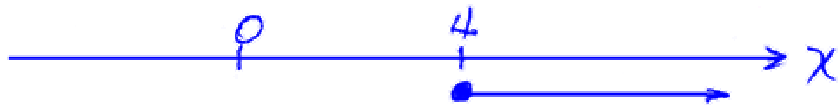


Read the symbol, ≥ , "greater than or equal to."

Read the symbol, ≤ , "less than or equal to."

Graph with a **solid circle** as illustrated in the example below.

Example 4: Sketch the graph of x ≥ 4 on a number line.



Adding (or subtracting) a number to both sides of an inequality:

Suppose *a* and *b* are related by the inequality, then

$a > b$

When the quantity *c* is added to both sides, the result is

$a + c > b + c$

Multiplying (or dividing) a number times both sides of an inequality:

Suppose *a* and *b* are related by the inequality, then

$a > b$

When the quantity *c* is multiplied by both sides, the result is

$a(c) > b(c)$ if *c* is a **positive** number.

$a(c) < b(c)$ if *c* is a **negative** number (Note the **reversal** of the inequality symbol.)

Example 5: Rewrite the inequality f < g after subtracting 3 from both sides.

$f - 3 < g - 3$

Example 6: Rewrite the inequality m ≥ n after multiplying 4 times both sides.

$4m \geq 4n$

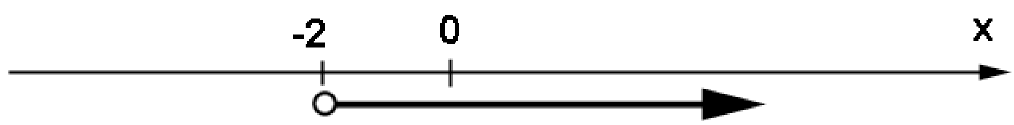
Example 7: Rewrite the inequality x ≥ y after dividing both sides by – 6.

$x/(-6) \leq y/(-6)$

Example 8: Rewrite the inequality p ≤ q after adding 2 to both sides.

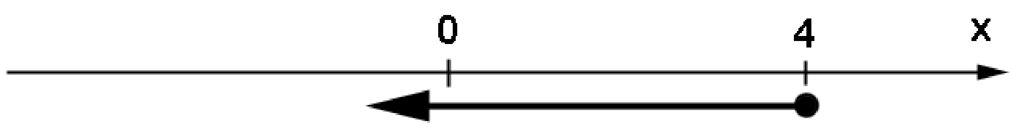
$p + 2 \leq q + 2$

Example 9: Write the inequality that describes this graph.



$x > -2$

Example 10. Write the inequality that describes this graph.



$x \leq 4$

Example 11: Which of the following set of *x* values is a solution to the inequality of Example 10? {–6, – 5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6}

$\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Assignment:

1. Express "*f* is less than or equal to *m*" in mathematical symbols.

2. Express "*z* is greater than *v*" in mathematical symbols.

3. Express $x \leq k$ in words.

4. Express $w > z$ in words.

5. Rewrite the inequality $f < g$ after subtracting 3 from both sides.

6. Rewrite the inequality $x \geq y$ after dividing both sides by – 6.

7. Rewrite the inequality $m \geq n$ after multiplying 4 times both sides.

8. Rewrite the inequality $p \leq q$ after adding 2 to both sides.

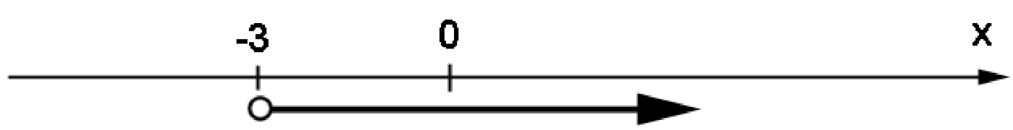
9. Sketch the graph of $x < 6$ on a number line.

10. Sketch the graph of $x \geq -7$ on a number line.

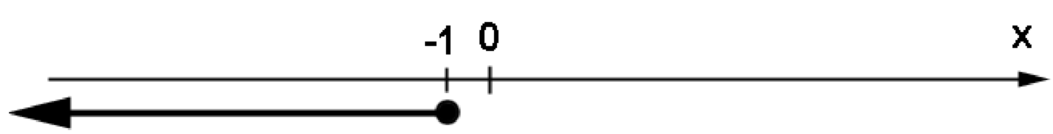
11. Sketch the graph of $x \geq 2.5$ on a number line.

12. Sketch the graph of $x < -3$ on a number line.

13. Write the inequality that describes this graph.



14. Write the inequality that describes this graph.



15. Which of the following set of *x* values is a solution to the inequality of problem 13?

{– 5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5}

16. Which of the following set of *x* values is a solution to the inequality of problem 14?

{– 5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5}

17. Express “p could be equal to 5; however, it could also be less than 5” in mathematical symbols.

18. Rewrite $2g \geq 11f$ after dividing both sides by -13.

19. Express $m \leq n$ in words.

20. Sketch the graph of $x < 0$ on a number line.

Unit 3:
Lesson 02

Inequality phrases
Solving linear inequalities

The phrases

"at least",
"no more than",
"don't exceed",
"in excess of",
or their equivalents

in a statement all lead to **inequality** statements.

Example 1: Write the inequality expressed by the statement, "This year's profit is **at least** last year's profit."

$typ \geq lyp$

Example 2: Write the inequality expressed by the statement, "Make sure the expenses are **no more than** \$100."

$e \leq 100$

Example 3: Write the inequality expressed by the statement, "My speed did not exceed 70 mph."

$s \leq 70$

Example 4: Write the inequality expressed by the statement, "My speed was in excess of 50 mph."

$s > 50$

Solving an inequality involves exactly the same steps as when solving an equation with the following **exception**:

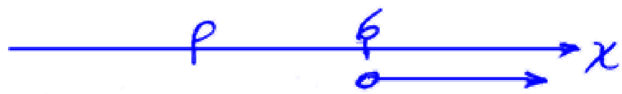
If both sides of the inequality are multiplied (or divided) by a **negative** number, the **inequality symbol must be reversed.**

Example 5: Determine the inequality solution to x – 4 > 2. Express the answer both symbolically and as a graph on a number line.

$$x - 4 > 2$$

$$x \cancel{-4} \cancel{+4} > 2 + 4$$

$$\boxed{x > 6}$$



Example 6: Determine the inequality solution to –3x + 2 ≤ – 7. Express the answer both symbolically and as a graph on a number line.

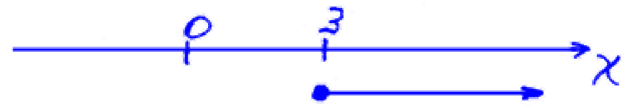
$$-3x + 2 \le -7$$

$$-3x \cancel{+2} \cancel{-2} \le -7 - 2$$

$$-3x \le -9$$

$$\frac{\cancel{-3}x}{\cancel{-3}} \ge \frac{-9}{-3} \quad \text{reversed}$$

$$\boxed{x \ge 3}$$



Example 7: Determine the inequality solution to 3x – 5 > x + 6. Express the answer both symbolically and as a graph on a number line.

$$3x - 5 > x + 6$$

$$3x \cancel{-5} \cancel{+5} > x + 6 + 5$$

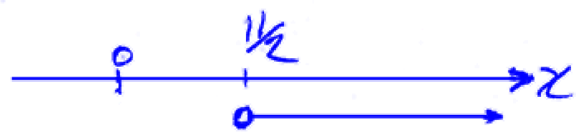
$$3x > x + 11$$

$$3x - x > \cancel{x} + 11 \cancel{-x}$$

$$2x > 11$$

$$\frac{\cancel{2}x}{\cancel{2}} > \frac{11}{2}$$

$$\boxed{x > \frac{11}{2}}$$



Example 8: Determine the inequality solution to 3(x + 2) > 7x – 10. Express the answer both symbolically and as a graph on a number line.

$$3(x+2) > 7x-10$$

$$3x+6 > 7x-10$$

$$3x+\cancel{6}\cancel{-6} > 7x-10-6$$

$$3x > 7x-16$$

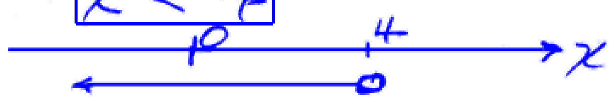
$$3x-7x > \cancel{7x}-16\cancel{-7x}$$

$$-4x > -16$$

reversed

$$\frac{\cancel{-4}x}{\cancel{-4}} < \frac{-16}{-4}$$

$$\boxed{x < 4}$$



Assignment:

1. Write the inequality expressed by the statement, “Unfortunately, Richard’s grade **did not exceed** 70.”	2. Write the inequality expressed by the statement, “The government says the work-day should be **no more than** 8 hours.”
3. Write the inequality expressed by the statement, “When I graduate, I want to make **at least** \$50,000 per year.”	4. Write the inequality expressed by the statement, “The number of calories in that meal was definitely **in excess of** 2000.”
5. Write the inequality expressed by the statement, “The probability of me passing Algebra is **not less than** 80%.”	6. Write the inequality expressed by the statement, “The score made by the Eagles will likely **not exceed** 10 more than the Bobcat’s score.”

Determine the inequality solution to the following problems. Express the answer both symbolically and as a graph on a number line.

7. $x + 17 \leq 4$	8. $4 - x < 11$

9. $7x + 2 > x - 9$

---

10. $-3 \leq x + 7 + 4x$

---

11. $4(x + 12) + 1 < x + 8$

12. $4x - 6 > 7x$

13. $4(2 + x) - 5x < x + 12$

*14. $6x - 2 + x \geq 7x - 13$

Unit 3:
Cumulative Review

1. Use three successive unit multipliers to convert from 156,000 centimeters to miles. First convert from centimeters to inches, then from inches to feet, and finally from feet to miles. (1 in = 2.54 cm, 12 in = 1 ft, 5280 ft = 1 mi)

2. Simplify this expression: $1 + 8 \cdot 6 \div 2 - 10$	3. Simplify $\dfrac{10 \cdot 2 + 1 \cdot 12}{1 + 2 \cdot 3 - 3}$
4. Simplify $\vert 8 - 6 - 5 \vert$	5. What is the opposite of 10? What is the absolute value of the opposite of 10?
6. Simplify $-3(-4)(-2) + 1$	7. Simplify $-200/(-4)/(-25)$

8. Evaluate $|3x - 2y + 1|$ if $x = 2$ and $y = -6$.

9. Simplify $3x + 2 - 11x + 9$ and then evaluate when $x = -1$.

10. Simplify $(1/2)x - (1/5)x + (2/7)y - (3/8)y$

11. Solve $8 = -6k - 4 + 2k$

12. Solve $-9 + 6x + 1 + 14x = 32$

13. Solve $6(y + 4) - 4 = y - 9(3y + 2)$

14. Simplify $1/2 + 1/3 - 1/4 + 2$

Alg 1, Unit 4

Word problems (area, perimeter, percent)

Solving abstract equations

Unit 4: Lesson 01

Word expressions and statements
Solving simple word problems

When encountering an algebraic word expression:

- First, define the variable.
- Write the expression using the symbols of algebra.

In the following examples, define the variable, and then write the expression algebraically.

Example 1: "three less than a number"

n = the number

$n - 3$

Example 2: "10 more than the cost"

c = cost

$c + 10$

Example 3: "the difference between the population of the city and 22"

p = city population

$p - 22$

Example 4: "8 decreased by three times the score"

s = score

$8 - 3s$

Example 5: "ten times the sum of the weight of the man and three"

w = man's weight

$10(w + 3)$

Example 6: "the sum of ten times the weight of the man and three"

w = man's weight

$10w + 3$

Example 7: "four times the difference of the volume and two"

v = volume

$4(v - 2)$

Example 8: "five less than nine times the river's width"

w = river's width

$9w - 5$

All of the examples above were algebraic **expressions**.

An algebraic **statement** is when an algebraic expression is set equal to another expression or number. The result is an equation than can be solved. (Typically, some form of the word "**is**" is replaced with "**=**".)

In examples 9 and 10, define the variable, set up the equation, and then solve for the variable.

Example 9: If sixteen is subtracted from the weight, the result is twelve.

w = weight

$w - 16 = 12$

$w \cancel{-16} \cancel{+16} = 12 + 16$

$w = \boxed{28}$

Example 10: Four less than one-half of a number is six.

n = the number

$\frac{1}{2}n - 4 = 6$

$\frac{1}{2}n \cancel{-4} \cancel{+4} = 6 + 4$

$\frac{1}{2}n = 10$

$\cancel{2} \cdot \frac{1}{2}n = 10 \cdot 2$

$n = \boxed{20}$

Assignment: In problems 1 – 8, define the variable(s), and then write the expression algebraically.

1. "six more than the number"	2. "twenty-nine less than his age"
3. "eighteen decreased by 4 times the weight of the package"	4. "two times the difference of the width of the box and 8"
5. "the difference between the depth of the pool and 11"	6. "nine less than five times the height of the water tower"
*7. "eleven times the sum of two different numbers"	8. "six times the difference of a number and 3"

In the following problems, define the variable, set up the equation, and then solve for the variable.

9. If eight is added to the diameter of the asteroid, the result is eleven.

---

10. Four times the sum of a number and -8 gives a value of sixteen.

---

11. When six is decreased by four times the number of calories, the result is negative eighteen.

*12. Two more than one-fifth the altitude is the same as the altitude.

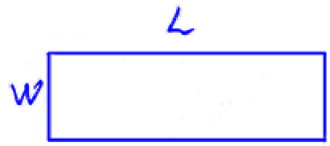
13. Twice the number of pears increased by three times the number of pears is exactly 125.

14. If the teacher would just give me 8 more points, I would be passing with a 70.

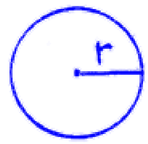
\

Unit 4: Lesson 02 Solving perimeter and area word problems

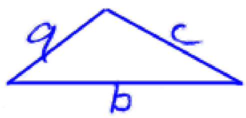
Perimeter is the total distance around a closed figure.

Perimeter of a rectangle: 

$P = 2W + 2L$

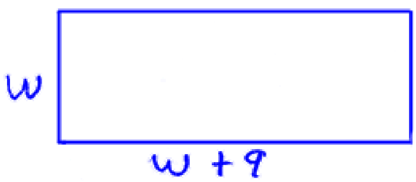
Perimeter (circumference) of a circle: 

$C = 2\pi r$

Perimeter of a triangle: 

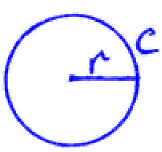
$P = a + b + c$

Example 1: The length of a rectangle is 9 ft more than its width. Find the length if the perimeter of the rectangle is 54.



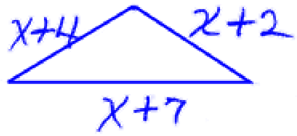
$$2w + 2(w+9) = 54$$
$$2w + 2w + 18 = 54$$
$$4w + \cancel{18} - \cancel{18} = 54 - 18$$
$$4w = 36$$
$$\frac{\cancel{4}w}{\cancel{4}} = \frac{36}{4} \quad ; \quad w = 9$$
$$L = w + 9 = 9 + 9 = \boxed{18}$$

Example 2: The circumference of a circle is 10 inches more than its radius. What is its radius?



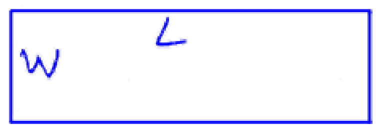
$$2\pi r = r + 10$$
$$2\pi r - r = \cancel{r} + 10 - \cancel{r}$$
$$2(3.14)r - r = 10$$
$$6.28r - 1r = 10$$
$$5.28r = 10$$
$$\frac{\cancel{5.28}r}{\cancel{5.28}} = \frac{10}{5.28} \quad ; \quad r = \boxed{1.89\overline{39}\text{ in}}$$

Example 3: The three sides of a triangle are x + 2, x + 4, and x + 7. If the perimeter of the triangle is 49, what is *x*?



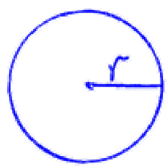
$$x+7+x+4+x+2=49$$
$$3x+13=49$$
$$3x+\cancel{13}-\cancel{13}=49-13$$
$$3x=36$$
$$\frac{\cancel{3}x}{\cancel{3}}=\frac{36}{3}$$
$$x=\boxed{12}$$

Area of a rectangle:



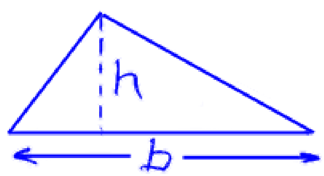
$A=L\cdot W$

Area of a circle:



$A=\pi r^2$

Area of a triangle:



$A=\frac{1}{2}b\cdot h$

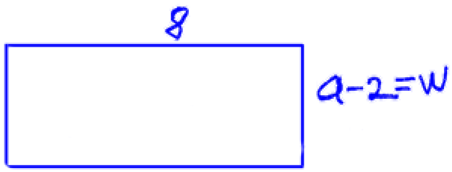
Example 4: Find the radius of a circle if its area is 32π.

$$\pi r^2=32\pi$$
$$\frac{\cancel{\pi}r^2}{\cancel{\pi}}=\frac{32\cancel{\pi}}{\cancel{\pi}}$$
$$r^2=32$$

To find, take $\sqrt{\ }$

$$r=\sqrt{32}=\boxed{5.656854}$$

***Example 5:** A rectangle's length is 8 and its width is 2 less than its area. What is its width?



$L \cdot w = area$ $\quad a = area$

$8(a-2) = a$

$8a - 16 = a$

$8a - \cancel{16} + \cancel{16} = a + 16$

$8a - a = \cancel{a} + 16 - \cancel{a}$

$7a = 16$

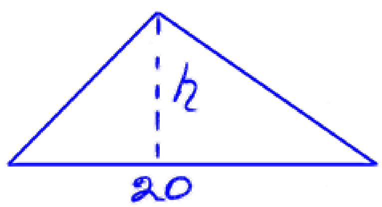
$\frac{\cancel{7}a}{\cancel{7}} = \frac{16}{7}; \; a = \frac{16}{7}$

$w = a - 2$

$w = \frac{16}{7} - \frac{2}{1} \cdot \frac{7}{7}$

$w = \boxed{\frac{2}{7}}$

Example 6: What is the height of a triangle if its base is 20 and its area is 100?



$\frac{1}{2} \, b \cdot h = area$

$\frac{1}{2} \, 20 h = 100$

$10h = 100$

$\frac{\cancel{10}h}{\cancel{10}} = \frac{100}{10}$

$h = \boxed{10}$

Assignment:

1. The width of a certain rectangle is 2 less than its length. If its perimeter is 96, what is its length?

2. The circumference of a circle is 2 more than three times its radius. What is the radius?

3. The length of one side of a triangle is x. Another side is two more than x while the third side is one less than x. If the perimeter is 13, what is x?

4. A rectangle's width is 4 less than its area while its length is 19. What is its width?

5. What is the base of a triangle whose height is 10 and whose area is 160?

6. If a circle's area is 180π, what is its radius?

7. If one side of a rectangle is $4x + 1$ and an adjacent side is $3x + 4$. What is the value of x if the perimeter of the rectangle is 38?

---

8. If all three sides of a triangle are equal (it's an equilateral triangle) and the perimeter of the triangle is 180, what is the length of each side?

---

*9. The perimeter of a particular circle is equal to its area. Write the equation that expresses this equality and then solve for r.

Unit 4: Lesson 03 Percent problems

To convert a % into a decimal fraction, move the decimal point two places to the left.

To convert a decimal fraction to a percent, move the decimal point two places to the right.

Example 1: Convert 38.2% to a decimal fraction.

38.2% = .382

Example 2: Convert .453 to a percent.

.453 = 45.3%

Percent word problems can many times be expressed as:

p percent of (quantity #1) is (quantity # 2)

In this context, "of" means to multiply, so the statement can be written mathematically as follows:

p (q1) = q2 where *p* is a decimal fraction representation of the percent.

Example 3: An example of ***p*** being the unknown variable in p(q1) = q2 is, "25 is **what** percent of 78?"

Reword this in reverse as, "p percent of 78 is 25."

$$p \cdot 78 = 25$$

$$\frac{\cancel{78}p}{\cancel{78}} = \frac{25}{78}$$

$$p = .3205 \rightarrow \boxed{32.05\%}$$

Example 4: An example of ***q1*** being the unknown variable in p(q1) = q2 is, "38% of **what** is 175?"

$$38\% = .38$$

$$p \cdot q1 = q2$$

$$.38(q1) = 175$$

$$\frac{\cancel{.38}q1}{\cancel{.38}} = \frac{175}{.38}$$

$$q1 = \boxed{460.526}$$

Example 5: An example of ***q2*** being the unknown variable in p(q1) = q2 is, "4.11% of 160 is **what**?"

$$4.11\% = .0411$$

$$p \cdot q1 = q2$$

$$.0411(160) = q2$$

$$\boxed{6.576} = q2$$

Assignment:

1. Convert 14% to a decimal fraction.

2. Convert .189 to a percent.

3. Convert 5.22 to a percent.

4. Convert .03% to a decimal fraction.

5. Convert 1500% to a decimal fraction.

6. Convert 1500 to a percent.

7. 18 is what percent of 36?

8. What percent of 6 is 18?

9. 19 percent of what is 23.1?

10. What is 58.2 percent of 75?

11. .029 % of 600 is what?

12. .052 percent of what is .001?

13. Big Bob wants to buy a car that sells for $21,000. If the sales tax is 8.25%, what will be the total cost of the car including tax?

14. Mooky bought a new laptop computer and is paying $95 per month for it. If his monthly income is $852, what percent of his monthly income is he paying each month for the computer?

15. Mr. Cleaver gets a 6% commission as a real estate agent for the selling price of a house. If his commission on a sale is $8280, what was the selling price of the house?

*16. Lisa bought a new dress on sale at "Teen Hottie" and paid only $75. If it had previously been listed at $125, what percent discount did Lisa receive on the dress?

17. Jimmy was surprised when the waitress presented a bill of $7.02 for his lunch because his plate was listed for $6.50 on the menu. If the additional cost was due to the tax, what was the sales tax rate?

18. The Eagles only completed 25% of their passes this past season. If they completed 20 passes, how many passes were attempted?

19. What is 13.2% of 187.02?

20. 18.09 is what percent of 11?

Unit 4:
Lesson 04

*More area, perimeter, and percent problems

Assignment:

1. A certain triangle has sides z + 1, z + 2, and z + 8. What is z if the perimeter is 44?

2. If the area of a semicircle is 46π, what is its radius?

3. 146 is what percent of 192?

4. What is 1.1% of .016?

5. The sides of a rectangle are all exactly equal and its area is 144. How long are the sides?

---

6. Joe receives a commission for selling used cars. If he was paid $65 for the sale of an old clunker, what was his commission rate if the selling price of the car was $475?

---

7. The length of one base of a trapezoid is 7 inches while its area is 57 inches^2. Find the length of the other base if the height is 6 inches. (The area of a trapezoid is .5(b1 + b2)h.)

*8. The length of a rectangle is 8 and its width is 2 less than its length. What percentage of the area of the rectangle is the width?

*9. Find the length of a side of a cube if its total surface area is 54 cm^2.

10. If the circumference of a circle is 110 inches, what is the diameter?

11. If the Bearcats won 75% of their games (they won 15 games) over the past two years, what was the total number of games they played in those two years?

12. The area of a triangle is 182 cm^2. What must its height be if its base is 42 cm?

13. What is the radius of a circle whose area is 19?

14. What is 125% of 536?

15. Convert 117.20% to a decimal fraction.	16. Convert 126/179 to a percent.
17. 11.2 is what percent of 146.9?	18. What is .18% of 4000?

Unit 4: Lesson 05 **Solving abstract equations**

Previously, we easily solved equations like

$4x + 2 = 10$

and were able to get specific answers (x = 2 in this case).

In the above problem, consider replacing 2 with *y* and 10 with 3*z* to produce

$4x + y = 3z$

Could the problem still be solved for *x*?

The answer is, "yes"; however, the answer would not be specific. Rather, it would be in terms of *y* and *z*.

Example 1: Solve $4x + y = 3z$ for *x*.

$$4x + y = 3z$$
$$4x + \cancel{y} \cancel{- y} = 3z - y$$
$$4x = 3z - y$$
$$\frac{\cancel{4}x}{\cancel{4}} = \frac{3z - y}{4} \quad ; \quad \boxed{x = \frac{3z - y}{4}}$$

Example 2: Solve $4x + y = 3z$ for *y*.

$$4x + y = 3z$$
$$\cancel{4x} + y \cancel{- 4x} = 3z - 4x$$
$$y = \boxed{3z - 4x}$$

Example 3: Solve 4x + y = 3z for *z*.

$$4x+y=3z$$

$$\frac{4x+y}{3}=\frac{\cancel{3}z}{\cancel{3}}$$

$$\boxed{\frac{4x+y}{3}=z}$$

Example 4: Solve B = J(1 + 4t) for *t*.

$$B=J(1+4t)$$

$$B=J+4Jt$$

$$B-J=\cancel{J}+4Jt\cancel{-J}$$

$$\frac{B-J}{4J}=\frac{\cancel{4J}t}{\cancel{4J}}$$

$$\boxed{\frac{B-J}{4J}=t}$$

***Example 5:** Solve (4p + 3q)7 = 2q for *q*.

$$(4p+3q)7=2q$$

$$28p+21q=2q$$

$$28p+\cancel{21q}\cancel{-21q}=\underbrace{2q-21q}$$

$$28p=-19q$$

$$\frac{28p}{-19}=\frac{\cancel{-19}q}{\cancel{-19}}\;;\;\boxed{-\frac{28p}{19}=q}$$

Assignment: Solve each equation for the indicated variable.

1. $y = mx + b$ for x

2. $y = mx + b$ for m

3. $y = mx + b$ for b

4. $ax + by = c$ for x

5. $5(2p + 3q) = 9$ for q

6. $A = \frac{1}{2} b(h)$ for b

7. $A = \frac{1}{2}(b_1 + b_2)\ h$ for h

8. $A = \frac{1}{2}(b_1 + b_2)\ h$ for b_2

9. $C = (5/9)(F - 32)$ for F

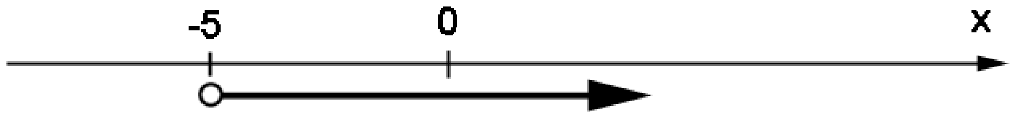
10. C = (5/9)(F – 32) for *C*

*11. (11x + 3c)4 = 5x for *x*

*12. (11x + 3c)4 = 6c for *c*

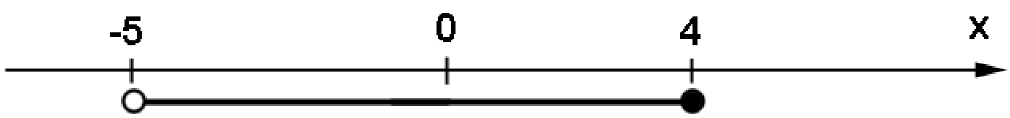
Unit 4: Cumulative Review

1. Write the inequality that is represented by this graph.



2. Which of the following satisfy the inequality of problem 1?
{ -6, -5.01, -4.9, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 }

3. What are the integers that satisfy this compound inequality?



4. Solve this inequality and give you answer both algebraically and as a graph on a number line: $5(x + 2) < 18$

5. Solve this inequality and give you answer both algebraically and as a graph on a number line: $-x - 2 \le 11$

6. Solve $(x + 2) – x – (4 – 5x) = 12$.

7. Solve $2(v + 1) – 1 = 3(v – 4)$.

8. Use two unit multipliers to convert 8 gallons to pints. (4 quarts = 1 gallon, 2 pints = 1 quart)

9. Simplify 3/5 – 1/7 + 6.

10. Convert 76.21 to percent.

11. Convert 3/8 to percent.

12. Convert .23% to a decimal fraction.

13. 150 is what percent of 200?

14. Simplify (2/3)(5/8) ÷ 3/4.

15. Work this problem on a calculator:

$$\frac{.5 - 27}{-11 + 2.09}$$

Unit 4: Review

In problems 1 – 4, define the variable, and then write the expression algebraically.

1. “the sum of the score and 13”

2. “four times the difference of the height and 79”

3. “22 decreased by 8 times the amount of rainfall”

4. “the product of the volume and 4π”

In problems 5 - 7, define the variable, set up the equation, and then solve for the variable.

5. Five less than one-third the population is 11.

6. Four times the sum of a number and 5 gives a value of sixty.

7. Twice the number of zebras increased by four times the number of zebras is exactly 72.

8. The lengths of the bases of a trapezoid are 3x + 1 and 2x + 1. The other two sides are both x. What is the length of the longest base if the perimeter of the trapezoid is 72?

9. The length of a rectangle is 3 more than the width. What are the dimensions of the rectangle if its perimeter is 23?

---

10. A triangle's base is 8 and its height is 2 less than its area. What is its height?

---

11. What is the length of a rectangle if its width is 12 and its area is 120?

12. What is the rate of commission on the sale of a house if a real estate agent makes $12,000 on a house that sold for $240,000?

13. 25 is what percent of 78?	14. 38% of what is 175?
15. 4.11% of 160 is what?	16. Convert 1.02% to a decimal fraction.

17. If the area of a quarter circle is 46π, what is its radius?

18. Solve for *x* from 3x + 2y = 29.

19. Solve for *a* from 4(3a + 9b) = 11.

20. In the equation y = mx + b, *m* is the slope of a line and *b* is the y-intercept. Solve for the y-intercept.

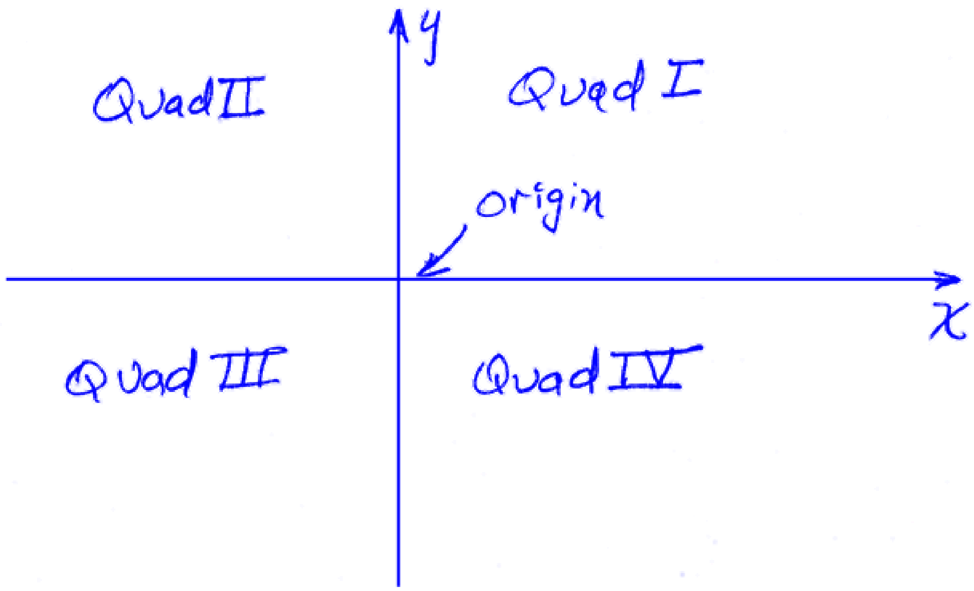
21. Solve for *p* from 3q + 8p = 4(11 + p)

Alg 1, Unit 5

Relations and Functions

Unit 5: Lesson 01 — The coordinate plane, reflections, and translations

The coordinate plane is divided into four **quadrants** by the coordinate axes.



Notice that the **arrow heads** show the directions of the positive x-axis and y-axis.

Example 1: Give the coordinates of the following points. Also give the quadrant in which the point resides. If a point lies on an axis, then state which axis.

A. *(-4, 2), quad II*

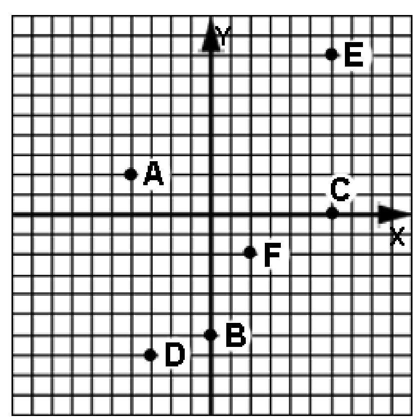
B. *(0, -6), y-axis*

C. *(6, 0), x-axis*

D. *(-3,-7), quad III*

E. *(6, 8), quad I*

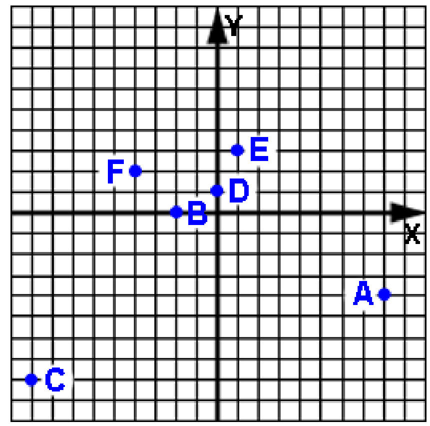
F. *(2, -2), quad IV*



Example 2: Plot and label the following points on the provided coordinate plane.

A. (8, -4) B. (-2, 0) C. (-9, -8)

D. (0, 1) E. (1, 3) F. (-4, 2)

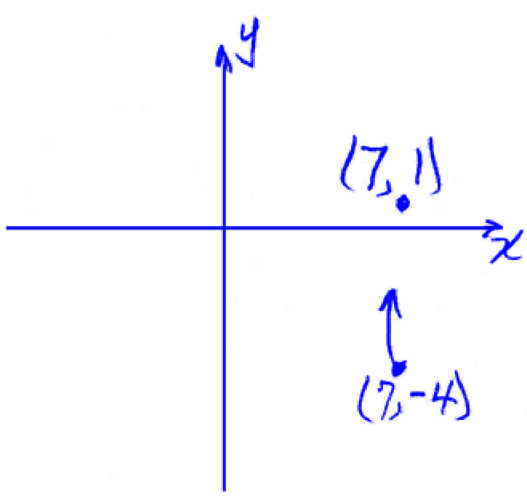


To **translate** a point means to **move** it.

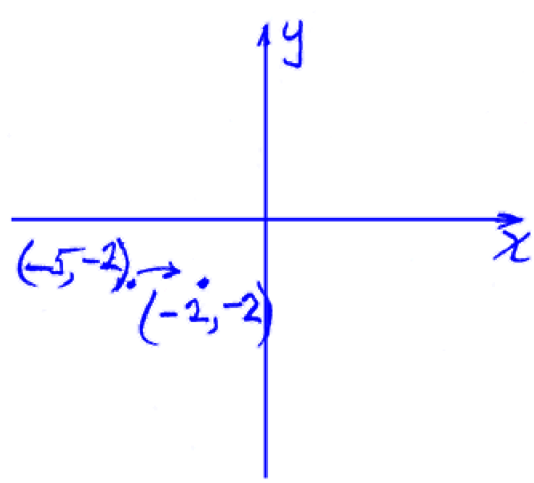
To translate a point **left or right**, add or subtract the appropriate amount from the **x-axis** coordinate.

To translate a point **up or down**, add or subtract the appropriate amount from the **y-axis** coordinate.

Example 3: Plot and label the point (7, -4) on a coordinate plane and then plot and label another point that is translated up 5 units.



Example 4: Plot and label the point (-5, -2) on a coordinate plane and then plot and label another point that is translated right 3 units.



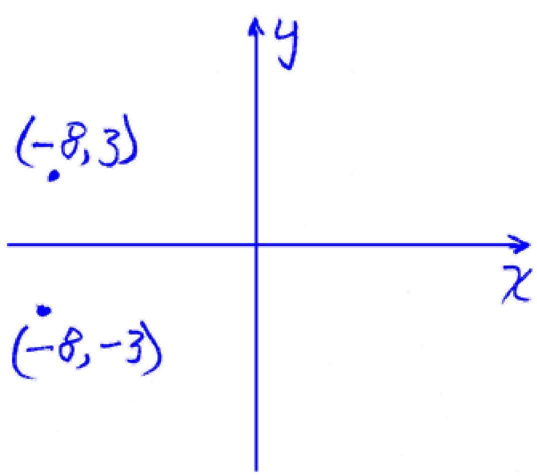
Reflection across the x- axis:

To reflect a point across the x-axis, draw its **mirror image** across the x-axis. The reflected point will have the same coordinates as the original point except the **sign of the y-coordinate will be changed**.

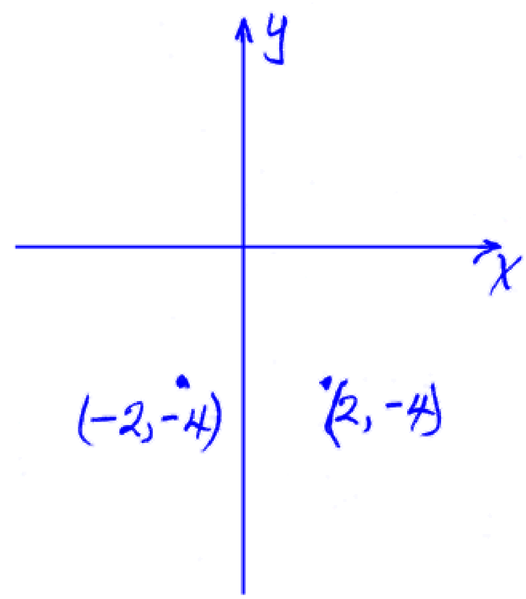
Reflection across the y-axis:

To reflect a point across the y-axis, draw its **mirror image** across the y-axis. The reflected point will have the same coordinates as the original point except the **sign of the x-coordinate will be changed**.

Example 5: Plot and label the point (-8, 3) on a coordinate plane and then plot and label another point that is the reflection of that point across the x-axis.



Example 6: Plot and label the point (2, -4) on a coordinate plane and then plot and label another point that is the reflection of that point across the y-axis.



Assignment:

1. Give the coordinates of the following points. Also give the quadrant in which the point resides. If a point lies on an axis, then state which axis.

A. ________________________

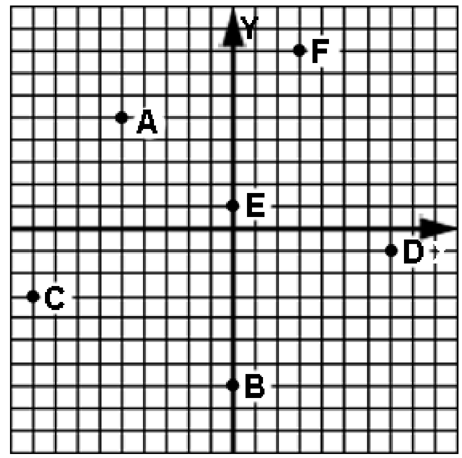
B. ________________________

C. ________________________

D. ________________________

E. ________________________

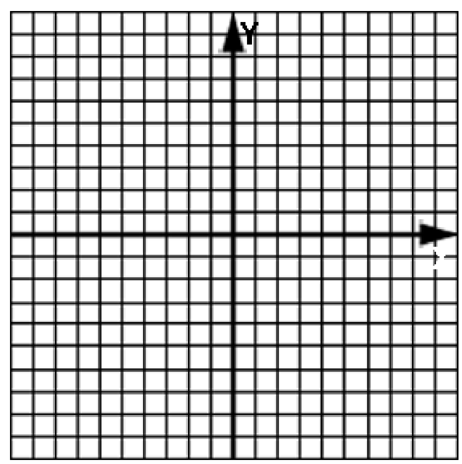
F. ________________________



2. Plot and label the following points on the provided coordinate plane.

A. (0, 5) B. (-4, -6) C. (3, -2)

D. (7, 2) E. (-9, 1) F. (2, 0)



3. Which of the points in problem 2 are in the 2nd quadrant?

4. Which of the points in problem 2 are in no quadrant?

5. What is the x-coordinate of point E in problem 2?

6. What is the y-coordinate of point B in problem 2?

7. What is the x-coordinate of any point on the y-axis?

8. What are the coordinates of the origin of a plane coordinate system?

9. Plot and label the point (8, 3) on a coordinate plane and then plot and label another point that is the reflection of that point across the y-axis.

10. Plot and label the point (-10, 4) on a coordinate plane and then plot and label another point that is the reflection of that point across the x-axis.

11. Plot and label the point (0, -8) on a coordinate plane and then plot and label another point that is the reflection of that point across the x-axis.

12. Plot and label the point (0, 1) on a coordinate plane and then plot and label another point that is the reflection of that point across the y-axis.

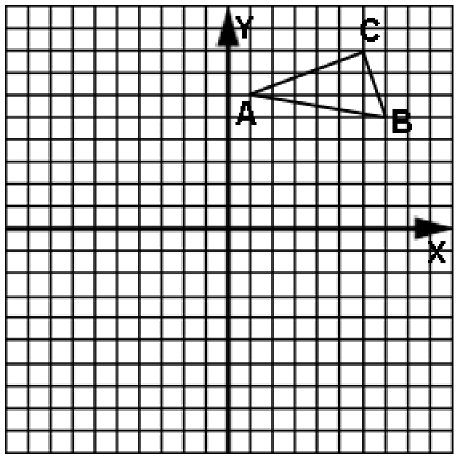
13. Plot and label the point (2, -8) on a coordinate plane and then plot and label another point that is translated to the left 6 units.

14. Plot and label the point (-3, 5) on a coordinate plane and then plot and label another point that is translated down 2 units.

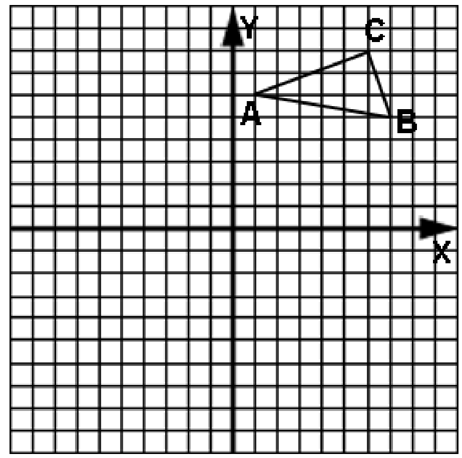
15. Plot and label the point (4, -4) on a coordinate plane and then plot and label another point that is translated up 5 units.

16. Plot and label the point (0, -3) on a coordinate plane and then plot and label another point that is translated to the right 4 units.

17. Draw triangle ABC reflected across the x-axis.



18. Draw triangle ABC reflected across the y-axis.



19.What are the new coordinates of C in problem 17 after the reflection?

20. What are the new coordinates of A in problem 18 after reflection?

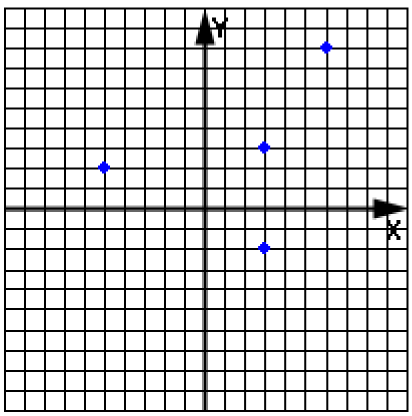
Unit 5: Lesson 02 Relations: domain and range

Relation definition: A relation is a collection of points (officially called a **set** of points). There are **five ways** to show such a collection (set).

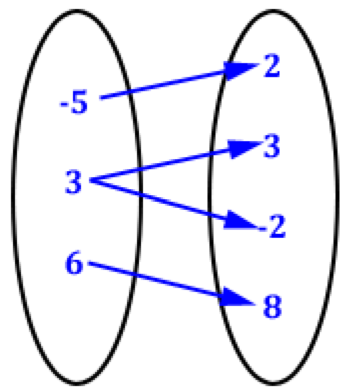
1st way: Let's begin with the most familiar, a list of **ordered pairs**:

{ (–5, 2), (3, 3), (3, –2), (6, 8) }

2nd way: Graph



3rd way: Mapping



4th way: Table

x	y
-5	2
3	3
3	-2
6	8

5th way: Formula rule (More on this later when we study functions)

Domain:

The domain of a relation consists of all the **first coordinates** (all the **x's that are used**).

Range:

The range of a relation consists of all the **second coordinates** (all the **y's that are used**).

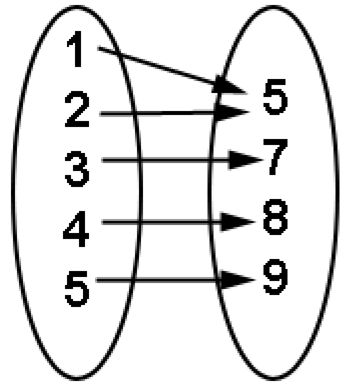
Example 1: State the domain and range of this relation:

{ (-9, 6), (7, 8), (6, -4), (7, 11), (0, 5) }

Domain: { -9, 7, 6, 0 }

Range: { 6, 8, -4, 5 , 11}

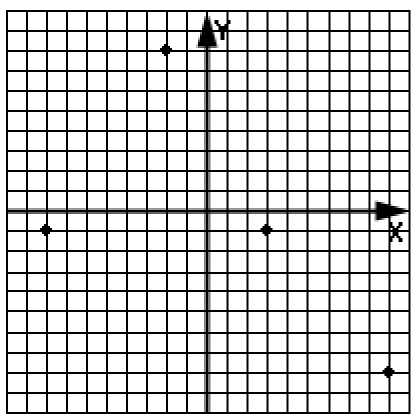
Example 2: State the domain and range of this relation:



Domain: { 1, 2, 3, 4, 5 }

Range: { 5, 7, 8, 9 }

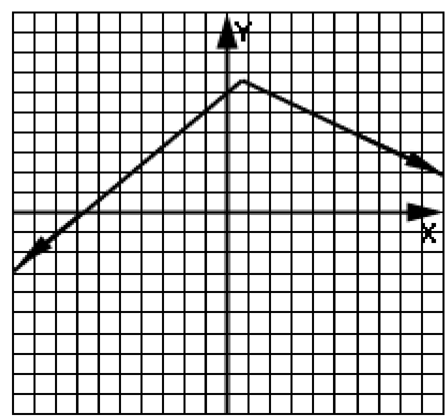
Example 3: State the domain and range of this relation:



Domain: { -8, -2, 3, 9 }

Range: { -1, 8, -8 }

Example 4: State the domain and range of this relation:



Domain: all real x

Range: $y \leq 6.5$

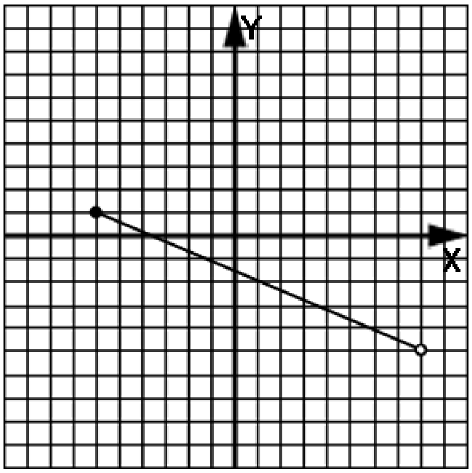
Example 5: State the domain and range of this relation:

Domain: { -5, 3, 6 }

Range: { 2, 3, -2, 8 }

x	y
-5	2
3	3
3	-2
6	8

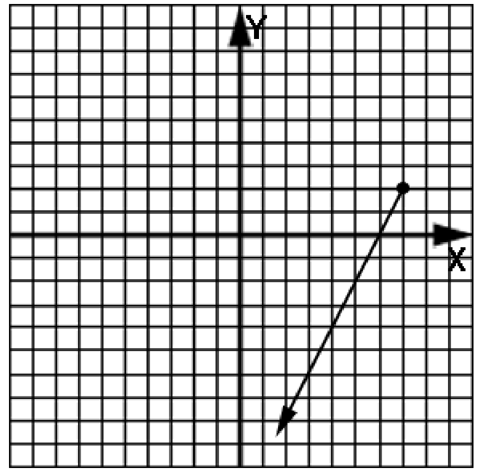
Example 6: State the domain and range of this relation:



Domain: $-6 \leq x < 8$

Range: $-5 < y \leq 1$

Example 7: State the domain and range of this relation:



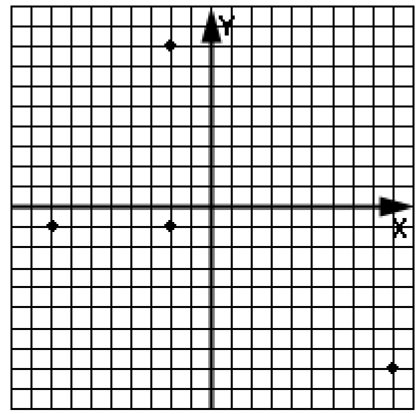
Domain: $x \leq 7$

Range: $y \leq 2$

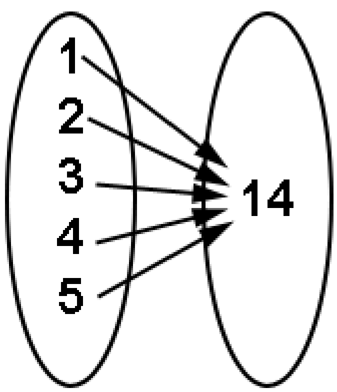
Assignment:

1. Express the relation represented by this list of ordered pairs as a mapping:
{ (4, 7), (-5, 11), (2, 9), (-1, 0) }

2. . Express the relation represented by this graph as a list of ordered pairs:



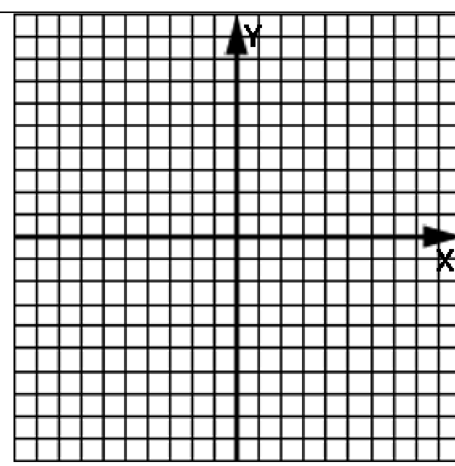
3. Express the relation represented by this mapping as a table:



x	y

4. Express the relation represented by this table as a graph:

x	y
0	1
5	7
8	2
-1	8
-3	4



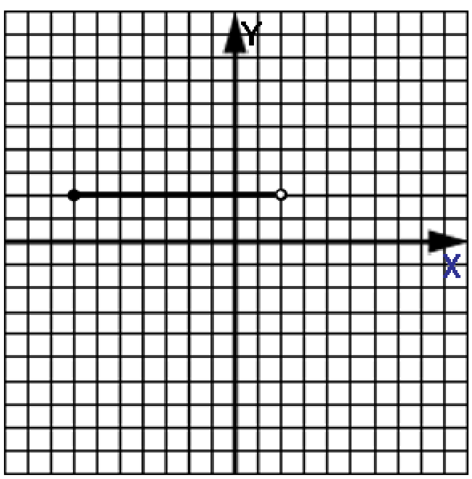
5. State the domain and range of the relation in problem 1.

6. State the domain and range of the relation in problem 2.

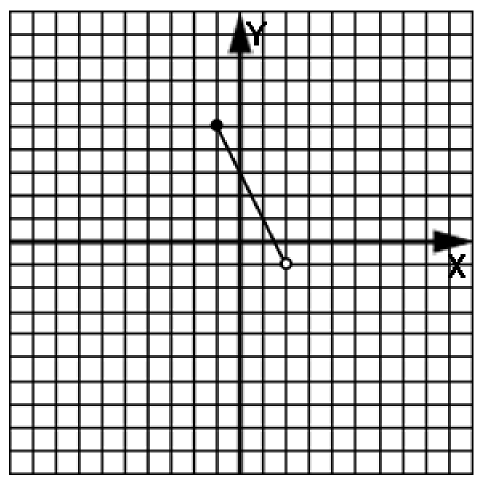
7. State the domain and range of the relation in problem 3.

8. State the domain and range of the relation in problem 4.

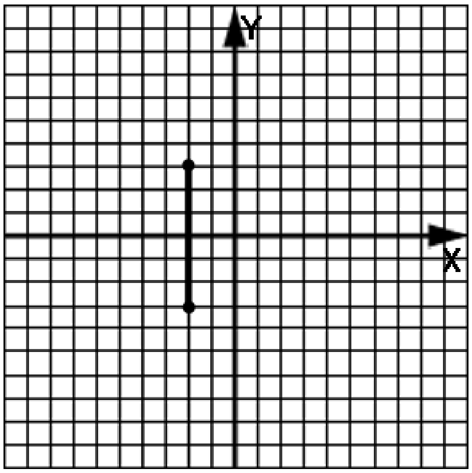
9. State the domain and range of the relation shown here.



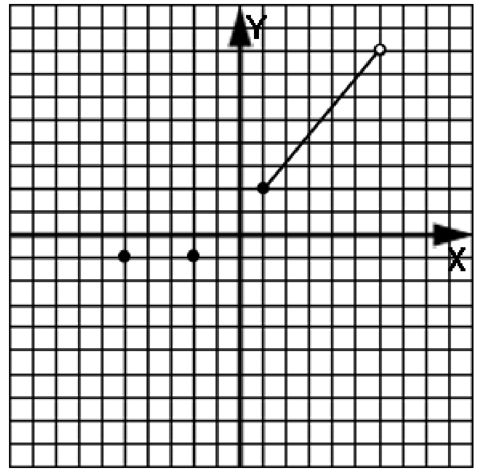
10. State the domain and range of the relation shown here.



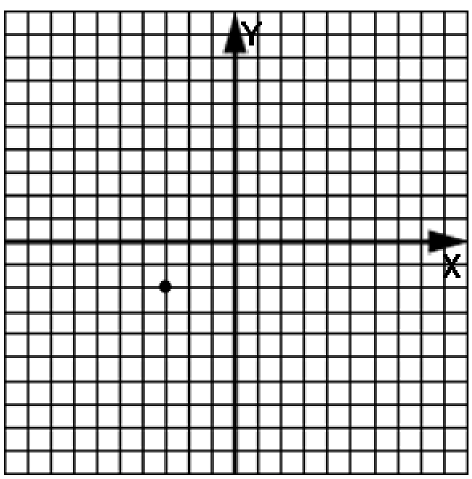
11. State the domain and range of the relation shown here.



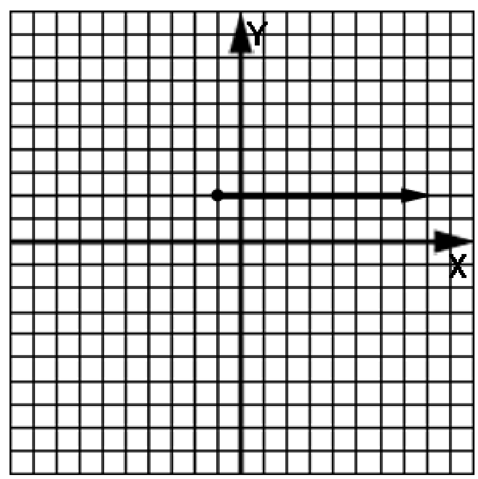
12. State the domain and range of the relation shown here.



13. State the domain and range of the relation shown here.



14. State the domain and range of the relation shown here.



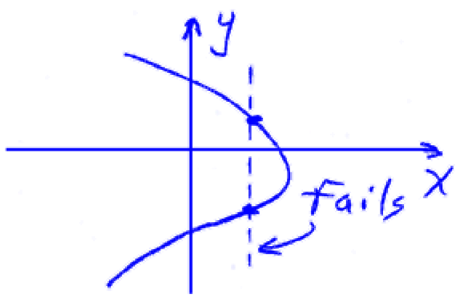
Unit 5: Lesson 03 Functions, function notation

Graphical function definition:

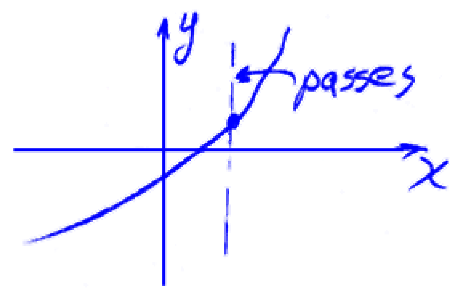
A function is a relation that passes the **vertical line test**:

When a vertical line is drawn anywhere on a graph of the relation, it must touch no more than one point of the relation.

Example 1: Draw an example of a relation that does not pass the vertical line test.



Example 2: Draw an example of a relation that passes the vertical line test.



What about relations that are not given as graphs (ordered pair lists, tables, or mappings)? How do we determine if they "pass the vertical line test?"

Abstract function definition:

For a relation to be a function, no two first-coordinates (the *x* values) can be the same.

Example 3: Is the relation given by { (3, 5), (8, -9), (3, 14) } a function? Why?

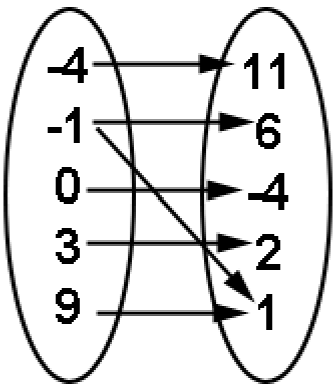
No, it is not a function. The x-coordinate, 3, is repeated.

Example 4: Is the relation given by this table a function? Why?

x	y
5	6
-3	11
8	-9
0	11
2	22

Yes, it is a function. All the x-coordinates are different.

Example 5: Is the relation given by this mapping a function? Why?



No, it is not a function. The ordered pairs this mapping represents are { (-4, 11), (-1, 6), (-1, 1), (0, -4), (3, 2), (9, 1) }.

Notice that the x-coordinate, -1, is repeated.

A function can be represented as a **rule** (a mathematical formula):

Think of the rule as a factory and as a given domain (the *x's*) as the raw material input to the factory. The factory (the rule) then produces *y* values (the values of the range).

Example 6: Given the function rule, y = 3x + 1, whose domain is { -2, 1, 5, 7 }, produce the corresponding range.

X (input)	Y (output)
-2	**-5**
1	**4**
5	**16**
7	**22**

$y = 3(-2) + 1 = -6 + 1 = -5$

$y = 3(1) + 1 = 3 + 1 = 4$

$y = 3(5) + 1 = 15 + 1 = 16$

$y = 3(7) + 1 = 21 + 1 = 22$

$\boxed{\text{Range: } \{-5, 4, 16, 22\}}$

Function notation:

One way to write a function rule is, for example, y = -2x + 6.

Another way to write the same rule is, f(x) = -2x + 6.

Read f(x) as **"f of *x*"**; however, just think of f(x) as meaning y.

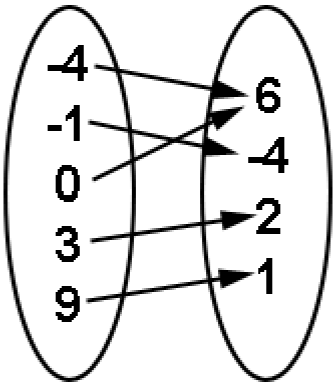
When we see something like f(-5), this just means to replace all the *x's* in the function rule with -5 and then simplify.

Example 7: Evaluate f(-4) when f(x) = 6x + 3 – x.

$$f(x) = 6x + 3 - x$$
$$f(-4) = 6(-4) + 3 - (-4)$$
$$= -24 + 3 + 4$$
$$= -21 + 4 = \boxed{-17}$$

Assignment: In problems 1-12, decide if the given relation is a function or not. Justify your answer.

1.



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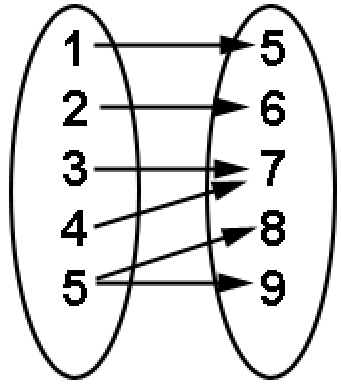
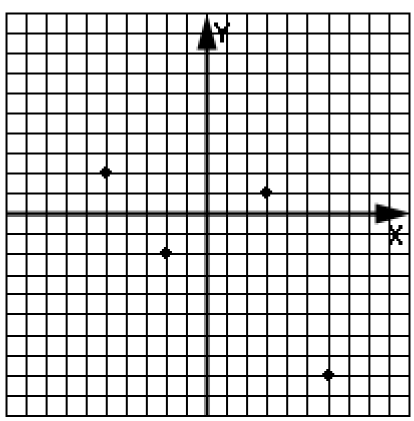


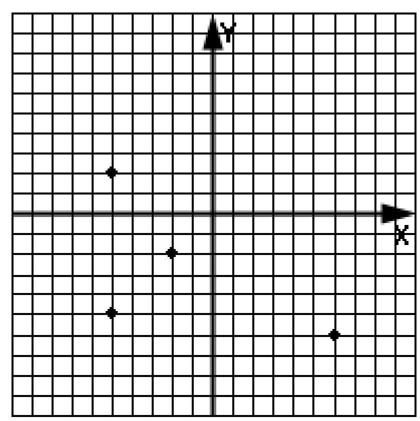
3. { (11, 2), (–4, 13), (11, –1), (3, 18) }

4. { (12, –2), (–4, –2), (11, –2), (16, –2) }

5.



6.



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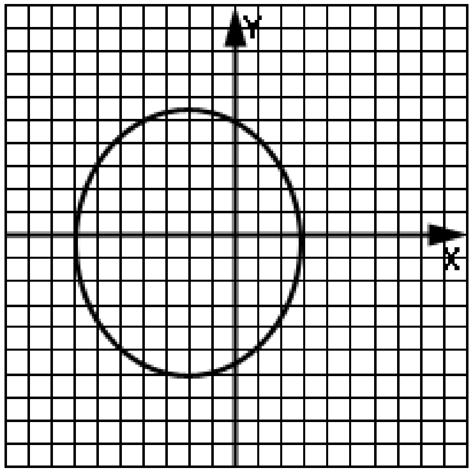

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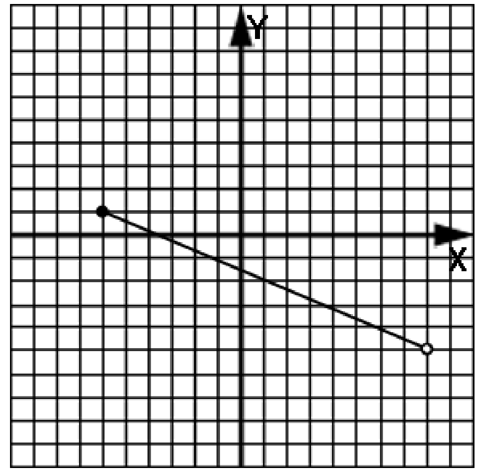

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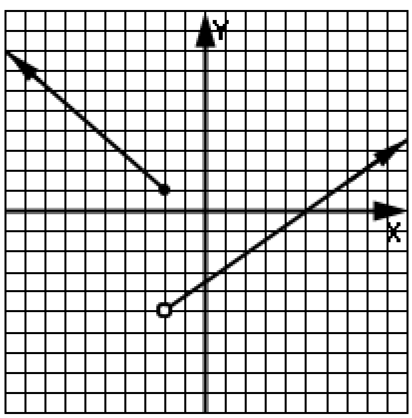
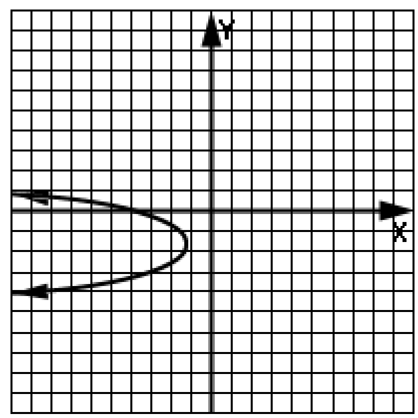


10.



11.

X (input)	f(x) (output)
-3	-5
9	4
-3	11
7	23

12.

X (input)	f(x) (output)
-3	8
9	8
-4	11
7	8

13. Find f(11) when f(x) = (x + 3)/ (x – 22).

---

14. Given the function rule f(x) = x + 9, find the range corresponding to the domain, { -3, 4, 6, 8 }. Fill in the table below with the domain and range.

X (input)	**f(x) (output)**

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15. Find the range for f(x) = x^2 + 3 if its domain is { -4, 6, 10 }.

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16. Give the domain and range for the relation in problem 1.

17. Give the domain and range for the relation in problem 2.

18. Give the domain and range for the relation in problem 6.

19. Give the domain and range for the relation in problem 12.

20. Evaluate $f(x) = x^2 + 3x + 1$ at $x = -2$.

Unit 5: Lesson 04 More practice with functions

Function notation does not necessarily have to use the letter *f*. A function could be called by another letter: for example, g(x).

In examples 1 & 2, use the functions $f(x) = 3x + 2$ and $g(x) = x^2 + 1$.

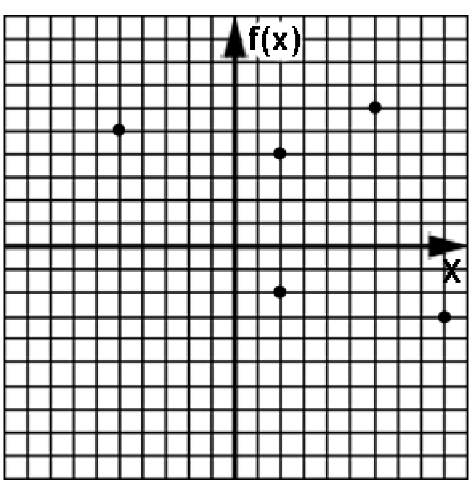
Example 1: Find f(-9) + g(3)

$$f(-9) + g(3) = 3(-9) + 2 + (3)^2 + 1$$
$$= -27 + 2 + 9 + 1$$
$$= -25 + 10 = \boxed{-15}$$

Example 2: Evaluate 2 f(4) + 5 g(1)

$$2[f(4)] + 5[g(1)] = 2[3 \cdot 4 + 2] + 5[1^2 + 1]$$
$$= 2[12 + 2] + 5[1 + 1]$$
$$= 2[14] + 5[2]$$
$$= 28 + 10 = \boxed{38}$$

Example 3: Use the relation given by f(x) to answer these questions.



List the ordered pairs in this relation.

(-5, 5), (2, 4), (2, -2), (6, 6), (9, -3)

What is the domain?

D: { -5, 2, 6, 9 }

What is the range?

R: { 5, 4, -2, 6, -3 }

Is it a function? If not, what points could be removed to make it a function?

No. Remove either (2, 4) or (2, -2)

f(-5) = ?	f(6) = ?	f(9) = ?	f(1) = ?
5	*6*	*-3*	*undefined*

Assignment: Use $p(x) = 4x – 7$ and $q(x) = -x^2 + 2$ in problems 1-4.

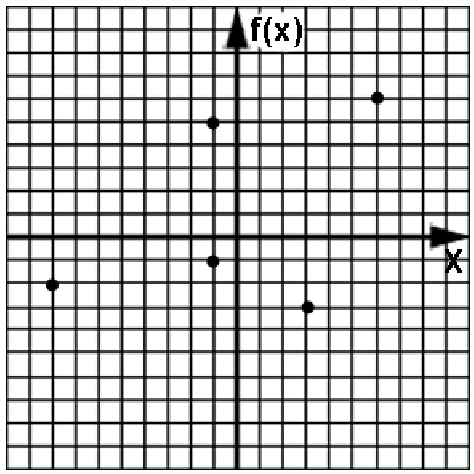
1. Find $p(2) – q(3)$.

2. Find $3q(1) + p(5)$

3. Evaluate $p(-6)$ [$q(1)$].

4. Evaluate $q(5)/p(-11)$

5. Use the relation given by f(x) to answer these questions.



List the ordered pairs in this relation.

What is the domain?

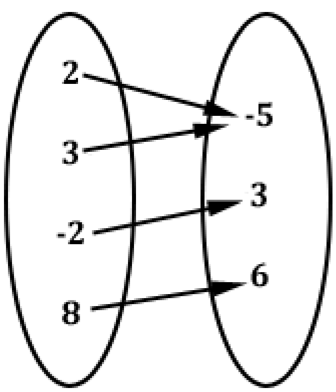
What is the range?

Is it a function? If not, what points could be removed to make it a function?

f(-8) = ? f(3) = ? f(2) = ? f(6) = ?

---

6. Use the relation given by this mapping, h(x) to answer these questions.



List the ordered pairs in this relation.

What is the domain?

What is the range?

Is it a function? If not, what points could be removed to make it a function?

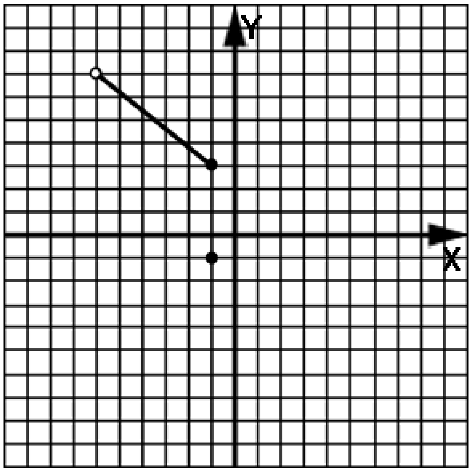
h(3) = ? h(-2) = ? h(1) = ? h(2) = ?

---

7. Find the range for function $t(x) = 4x - x^2$ if the domain is { 1, 2, 3 }.

In problems 8-11, state the domain and range of the given relation. Also state if the relation is a function.

8.



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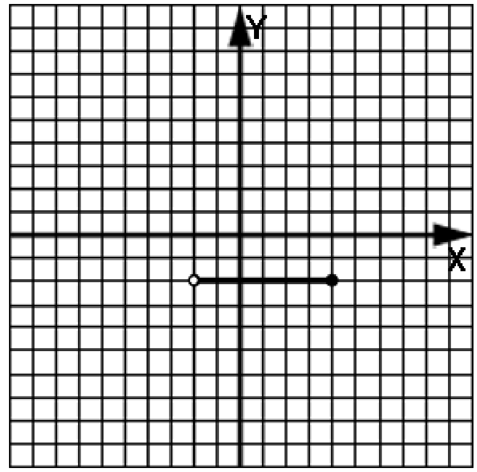

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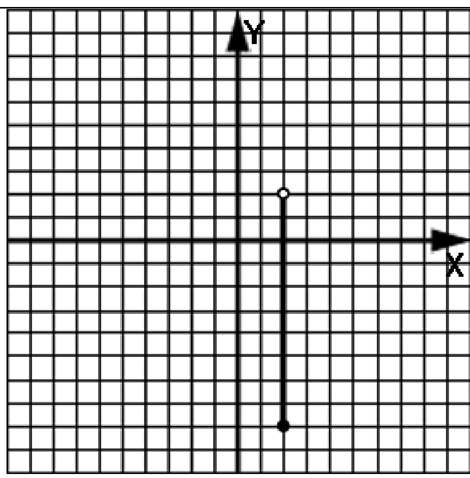

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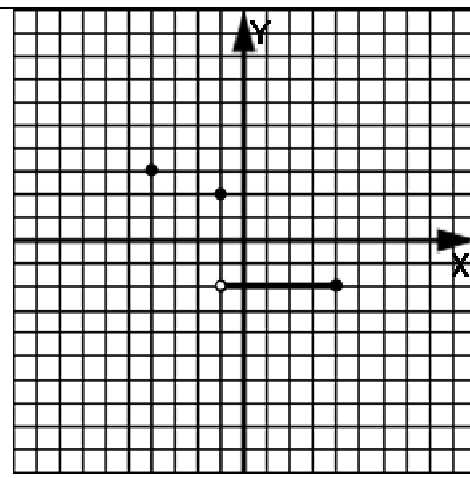


12. Evaluate g(x) at x = -7 where $g(x) = 6x^2 + x – 2$.

*13. Evaluate h(x) at g(2) when $g(x) = 3x^2$ and $h(x) = 11x + 9$.

Unit 5: Lesson 05 Function word problems
Constant rates of change

When a quantity is changing at a constant rate (either increasing or decreasing) the quantity at any time *t* is given by the function Q(t):

Q(t) = b + rate(t) where *b* is the starting quantity and *rate* is the rate of change.

Example 1: Larry Large currently weighs 380 pounds. He plans to go on a diet and lose 4 lbs per week. What is the starting quantity? What is the rate? Write his weight as a function of time, w(t).

t, weeks	w(t), lbs
0	380 – 4(0) = 380
1	380 – 4(1) = 376
2	380 – 4(2) = 372
3	380 – 4(3) = 368

starting quantity = 380 lbs
rate = -4 lbs/week

$\boxed{w(t) = 380 - 4t}$

Example 2: After how many weeks will Larry Large weigh 300 lbs?

$w(t) = 380 - 4t$
$300 = 380 - 4t$
$300 - 380 = \cancel{380} - 4t \cancel{-380}$
$-80 = -4t$
$\frac{-80}{-4} = \frac{\cancel{-4}t}{\cancel{-4}}$; $\boxed{t = 20 \text{ weeks}}$

Example 3: How much will Larry Large weigh after 38 weeks?

$w(t) = 380 - 4t$
$w(38) = 380 - 4 \cdot 38$
$w(38) = 380 - 152$
$w(38) = \boxed{228 \text{ lbs}}$

Example 4: Sally Sadsack is leaving home to see the world. Her old junk VW will only do 45 mph. Specify the starting quantity, and the rate. Then write her distance from home (in miles) as a function of time (in hours), d(t).

t, hrs	d(t), miles
0	$0 + 45(0) = 0$
1	$0 + 45(1) = 45$
2	$0 + 45(2) = 90$
3	$0 + 45(3) = 135$

starting quantity = 0 miles
rate = 45 mi/hr

$d(t) = 0 + 45t$
$\boxed{d(t) = 45t}$

Example 5: What is Sally's distance from home after 8 hrs?

$d(t) = 45t$
$d(8) = 45 \cdot 8$
$d(8) = \boxed{360 \text{ miles}}$

Example 6: After how many hours will Sally be exactly 287 miles from home?

$d(t) = 45t$
$287 = 45t$
$\frac{287}{45} = \frac{\cancel{45}t}{\cancel{45}}$; $\boxed{t = 6.3\overline{7} \text{ hrs}}$

In both examples above the rate was with respect to **time**. For example, lbs per **week** and miles per **hour**.

Notice that when we say **lb per week**, we can also write it as **lb/week**. Notice when we say **miles per hour**, we can also write it as **miles/hour**.

Notice that whatever follows the word **"per"** goes on the **bottom**.

Anything written in the form (something)/(something else) is specifying a **rate**.

Rates are very often with respect to time …some quantity per some unit of time. However, as the next example shows, rates do not necessarily have to be with respect to time.

Example 7: A certain football team averages 4.5 yards/play. They start a drive on their own 20 yard line. Write a function that tells how far *d* from their own goal line they will be as a function of plays *p*. Specify a starting quantity and the rate.

p, plays	d(p), yards
0	$20 + 4.5(0) = 20$
1	$20 + 4.5(1) = 24.5$
2	$20 + 4.5(2) = 29$
3	$20 + 4.5(3) = 33.5$

starting quantity = 20 yd
rate = 4.5 yd/play

$\boxed{d(p) = 20 + 4.5p}$

Example 8: How far will the team be from their own goal line after 7 plays?

$d(p) = 20 + 4.5p$
$d(7) = 20 + 4.5(7)$
$d(7) = 20 + 31.5 = \boxed{51.5\text{ yd.}}$

Example 9: How many plays will it take for the team to be 65 yards from their own goal line?

$d(p) = 20 + 4.5p$
$65 = 20 + 4.5p$
$65 - 20 = \cancel{20} + 4.5p - \cancel{20}$
$45 = 4.5p$
$\frac{45}{4.5} = \frac{\cancel{4.5}p}{\cancel{4.5}}$; $\boxed{p = 10\text{ plays}}$

Assignment:

1. A water tank initially has 120 gallons of water in it. A pipe delivers water into the tank at the rate of 18 gal/min. Specify the starting quantity, the rate, and the water in the tank as a function of time.

t, min	**w(t), gal**

2. Referring to problem 1, after how many minutes will there be 233 gal of water in the tank?

3. Referring to problem 1, how many gallons of water will be in the tank after 20 minutes?

4. The Cats have a tremendous team this year and score points at the rate of 52 points per game. A new sports writer joins a newspaper midway through the season after the Cats have already racked up 156 points. He wants to write a function to know how many points *p* to expect them to have scored after *g* more games. Write this function along with a starting quantity and rate.

g, games	**p(g), points**

5. Referring to problem 4, how many more games will it take for the Cats to bring their total to 312 points?

---

6. Referring to problem 4, how many points will the Cats have scored by the end of 8 more games?

---

7. Miss Carol Spendmore is not very good at budgeting her money. She notices that her bank account is declining by $44 dollars per day. If she initially had $500 in the bank, write a function that describes her bank balance *b* as a function of time *t* in days. Specify a starting amount and rate,

t, days	**b(t), dollars**

---

8. How many days will it be until Miss Spendmore of problem 7 is flat broke?

9. At the rate Miss Spendmore of problem 7 is spending money, how much will her bank balance be at the end of 8 days?

---

10. Farmer Jones sells each apple for 32 cents. Write a function for the cost *c* for a box of *a* apples. Specify the starting quantity and the rate.

a, apples	**c(a), dollars**

---

11. How many apples does Farmer Jones of problem 10 need to sell in order to make $6.08?

---

12. If Farmer Jones of problem 10 sells 302 apples, how much money will he make?

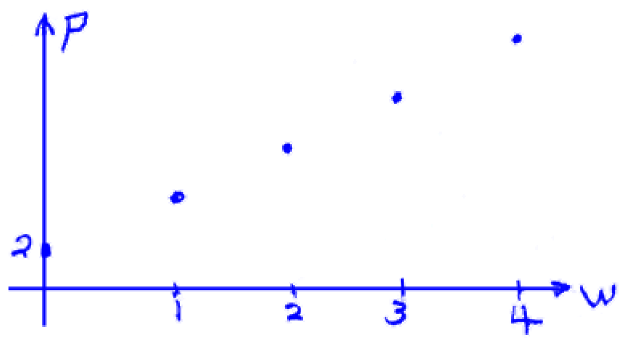
13. Express the ratio form of the rate 4 lb/day in“per” form.

14. Express the rate “ 18.3 counts per day” in ratio form.

Unit 5: Lesson 06 Graphical representations of functions Independent and dependent variables

Example 1: For the function p(w) = 3w + 2, produce the *p* values (the range) when the domain is { 0, 1, 2, 3, 4 }. Plot the resulting points on a coordinate plane.

w	p(w) = 3w + 2
0	$p = 3(0) + 2 = 2$
1	$p = 3(1) + 2 = 5$
2	$p = 3(2) + 2 = 8$
3	$p = 3(3) + 2 = 11$
4	$p = 3(4) + 2 = 14$



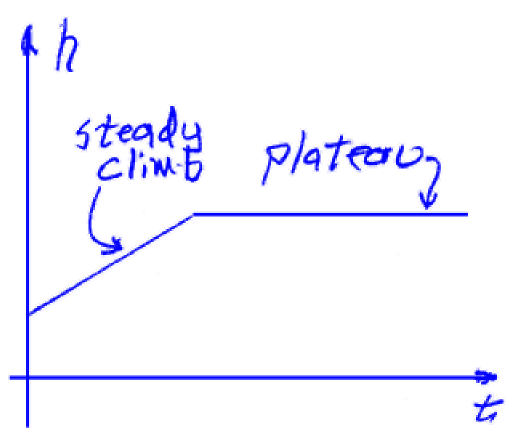
In reference to Example 1 above, fill in the following blanks.

The value of __*p*__ **depends** on the value of __*w*__.

The **dependent** variable_*p*_**depends** on the **independent** variable _*w*_

_*p*_ is a function of _*w*_ . The functional notation is __*p(w)*__.

Example 2: Consider a truck climbing steadily up a hill to a level plateau. Sketch and label the graph of the height of the truck as it climbs and then proceeds on level ground. Label and explain the meanings of the various parts of the graph.



Which is the dependent variable?

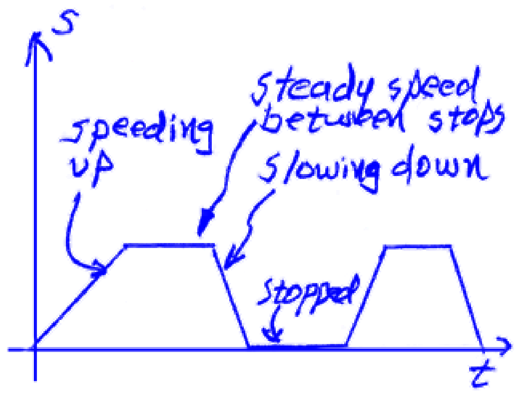
h... height

Which is the independent variable?

t... time

The _*height*_ is a function of _*time*_ and the functional notation is __*h(t)*__

Example 3: Sketch a graph of the speed of a car in stop and go traffic. Label and explain the meanings of the various parts of the graph.



Which is the dependent variable?

s … speed

Which is the independent variable?

t … time

The _*speed*_ is a function of _*time*_ and the functional notation is __*s(t)*__

Assignment:

1. Sketch the temperature of a cold object as it warms up after being dropped into a warm swimming pool. Label and explain the meanings of the various parts of the graph.

Which is the dependent variable?

Which is the independent variable?

The ________ is a function of _______
and the functional notation is _______

2. A caveman notices that when a rock is dropped on his foot, the pain it causes is related to how high the rock was before being dropped. Make a sketch of the pain factor as it is related to the height of the rock. Label and explain the meanings of the various parts of the graph.

Which is the dependent variable?

Which is the independent variable?

The _______ is a function of ________
and the functional notation is _______

3. In problem 2 above, does pain depend on height or does height depend on pain? Why?

4. Are the relations in problems 1 and 2 functions? Why?

---

5. Using a graph, describe the income of a person from birth through the end of a retirement. Start with an allowance as a child, assume that low paying jobs are available in the younger years, and upon graduating from college, a higher paying job is taken with steady raises until retirement. At retirement, only the lower (but steady) retirement pay is available. Label and explain the meanings of the various parts of the graph.

Which is the dependent variable?

Which is the independent variable?

The _______ is a function of ________
and the functional notation is _______

---

6. Consider the temperature of a cold house in winter. After sleeping in the cold house all night you get up in the morning, set the thermostat for heat, and wait a few hours for the house to heat up to a comfortable temperature that is maintained for the rest of the day. Sketch the graph of the temperature of the house throughout the cold night and until the end of the next day. Label and explain the meanings of the various parts of the graph.

Which is the dependent variable?

Which is the independent variable?

The _______ is a function of ________
and the functional notation is _______

7. A man jumps off a bridge with a bungee cord, hits the end of the cord, and then bobs up and down while slowly coming to a stop. Sketch a graph of the man's height in relation to the ground and the bridge as this all happens. Label and explain the meanings of the various parts of the graph.

Which is the dependent variable?

Which is the independent variable?

The _______ is a function of ________

and the functional notation is _______

8. Billy Bob sucks in a big lung-full of air and begins blowing up his balloon. He breathes in again and continues blowing up the balloon, but at a reduced rate. All of a sudden a prankster pops his balloon. Sketch a graph of the volume of the balloon over the time span of these events. Label and explain the meanings of the various parts of the graph.

Which is the dependent variable?

Which is the independent variable?

The ________ is a function of ________
and the functional notation is ________

9. Is the independent variable normally the horizontal or vertical axis when graphing a function?

10. “The grades on a test are generally related to the difficulty level of the questions on the test.” From this statement, define two variables and label each as either dependent or independent. Write the functional notation relating these two variables.

11. “The quality of the party determines the number of nerds that will be present.” From this statement, define two variables and label each as either dependent or independent. Write the functional notation relating these two variables.

12. Is the dependent variable normally associated with the x-axis or the y-axis?

Unit 5: Cumulative Review

1. Simplify 3+ 8/2 + 60(5) + 1

2. Simplify 3+ 8(2) + 60/5 + 1

3. Simplify -3(-7) – 5(6)/(-2)

4. Simplify -8/(6 – 8) + 1

5. Evaluate |z – x/2 + y| if x = 6, y = 10, z = 15

6. Simplify $\frac{a+b}{c-d}$ when a = 20, b = 5, c = 9, d = 4.

7. Simplify (11x – (5/4)x)/(3/2)

---

8. Find the solution to the equation 14x + 1 + 6x – 9 = 32.

---

9. Solve 4(y + 2) – 5y = y + 12

---

10. Use a unit multiplier to convert 26 nerds to geeks when it is known that exactly 13 nerds = 12 geeks.

11. Express "3y plus 5 is at least 5x minus 12" in mathematical symbols.

12. Determine the inequality solution to $4x – 6 > x – 1$. Express the answer both symbolically and as a graph on a number line.

13. From the expression "3 less than 10 times the weight", define a variable and then write the expression algebraically.

14. The three sides of a triangle are $z + 9$, $z + 1$, and $z + 2$. If the perimeter of the triangle is 48, what is the value of z?

*15. If the perimeter of a closed half circle is 37.09, what is the radius?

16. Bob sells used cars for a commission of 8%. What is the price of a car he sells if his commission is $130?

Unit 5: Review

1. Of the following points, which is in the 3rd quadrant?

   (4, -9), (-4, -9), (4, 9), (-4, 9)

2. Plot the point (8, 2) on a coordinate plane and show its new position after being translated 4 units down.

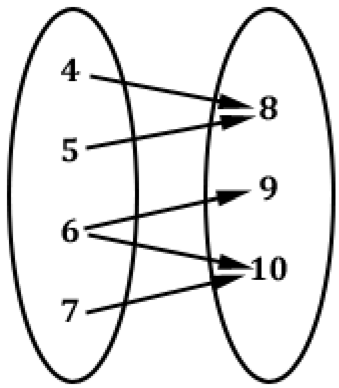
3. Plot the point (-6, 3) on a coordinate plane and show its new position after being translated 2 units to the right.

4. Plot the point (2, -5) on a coordinate plane and show its new position after being reflected across the x-axis.

5. Plot the point (2, -5) on a coordinate plane and show its new position after being reflected across the y-axis.

6. If $g(x) = 2x^2 + x + 1$, find $g(-3)$

7. Find the domain and range of the relation represented by this mapping. Is it a function? Why?



8. Find the domain and range of the relation represented by this table. Is it a function? Why?

x	y
5	–2
6	–2
7	17.5
2	0

9. Find the domain and range of the relation represented by this set of ordered pairs. Is it a function? Why?

{ (3, 7), (6, –2), (11, 9), (6, –7) }

10. If the function f(x) = 3x + 9 has the domain, { -1, 0, 3, 7 }, what is its range?

11. If $f(x) = -2x - 11$ and $g(x) = 7 - x$, find the value of $3f(8) - 2g(6)$.

12. Bob has already sold $200 worth of fruit from his fruit stand. If he continues selling at the rate of $80 per hour, write a function that will predict his total up to any later time.

13. After how much more time will Bob have sold $420 worth of fruit?

14. What will be Bob's total sales after another 8 hours?

15. Just as it comes out of the oven, a hot pie is placed in a refrigerator. Sketch a graph of the temperature of the pie over the next several hours.

Which is the dependent variable?

Which is the independent variable?

The _______ is a function of _______ and the functional notation is _______

Alg 1, Unit 6

Graphing Linear Functions

Unit 6: Lesson 01

Linear function definition
Plotting points and verifying with a graphing calculator

A linear function is one in which *x* and *y* and are both raised to the **1 power.**

Linear functions always produce a **straight line** when graphed.

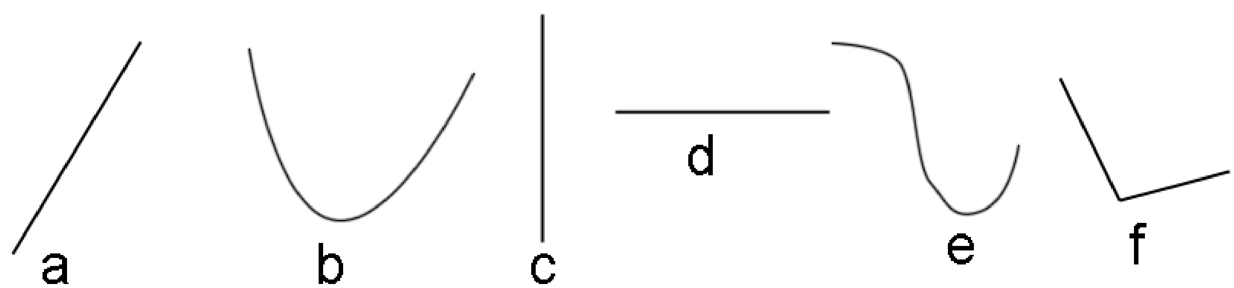
Example 1: Is y = 3x + 13 a linear function? Why?

Yes. Both x and y are both understood to be raised to the 1 power.

Example 2: Is y = 3x^2 + 9 a linear function? Why?

No, because x is raised to the 2 power.

Example 3: Of the following graphs, which represent linear functions?



a & d (note that c is not even a function…fails vert. line test)

When we are given a linear function, instead of something like

y = 4x + 2,

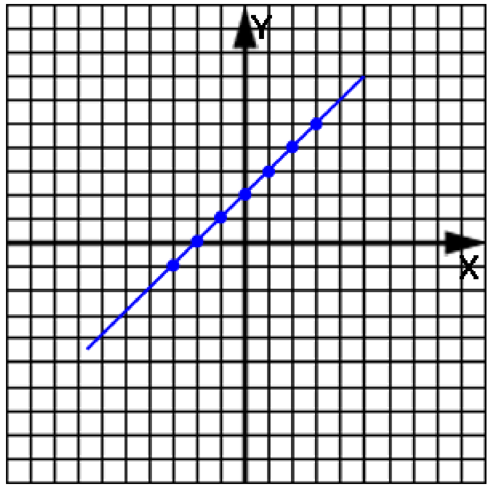
we will often be given the equivalent form in functional notation:

f(x) = 4x + 2

Both ***y* and f(x) are the same** and can be used interchangeably.

Example 4: Fill in the chart to produce the *y* values for the linear function f(x) = x + 2 corresponding to the domain { -3, -2, -1, 0, 1, 2, 3 }. Plot each point and then connect the points with a best fitting graph.

x	f(x) = x + 2	y
-3	f(-3) = -3 + 2 =	-1
-2	f(-2) = -2 + 2 =	0
-1	f(-1) = -1 + 2 =	1
0	f(0) = 0 + 2 =	2
1	f(1) = 1 + 2 =	3
2	f(2) = 2 + 2 =	4
3	f(3) = 3 + 2 =	5



Example 5: If the graph produced in Example 4 above is extended forever in both directions, the given domain is ignored, and assuming **all** points are used:

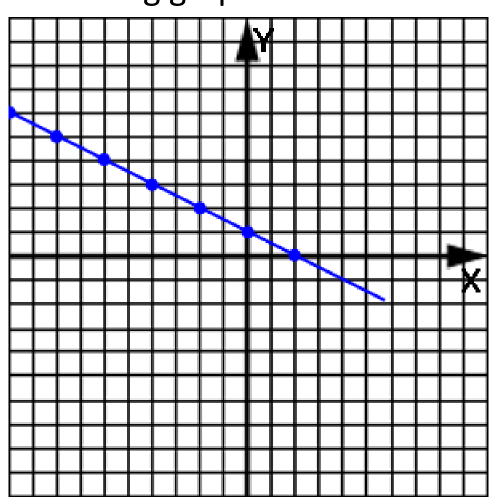
What would be the new domain of the function f(x)?
All real x's

What would be the new range of the function f(x)?
All real y's

What is the apparent shape of the graph?
A line

Example 6: Fill in the chart to produce the *y* values for the linear function f(x) = (-½)x + 1 corresponding to the domain { -10, -8, -6, -4, -2, 0, 2 }. Plot each point and then connect the points with a best fitting graph.

x	f(x) = (-½)x + 1	y
-10	f(-10) = (-½)(-10) + 1 =	6
-8	f(-8) = (-½)(-8) + 1 =	5
-6	f(-6) = (-½)(-6) + 1 =	4
-4	f(-4) = (-½)(-4) + 1 =	3
-2	f(-2) = (-½)(-2) + 1 =	2
0	f(0) = (-½)(0) + 1 =	1
2	f(2) = (-½)(2) + 1 =	0



Example 7: If the graph produced in Example 6 above is extended forever in both directions, the given domain is ignored, and assuming **all** points are used:

What would be the new domain of the function f(x)?

All real x's

What would be the new range of the function f(x)?

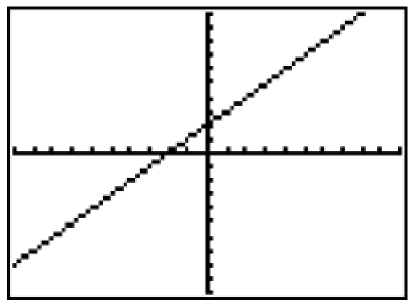
All real y's

What is the apparent shape of the graph?

A line

Example 8: We will now verify with a graphing calculator that the graph we produced in Example 4 is correct.

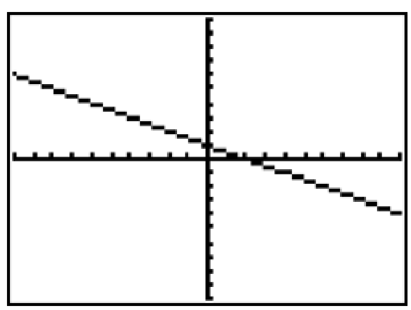
Using the linear function f(x) = x + 2, enter the *x* + 2 part as **Y1** and with the **ZOOM** button, specify 6. ZSTANDARD ZOOM. Press the **GRAPH** button and make a sketch of the calcultor display here.



If you are not famaliar with how to graph a function on a graphing calculator, see **Calculator Appendix D** and an associated video.

Example 9: We will now verify with a graphing calculator that the graph we produced in Example 6 is correct.

Using the linear function f(x) = (-½)x + 1, enter the (-½)x + 1 part as **Y1** and with the **ZOOM** button, specify **6. ZSTANDARD ZOOM**. Press the **GRAPH** button and make a sketch of the calcultor display here.



Assignment: In problems 1- 6, state whether the function would produce a line when graphed. Give the reason for your choice.

1. $y = -8x/2 + 11$

2. $f(x) = x^3 - 3$

3. $x^2 + y^2 = -20$

4. $9x - 2y = 0$

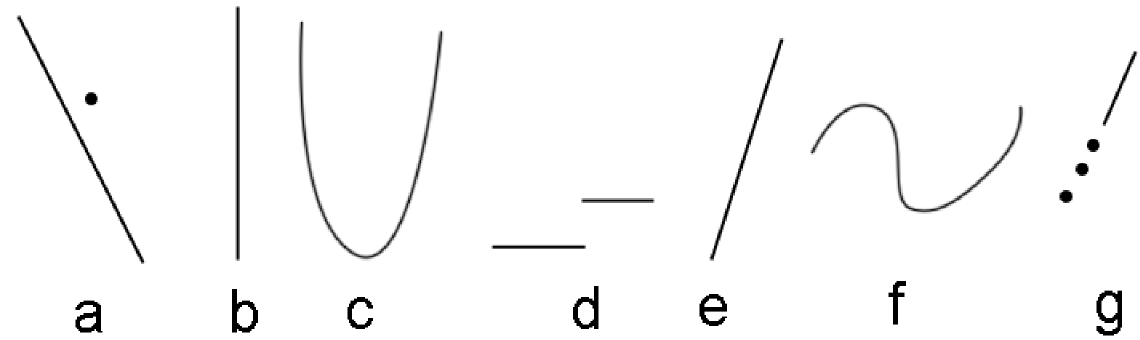
5. $2f(x) + 8x = 19$

6. $2[f(x)]^2 + 8x = 19$

7. Which of problems 1-6 are linear functions?

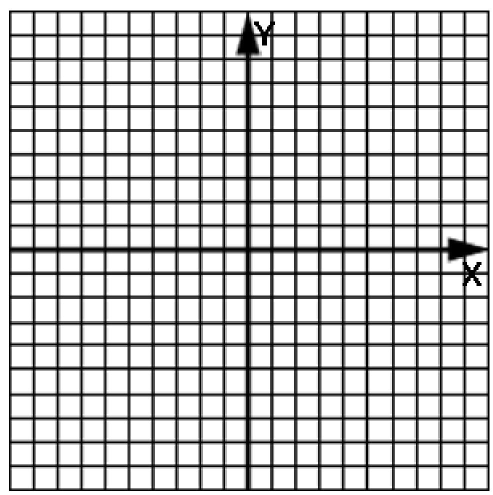
8. When a linear function is given in terms of f(x), with what variable do we commonly replace f(x)?

9. Which of the following graphs depict linear functions? Assume that the domain is "all real *x.*" (Hint: It not only has to be linear, it must also be a **function**.)



10. Fill in the chart to produce the *y* values for the linear function f(x) = 2x + 1 corresponding to the domain { -5, -3, -1, 0, 1, 3, 4 }. Plot each point and then connect the points with a best fitting graph.

x	**f(x) = 2x + 1**	**y**
-5		
-3		
-1		
0		
1		
3		
4		

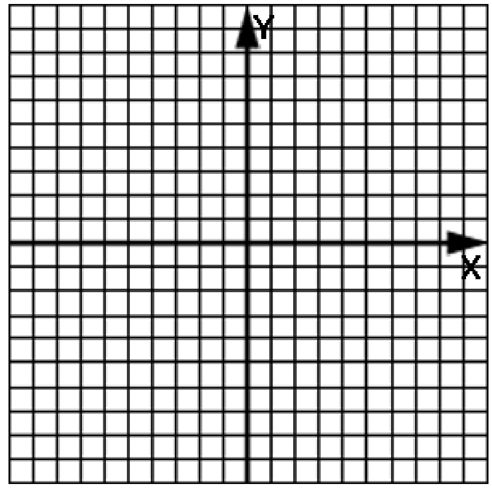


11. Disregarding the domain given above for problem 10, what would be the domain for this function if the graph is extended forever in both directions and **all** points on the line are used?

12. Disregarding the domain given above for problem 10, what would be the range for this function if the graph is extended forever in both directions and **all** points on the line are used?

13. Fill in the chart to produce the *y* values for the linear function
f(x) = (-1/3)x + 2 corresponding to the domain { -9, -6, -3, 0, 3, 6, 9 }. Plot each point and then connect the points with a best fitting graph.

x	**f(x) = (-1/3)x + 2**	**y**
-9		
-6		
-3		
0		
3		
6		
9		



14. Use a graphing calculator to graph y = 2x – 1. Make a sketch of the calculator display.

15. Use a graphing calculator to graph problem 13. Make a sketch of the calculator display.

Unit 6: Lesson 02 Slope

The slope of a line (denoted with the letter **m**) can be defined in several different ways; however, they are all consistent with each other.

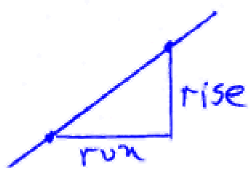
Definition #1:

The slope of a line can be thought of as the rate of vertical change with respect to the horizontal change.

$$m = \frac{\text{vert. change}}{\text{horiz. change}}$$

Definition # 2:

The slope of a line can be defined in terms of **rise** and **run**.



$$m = \frac{\text{rise}}{\text{run}}$$

Definition # 3:

Simplest of all, slope can be thought of as a measurement of the **steepness** of a line; the larger the slope, the steeper the line.

Definition # 4:

Given two points on a line (x_1, y_1) and (x_2, y_2) the slope m is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines that slope **up** when moving from left to right have a **positive slope**.

$m \rightarrow$ positive

Lines that slope **down** when moving from left to right have a **negative slope.**

$m \rightarrow$ neg slope

Horizontal lines have a slope of 0.

$m = 0$

Vertical lines have a steepness so great that it is infinity and is, therefore, **undefined**. In this case, we say **"no slope"**, or **does not exist**. (Note that this does not mean 0, it simply means the slope is so large, we can't say what it is.)

Example 1: If a line increases three units in the vertical direction for every two units of change in the horizontal direction, what is the slope?

$$m = \frac{\text{vert. change}}{\text{horiz. change}}$$

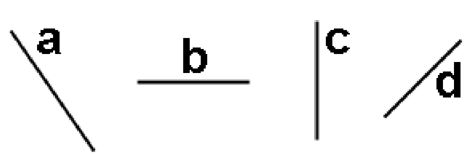
$$= \boxed{\frac{3}{2}}$$

Example 2: If a line decreases five units in the vertical direction for every six units of change in the horizontal direction, what is the slope?

$$m = \frac{\text{vert. change}}{\text{horiz. change}}$$

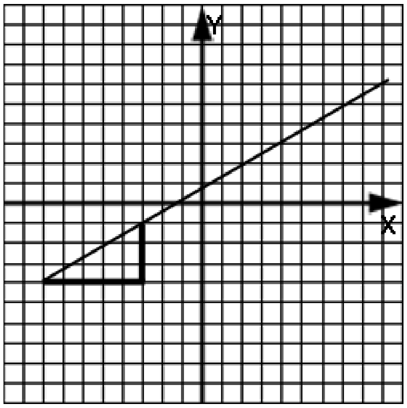
$$m = \boxed{\frac{-5}{6}}$$

Example 3: Identify the slopes of these lines as being positive, negative, 0, or no slope:



a is negative *b is 0* *c is No slope* *d is positive*

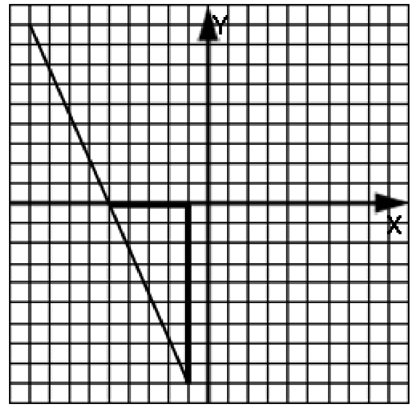
Example 4: Using the rise and run definition, what is the slope of this line?



$m = \text{rise}/\text{run}$

$= \boxed{3/5}$

Example 5: Using the rise and run definition, what is the slope of this line?



$m = \text{rise}/\text{run}$

$m = \boxed{-9/4}$

Example 6: Find the slope of a line passing through (-5, -2) and (6, -7).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-(-2)}{6-(-5)}$$

$$= \frac{-7+2}{6+5}$$

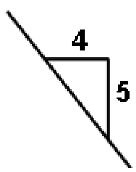
$$= \boxed{\frac{-5}{11}}$$

Assignment:

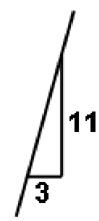
1. Define slope in terms of rise and run.

2. Give a slope formula in terms of two points (a, b) and (c, d).

3. Find the slope of this line.



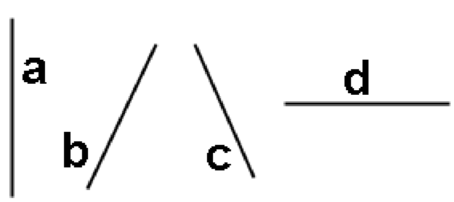
4. Find the slope of this line.

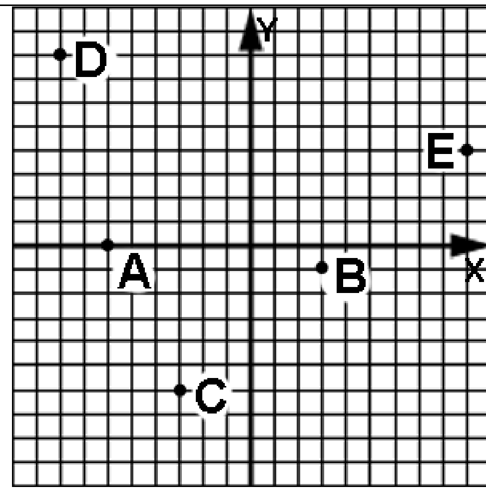


5. If a line decreases 22 units in the vertical direction for every four units of change in the horizontal direction, what is the slope?

6. If a line increases two units in the vertical direction for every two units of change in the horizontal direction, what is the slope?

7. What are the slopes of these lines pos., neg., 0, or “no slope”?



8. When working with linear functions the variable *m* usually means what?	9. Several different lines have slopes of m = 3, m = 2, m = 11, and m = 0. Which of the lines has the greatest steepness?
Use these points for problems 10 -18.	
10. Is the slope of the line between A & B positive or negative?	11. Is the slope of the line between C & E positive or negative?
12. The line between which two points has the steepest negative slope?	13. Between which two points is the slope zero?

14. What is the slope of the line passing through points A and E?

15. What is the slope of the line passing through points D and B?

16. What is the slope of the line passing through points D and the origin?

17. To what value should the *x* coordinate of point A be changed so that the slope of the line connecting A and C be undefined?

18. To what value should the *y* coordinate of point E be changed so that the slope of the line connecting E and B be zero?

Unit 6: Lesson 03

Graphing a line given a point and a slope
Slope-intercept form of a linear function

To graph a line given a point and a slope,

- Plot the point.
- Use the slope to identify the **rise** and **run** (**run is always positive**).
- Starting at the plotted point, move to the right by an amount equal to the run.
- Continue moving up (if the rise is positive), or down (if the rise is negative) and plot a new point.
- Draw a line connecting the two points. When drawing the line, show arrow heads on both ends.

  The arrow heads indicates that the line continues forever **(infinitely**) in both directions.

Example 1: Identify the rise and run when the slope is 3.

$m = \frac{3}{1} = \frac{rise}{run}$

$\boxed{\begin{array}{l} rise = 3 \\ run = 1 \end{array}}$

Example 2: Identify the rise and run when the slope is $-\frac{4}{3}$.

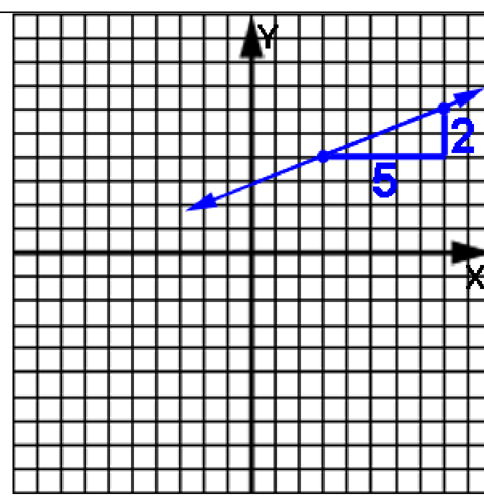
$m = -\frac{4}{3} = \frac{rise}{run}$

$\boxed{\begin{array}{l} rise = -4 \\ run = 3 \end{array}}$

Example 3: Graph the line passing through (3, 4) and having a slope of 2/5.

$m = \frac{2}{5} = \frac{rise}{run}$

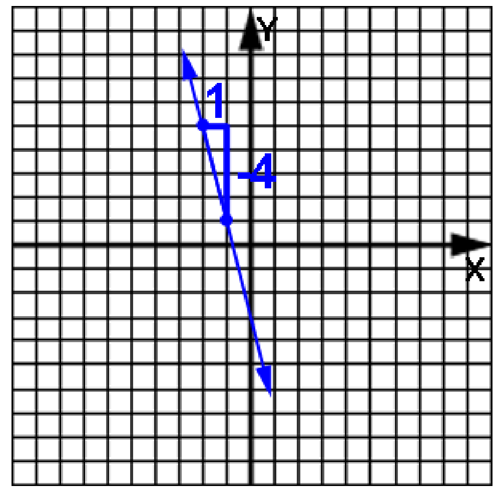
$\boxed{\begin{array}{l} rise = 2 \\ run = 5 \end{array}}$



Example 4: Graph the line passing through (-2, 5) and having a slope of -4.

$m = -4 = \frac{-4}{1} = \frac{rise}{run}$

$\boxed{\begin{array}{l} rise = -4 \\ run = 1 \end{array}}$



Slope-intercept form of a linear function:

y = mx + b

where *m* is the slope of the line, and

b is the y-intercept (where the line crosses the y-axis).

Example 5: Identify the slope (*m*) and the y-intercept (*b*) in the linear function y = -8x – 4.

$y = -8x - 4$

$y = mx + b$

$\boxed{m = -8 \quad b = -4}$

Example 6: Identify the slope (*m*) and the y-intercept (*b*) in the linear function f(x) = x + 4.

$f(x) = x + 4$

$y = 1x + 4$

$y = mx + b$

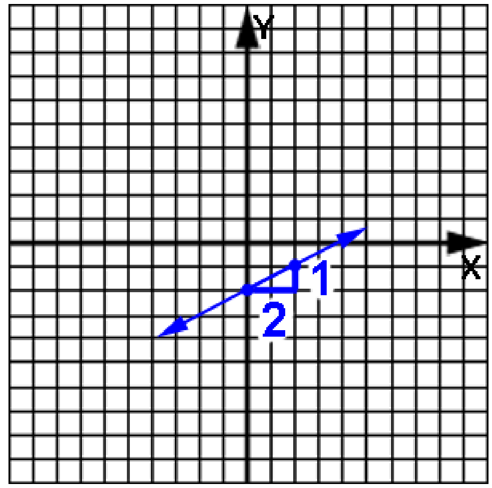
$\boxed{m = 1 \quad b = 4}$

When we are given the y-intercept (*b*) of a linear function, we are really being given the point (0, b).

Therefore, being given a **slope and y-intercept** is equivalent to being given a **slope and a point**.

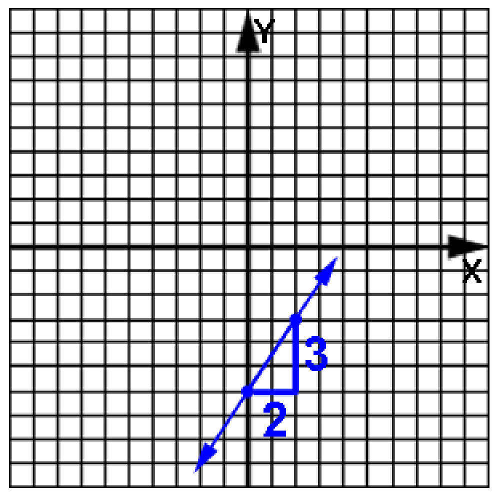
Example 7: Graph the linear function having a y-intercept of -2 and a slope of ½.

$m = \frac{1}{2} = \frac{rise}{run}$



Example 8: Graph the linear function y = (3/2) x – 6.

$m = \frac{3}{2} = \frac{rise}{run}$



Assignment:

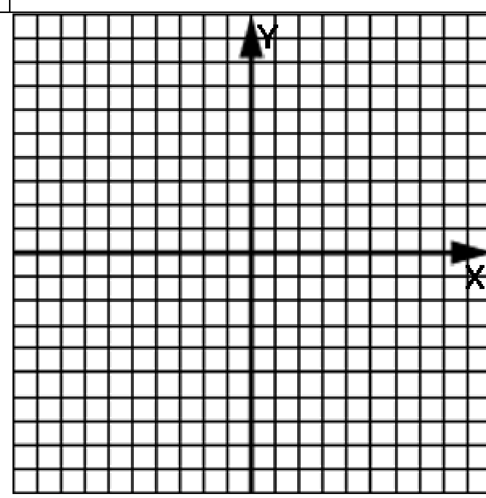
1. Identify the rise and run when the slope is -4.

2. Identify the rise and run when the slope is 7/5.

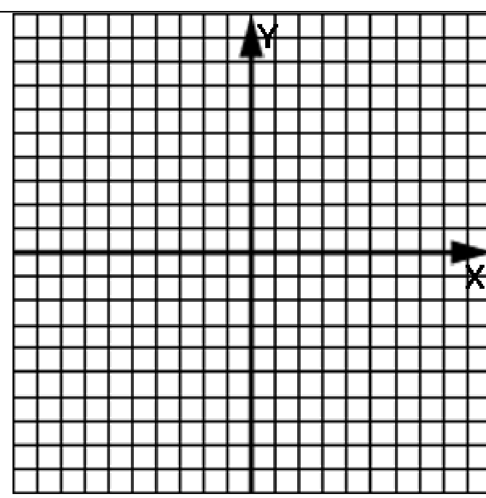
3. Identify the rise and run of the slope of a line having this equation:
   y = x – 19

4. Identify the rise and run of the slope of a line having this equation:
   f(x) = .75x + 2

5. Graph the line passing through (-2, -4) and having a slope of 6/5.



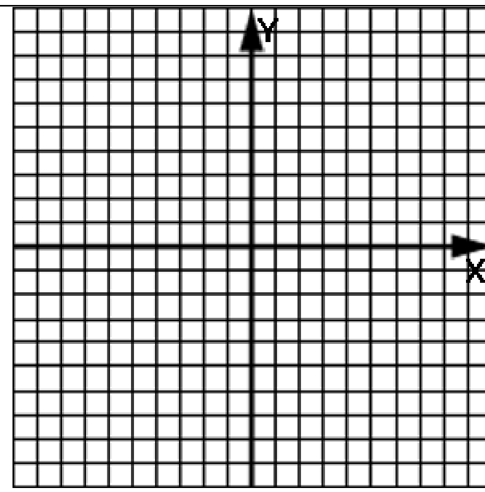
6. Graph the line passing through the origin and having a slope of -7/2.



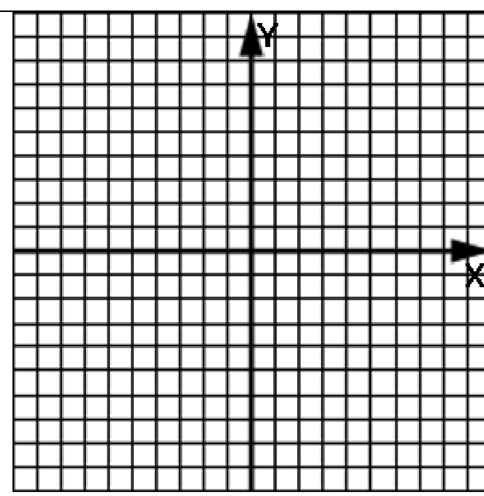
7. Identify the slope and y-intercept of the linear function f(x) = 8x + 9.

8. Identify the slope and y-intercept of the linear function y = -.5x – 18.

9. Graph the line given by y = 4x – 1.

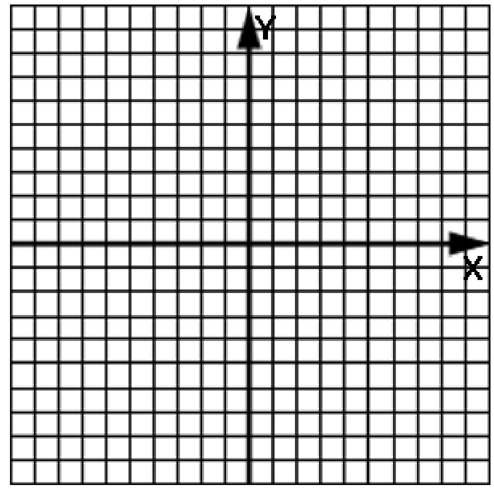


10. Graph the line given by y = mx + b where m = -3/8 and b = 6.



*11. Which quadrants does the line given by y = 4x – 9 touch?

12. Graph the line whose y-intercept is -5 and whose slope is -3.



13. Which quadrants does the line of problem 10 touch?

14. What is the slope and y-intercept of the linear function given by:

$y = -8 + 2x$

Unit 6: Lesson 04

Converting linear functions to y = mx + b form
Verifying solutions of linear equations

A shortcut:

When we solve an equation like y + 2= 3x for y we add -2 to both sides as follows:

$$y + \cancel{2} - \cancel{2} = 3x - 2$$
$$y = 3x - 2$$

Notice that the original +2 eventually shows up on the other side of the equation as -2. This leads to a new shortcut rule:

Any term of an equation can be **moved to the opposite side** of the equation if its **sign is reversed**.

Just remember this shortcut (sometimes called transposing) is **really adding or subtracting** a quantity to/from both sides.

Example 1: Solve y + 5 = x for y.

$$y + 5 = x$$
$$\boxed{y = x - 5}$$

Example 2: Solve y – 4x = 2 for y.

$$y - 4x = 2$$
$$\boxed{y = 4x + 2}$$

Notice that the linear equation y = mx + b has "*y* by itself on the left side." This means *y* "has been solved for."

Many times linear equations are encountered that are not in y = mx + b form. **Convert to y = mx + b** form by simply **solving for *y***.

Example 3: Convert 5x = 7 + 2y to y = mx + b form.

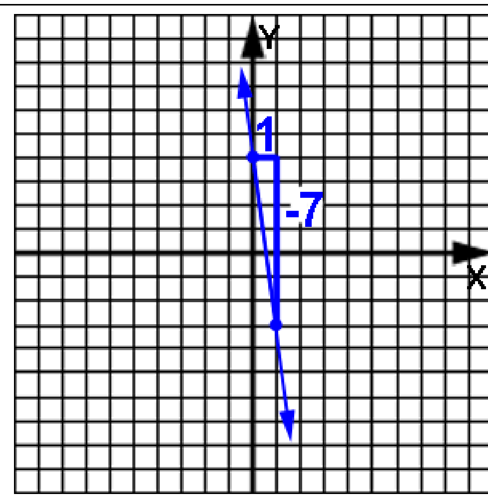
$$5x = 7 + 2y$$
$$-2y = -5x + 7$$
$$\frac{\cancel{-2}y}{\cancel{-2}} = \frac{-5x}{-2} + \frac{7}{-2}$$
$$\boxed{y = \frac{5}{2}x - \frac{7}{2}}$$

Example 4: Convert x + y – 11 = 0 to y = mx + b form.

$$x + y - 11 = 0$$
$$\boxed{y = -x + 11}$$

Example 5: Graph the linear function given by 7x + y – 4 = 0.

$$7x + y - 4 = 0$$
$$y = -7x + 4$$
$$y = mx + b$$
$$m = -7 \qquad b = 4$$



How can we know if any particular **point lies on a line**?

Obviously, we could plot the point, graph the line, and by a visual inspection, observe if the point is on the line.

> If the line and point are far away from each other, this technique works fine; however, what if they were very close? In that case it would be difficult to tell if the point was really on the line or not.

We need a better technique:

> **Substitute the coordinates for the point into the equation** for the line. If the equation is "**satisfied**", the point is on the line.

Example 6: Determine if the point (2, -5) is on the line given by:

$y + 3x - 7 = 0$

$y + 3x - 7 = 0$

$-5 + 3(2) - 7 = 0$

$-5 + 6 - 7 = 0$

$-1 - 7 = 0$

$-8 \neq 0$

$\boxed{\text{No, not on the line}}$

Example 7: Determine if the point (2, 1) satisfies this equation:

$y + 3x - 7 = 0$

$y + 3x - 7 = 0$

$1 + 3(2) - 7 = 0$

$1 + 6 - 7 = 0$

$7 - 7 = 0$

$0 = 0$ ✓

$\boxed{\text{yes it's satisfied}}$

Assignment:

1. Solve $4x + 3 = 2x$.

2. Solve $8p – 9q + p = 4$ for p.

3. Put $x + y + 2 = 0$ in slope-intercept form.

4. Put $4x – 9y = 11$ in $y = mx + b$ form.

5. Convert $(3/4)y + (1/2)x + 12 = 0$ to $y = mx + b$ form.

6. Put $x = y$ in slope intercept form.

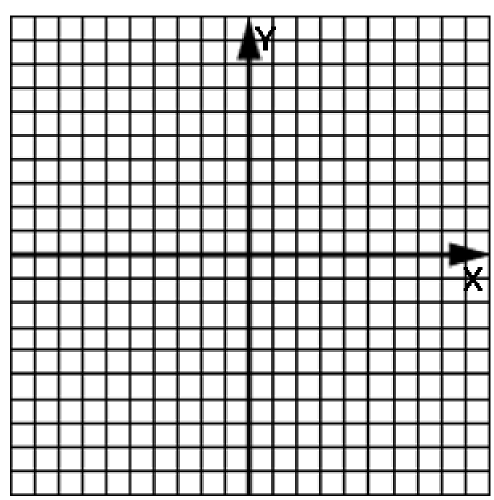
7. What is the slope of the line whose equation is y + x – 4 = 0?

8. What is the y-intercept of the line whose equation is 22x – 5y = 1?

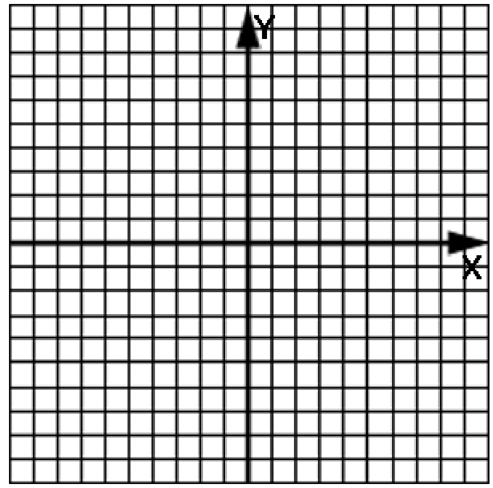
9. Where does 4 – 8x = f(x) cross the vertical axis?

10. If the points (3, -18) and (0, 6) are two points on a line, what is the y-intercept of the line?

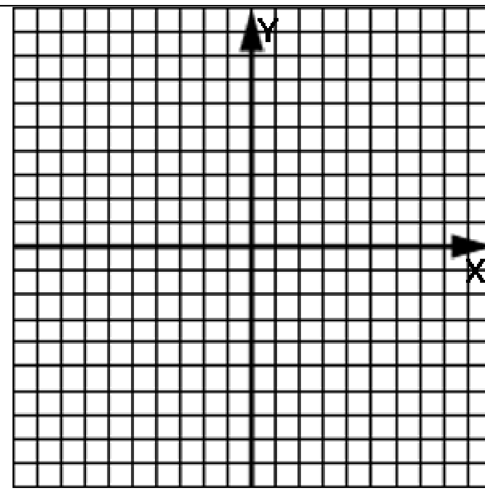
11. Graph the line given by the equation 4 = y – x + 1.



12. Graph the line given by 4x + 5y = 15



*13. Graph the line whose slope is -2 and whose y-intercept is four less than the y-intercept of the linear function given by y + x – 11 = 0.



14. Determine if the point (2, -5) is on the line given by: f(x) = 5x + 1

15. Is (6, 1) a solution to the equation 2x + 5y = 17?

16. Does the graph of the function given by $f(x) = 2x$ pass through the origin?

17. Does the point $(-8, 2)$ lie on the graph of $3x + 5y = -14$?

Unit 6: Lesson 05

Finding function rules given points in a chart
Special cases of linear functions (vert., horiz., b = 0)

Consider the case of being given several points on a line (we must be given at least two) and then finding the equation, y = mx + b, that produces the line that passes through the given points:

- Use any two of the points to produce the slope with the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Substitute this slope, and any point, for example, (x_1, y_1) into y = mx + b.
- This produces the equation $y_1 = mx_1 + b$ where only *b* is unknown. Solve for *b*.
- Use the *m* and *b* just produced to write the final equation of the line.

Example 1: All of the points in this chart lie on the same line. What is the equation of the line?

x	y
0	5
1	8
4	17
6	23

$(x_1, y_1) = (0,5)$ $(x_2, y_2) = (1,8)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-5}{1-0} = \frac{3}{1}$

$= 3$

$y = mx + b$

$y = 3x + b$ sub in (0,5) [or any other pt]

$5 = 3(0) + b$

$5 = b$

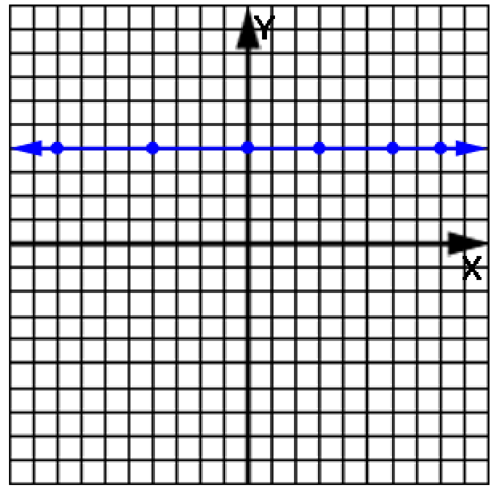
$y = mx + b$

$\boxed{y = 3x + 5}$

Example 2: Consider graphing the linear function y = mx + b where m = 0 and b = 4. Write the equation and simplify. Then fill in the chart below with several *x* values between -10 and 10, plot the points, and then connect with a line. How would you describe the graphed line?

$y = mx + b$
$y = 0x + 4$
$\boxed{y = 4}$

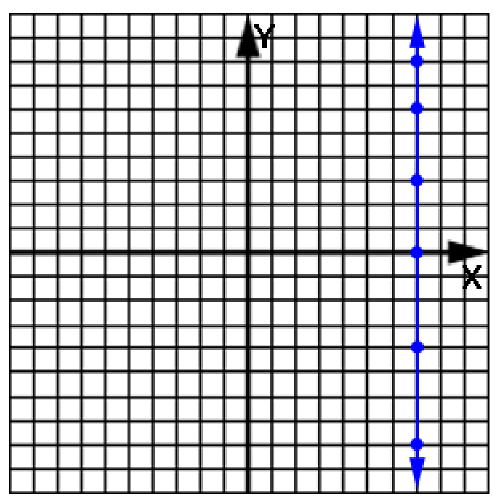
x	y = *4*
-8	*4*
-4	*4*
0	*4*
3	*4*
6	*4*
8	*4*



It's a horizontal line.

Example 3: Consider graphing a linear relation in which the *y* term is missing: for example, x = 7. Fill in the chart below in which we let *y* be the independent variable with several values between -10 and 10, plot the points, and then connect with a line. How would you describe the graphed line?

x = 7	y
7	*-8*
7	*-4*
7	*0*
7	*3*
7	*6*
7	*8*

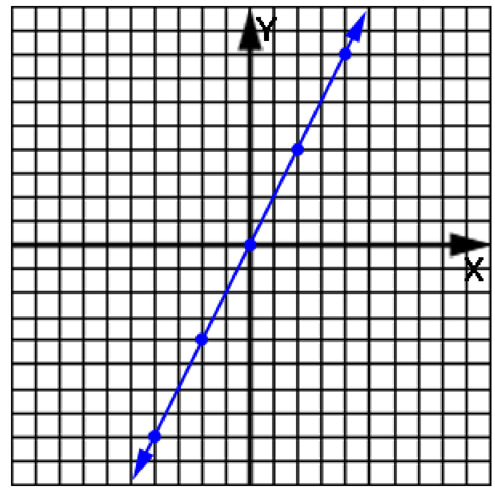


It's a vertical line.

Example 4: Consider the graph of y = mx + b when *b* is zero: for example, y = 2x + 0. Simplify this equation. Then fill in the chart below with several *x* values between -4 and 4, plot the points, and then connect with a line. Comment on any special thing you notice about the line.

$y = mx + b$
$y = 2x + 0$
$\boxed{y = 2x}$

x	y = *2x*
-4	*-8*
-2	*-4*
0	*0*
2	*4*
4	*8*



The line passes through the origin.

Summary of the last three examples:

An equation of the form **y = "a constant"**

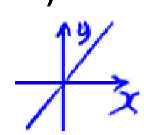
for example, y = 4, graphs as a **horizontal line**.

An equation of the form **x = "a constant"**

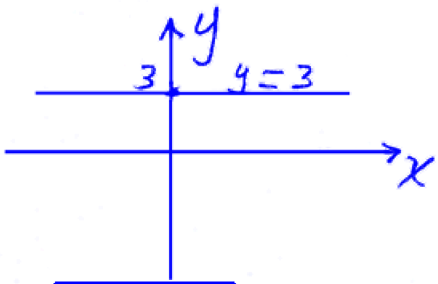
for example, x = 7, graphs as a **vertical line**.

An equation of the form **y = mx** (notice *b* is missing: it's 0)

for example, y = 2x, passes **through the origin**.

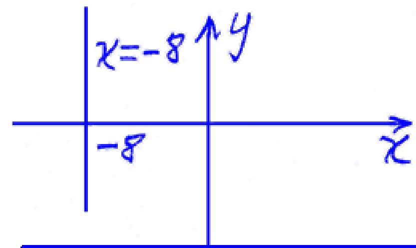


Example 5: Make a sketch of y = 3. What is its slope?



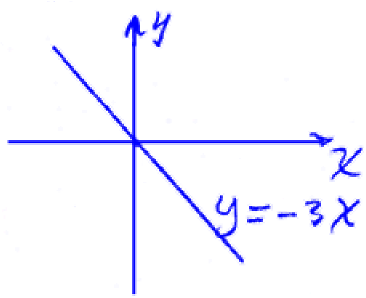
$\boxed{m = 0}$

Example 6: Make a sketch of x = -8. What is its slope?



$\boxed{\text{no slope; undefined}}$

Example 7: Make a sketch of y = -3x. What is its slope?

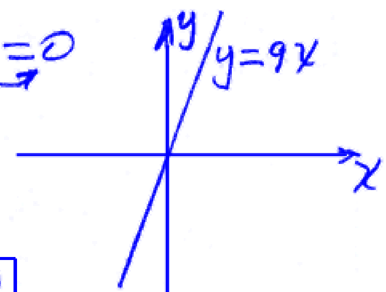


$\boxed{m = -3}$

Example 8: Make a sketch of y – 9x = 0. What is its slope?

$y - 9x = 0$

$y = 9x$



$\boxed{m = 9}$

Assignment:

1. All of the points in this chart lie on the same line. What is the equation of the line?

x	y
-1	2
5	-4
6	-5
10	-9

2. All of the points in this chart lie on the same line. What is the equation of the line?

x	y
-4	14
-1	8
4	-2
8	-10

3. All of the points in this chart lie on the same line. What is the equation of the line?

x	y
20	-20
30	-30
40	-40
100	-100

4. Sketch $y = -2$. What is its slope?

5. Sketch $f(x) = 12$. What is its slope?

6. Sketch $y = -4x$. What is its slope?

7. Sketch $f(x) = 5x$. What is its slope?

8. Sketch $x = 2$. What is its slope?

9. Sketch $x + 9 = 0$. What is its slope?

10. Sketch $y - 4x = 0$. What is its slope?

11. Sketch $y - 6 = 0$. What is its slope?

12. Sketch and label both y = 6 and x = -5 on the same coordinate system.

13. Sketch and label both y + 2 = 0 and 7 – x = 0 on the same coordinate system.

14. Sketch and label both y = 3x and y – 6x = 0 on the same coordinate system. Which has the steepest slope?

Unit 6: Lesson 06 Putting it all together: interpreting linear graphs

Another shortcut:

Consider the solution to 3y = 4x + 7: divide both sides by 3.

$3y = 4x + 7$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{4x}{3} + \frac{7}{3}$

$\boxed{y = \frac{4x}{3} + \frac{7}{3}}$

Notice that ***y* winds up by itself** and the **3 winds up in the denominator** of all the other terms.

As a short cut, in the future we will **just think of moving the 3 to the denominators of all the other terms**.

$3y = 4x + 7$

$\boxed{y = \frac{4x}{3} + \frac{7}{3}}$

Example 1: Solve 9y = 11x – 1 for *y*.

$9y = 11x - 1$

$\boxed{y = \frac{11x}{9} - \frac{1}{9}}$

Example 2: Solve 4x – 5y = 7 for *y*.

$4x - 5y = 7$

$-5y = -4x + 7$

$y = \frac{-4x}{-5} + \frac{7}{-5}$

$\boxed{y = \frac{4x}{5} - \frac{7}{5}}$

Example 3: Assuming that (x, 2) is a solution to the linear function y = x – 12, what is the value of *x*?

$y = x - 12$

$2 = x - 12$

$x - 12 = 2$

$x = 2 + 12$

$\boxed{x = 14}$

Example 4: Determine the value of *a* when the line passing through (4, a) and (-3, -2) has a slope of -2.

$(x_1, y_1) = (4, a) \quad (x_2, y_2) = (-3, -2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

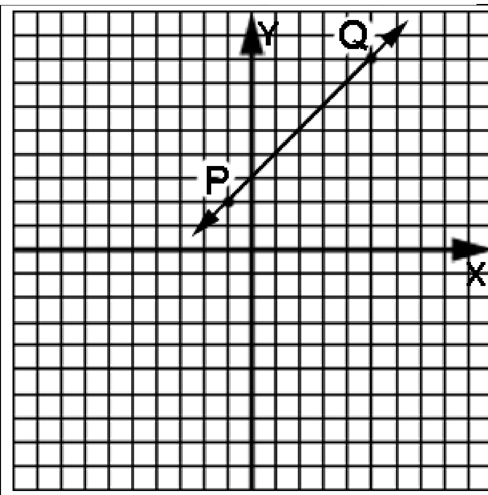
$\frac{-2}{1} = \frac{-2 - a}{-3 - 4}$ cross multiply

$-2 - a = -2(-3 - 4)$ $\qquad -a = 14 + 2$

$-2 - a = -2(-7)$ $\qquad -1a = 16$

$-2 - a = 14$ $\qquad a = \frac{16}{-1} = \boxed{-16}$

Use this drawing in Examples 5 – 8.



Example 5: Just from the graph, what appears to be the value of the function at x = 2?

5

Example 6: Just from the graph, when the value of the function is 7, what appears to be the corresponding x-value?

4

Example 7: What quadrants are touched by this line?

I, II, III

Example 8: What is the equation of a vertical line through point P?

y = -1

Assignment:

1. Solve $-2y = 8x + 7$ for *y*.

2. Solve $x + 19y - 14 = 0$ for *y*.

3. Solve $23x - 7z = p$ for *z*.

4. Solve $100y - 6 = x$ for *y*.

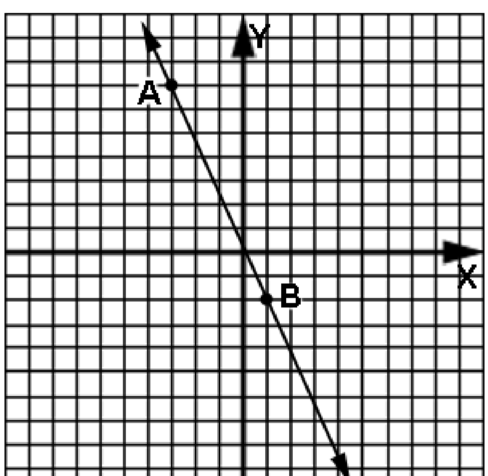
5. Assuming that $(x, -2)$ is a solution to the linear function $x = y - 12$, what is the value of *x*?

6. If $(11, y)$ is a solution to the linear function $4x = 5y - 12$, what is the value of *y*?

7. Determine the value of *a* when the line passing through (8, a) and (10, 0) has a slope of 7.

8. Determine the value of *b* when the line passing through (12, -7) and (b, -3) has a slope of -5.

Use the linear function shown here to answer questions 9-20.



9. Using the points A & B, what is the exact slope of the line?

10. Using points A & B, what is the exact equation of the line?

11. Just from the graph, what appears to be the y-intercept?	12. From the answer to problem 10, what is the exact y-intercept?
13. Which of the two answers in problems 11 & 12 is most trustworthy? Why?	14. Just from the graph, what appears to be the value of the function when x = -1?
15. Just from the graph, when the value of the function is 6, what appears to be the corresponding x value?	16. If the exact y-intercept is used, what quadrants are touched by the line?
17. Using the exact equation of problem 10, evaluate f(-4).	18. What is the equation of a vertical line through point A?
19. What is the equation of a horizontal line through point B?	20. Would the slope of a line drawn perpendicular to the one shown have a positive or negative slope?

Unit 6: Lesson 07

Comparing linear graphs with a graphing calculator
Evaluating linear functions with a calculator

In this lesson we will use the graphing calculator to simultaneously view the graphs of two lines. This will lead to an understanding of the effect of changing:

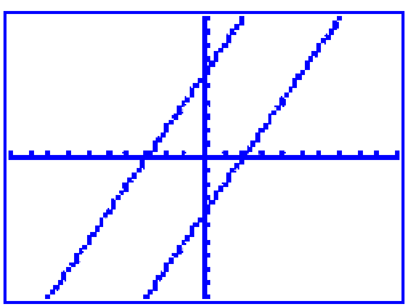
- the y-intercept
- the slope

See **Calculator Appendix E** and its associated video for how to graph and compare two lines with differing parameters.

Example 1: Use a graphing calculator to simultaneously graph these two linear functions:

Y1 = 2x – 4 and **Y2** = 2x + 6

Make a sketch of the calculator display and comment on what is different and what is the same about these two lines.

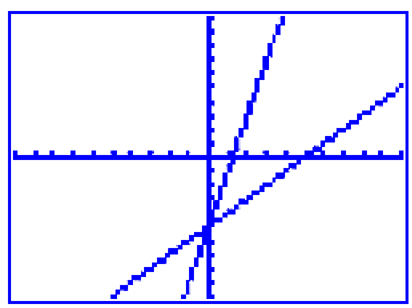


The two lines are parallel (because they have the same slope) but they cross the y-axis at different places because they have different y-intercept values.

Example 2: Use a graphing calculator to simultaneously graph these two linear functions:

Y1 = x – 5 and **Y2** = 4x – 5

Make a sketch of the calculator display and comment on what is different and what is the same about these two lines.



The two lines have the same y-intercept; however, Y2 is steeper because its slope, 4, is larger than the other slope, 1.

The graphing calculator can also be used to evaluate functions for a particular *x* value.

See **Calculator Appendix F** and its associated video for how to evaluate a function at a particular *x* value.

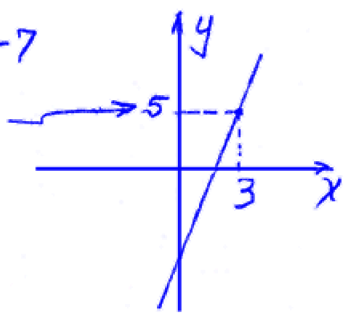
Example 3: Manually evaluate the function y = f(x) = 4x – 7 at x = 3. Make a sketch of the function labeling both x = 3 and f(3).

$f(x) = 4x - 7$

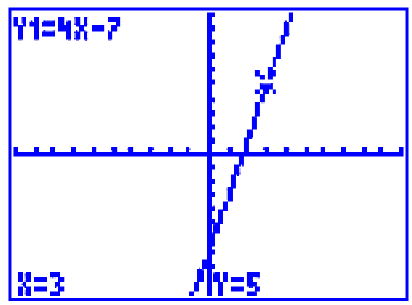
$f(3) = 4 \cdot 3 - 7$

$= 12 - 7$

$= \boxed{5}$



Example 4: Use a graphing calculator to graph **Y1** = 4x – 7. Use **2nd Calc | 1: Value** to evaluate this function at x = 3. Make a sketch of the calculator display including the answer it gives.



Assignment:

1. Use a graphing calculator to simultaneously graph these two linear functions:

Y1 = 2x + 1 and **Y2** = -2x +1

Make a sketch of the calculator display and comment on what is different and what is the same about these two lines.

2. Use a graphing calculator to simultaneously graph these two linear functions:

Y1 = x – 1 and **Y2** = x + 8

Make a sketch of the calculator display and comment on what is different and what is the same about these two lines.

3. Manually evaluate the function y = f(x) = -x + 5 at x = 2. Make a sketch of the function labeling both x = 2 and f(2).

4. Use a graphing calculator to graph **Y1** = -x + 5. Use **2nd Calc | 1: Value** to evaluate this function at x = 2. Make a sketch of the calculator display including the answer it gives.

5. Use a graphing calculator to simultaneously graph these two linear functions:

Y1 = -x + 4 and **Y2** = -2x + 4

Make a sketch of the calculator display and comment on what is different and what is the same about these two lines.

6. Use a graphing calculator to graph **Y1** = -6x – 2. Use **2nd Calc | 1: Value** to evaluate this function at x = -1. Make a sketch of the calculator display including the answer it gives.

Unit 6: Cumulative Review

1. Solve $3x – 5(x – 2) = x + 1$

2. Simplify $1/3 – 3/4 – 7/8$	3. Simplify $\lvert -5 – 3 + 1 \rvert – 6$
4. Solve $-3x – 2 < x + 1$ and show the answer both algebraically and on a number line.	5. Plot the point (6, -5) on a coordinate plane and locate its reflection across the y-axis.

6. If $h(x) = -3x^2 + x - 11$, find h(-2).

7. Draw a mapping for the relation represented by these ordered pairs:
{ (-2, 4), (4, 8), (-2, 1), (9, 6) }

8. Give the domain and range for the relation in problem 7. Is it a function? Why?

9. Draw a coordinate plane and label the quadrants. In which quadrant are both the x and y-coordinates negative?

10. A balloon initially filled with 3 Liters of air Is leaking air at the rate of .03 liters/minute. Write an expression for the *L*, the number of liters, at some later time, *t*, in minutes.

11. From problem 10, how many liters of air are left in the balloon after 15 minutes?

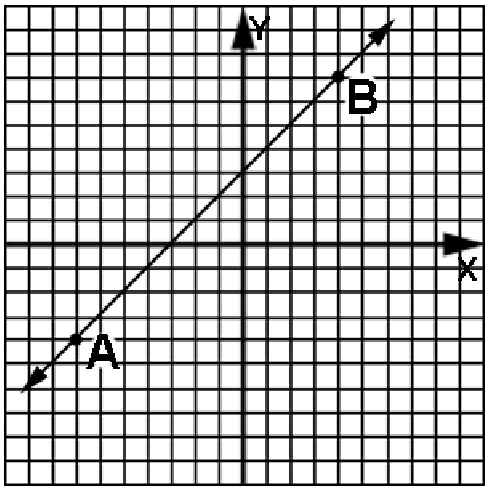
12. If the domain of $f(x) = 4x - 2$ is { -2, 1, 3, 4, 5, 6 }, what is the range?

13. If $g(x) = 8x - 2$ and $h(x) = x + 1$, find $2h(3) - 8g(7)$.

14. The width of a rectangle is 6 meters less than its length. If the perimeter is 132 meters, what are the dimensions of the rectangle?

Unit 6: Review

Use these two points and the line passing through them to work problems 1-11:



1. Using the points A and B, what is the exact slope of the line?

2. Using points A and B, what is the exact equation of the line?

3. Using the exact equation determined in problem 2, find f(-2).

4. Which quadrants does the line touch?

5. What is the y-intercept?

6. What is the equation of the horizontal line passing through point A?

7. What is the equation of the vertical line passing through point B?

8. Just from the appearance of the graph, what is an estimate for f(2.6)?

9. Just from the graph, when the value of the function is -2, what appears to be the corresponding value of *x*?

-

10. Would the slope of a line drawn from the origin to point A have a positive or negative slope?

11. Would the slope of a line drawn from (8, 0) to point B have a positive or negative slope?

12. Given the following ordered pairs as solutions to a linear function, find the function rule:

x	y
-2	0
0	8
2	16
6	32

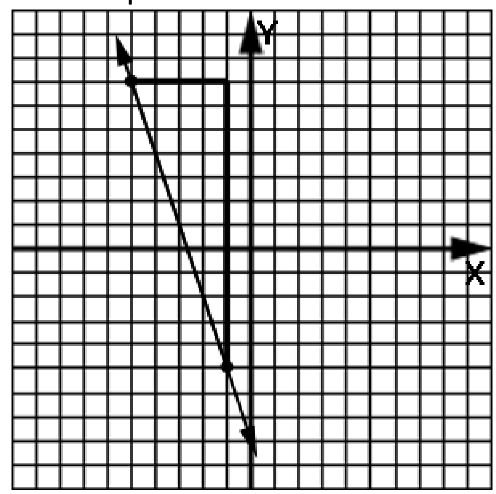
13. Sketch and give the equation of a horizontal line through (4, 5).

14. What is the equation of the vertical line connecting (-7, 9) and (-7, -6)?

15. Identify the slope and y-intercept of the linear function given by -4y + 2x = 11.

16. Find the slope of the line passing through (1, 2) and (10, 5).

17. Find the slope of the line shown here.



18. After losing power an airplane is descending along a straight path. Radar shows that the plane is losing altitude at the rate of 20 feet for every 80 feet the plane's shadow moves on the ground. Make a drawing of this situation and then find the slope of the line describing the glide path of the airplane.

Alg 1, Unit 7

More on Writing Linear Functions

Unit 7: Lesson 01

Writing the equation of a line given the slope and one other piece of information

In this unit we will write the equations of lines. Just as drawing a line is possible with a minimum of two points specified, to write its equation requires **at least two pieces of information**:

In this lesson, one piece of information will be the **slope** of the line. The second piece of information could be any of the following:

- a point
- a y-intercept
- an x-intercept (also called a root)

In the following examples, use the two given pieces of information about a line to find the equation of the line in **slope-intercept form** (y = mx + b).

Example 1: slope = 3; passes through the point (-8, 5)

$m = 3$

$y = mx + b$

$y = 3x + b$ sub in (-8, 5)

$5 = 3\cdot(-8) + b$

$5 + 24 = b$

$29 = b$

$y = mx + b$

$\boxed{y = 3x + 29}$

Example 2: m = -7; y-intercept = 4

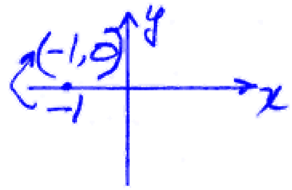
$m = -7 \quad b = 4$

$y = mx + b$

$\boxed{y = -7x + 4}$

Example 3: slope = 2; x-intercept = -1

$m = 2$



so x-intc = -1 really gives us the point (-1,0)

$y = mx + b$

$y = 2x + b$ (-1,0)

$0 = 2(-1) + b$

$2 = b$

$y = mx + b$

$\boxed{y = 2x + 2}$

Assignment: In the following problems, use the two given pieces of information about a line to find the equation of the line in **slope-intercept form** ($y = mx + b$).

1. slope = -3; passes through the points (5, 6)	2. slope = 5; y-intercept = 11
3. slope = 1; (7, 1) is on the line	4. slope = 1/3; x-intercept = -8
5. m = 10; the y-intercept is at the origin	6. slope = 4; (4, -3) is on the line

7. The line rises 3 for every 4 it runs and passes through the y-axis at -6.

8. The rate of change of *y* along a line with as x changes is 4/5. The line passes through (4, 1).

9. The line is horizontal and passes through (-2, -6).

10. Moving along a line results in it changing horizontally to the right 8 units for every 3 units it changes vertically upward. The line passes through (5, -7).

11. m = 4; x-intercept = 5

12. The ratio of rise to run is 11 and the line pass through (4, 5).

13. m = 1/8; passes through the origin	14. m = 6; b = 22
15. slope = -3/2; passes through the x-axis at x = 12	16. The line is horizontal and crosses the y-axis at y = 4.

Unit 7: Lesson 02 — Writing the equation of a line given two points / Writing the equations of horizontal & vertical lines

It is possible to write the equation of a line given **two points** on that line:

- Apply the two points to the slope formula to find the slope, *m.*
- Using the slope just found in y = mx + b, substitute in **either** point and solve for *b.*
- Use the *m* and *b* just found to write the equation.

(Notice that the last two steps were the technique of lesson 1.)

Example 1: Write the equation of the line passing through (4, -8) and (1, 5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-8)}{1-4} = \frac{5+8}{-3} = \frac{-13}{3}$$

$$y = mx + b$$

$$y = \frac{-13}{3}x + b \quad (1, 5)$$

$$5 = \frac{-13}{3}(1) + b$$

$$\frac{5}{1}\cdot\frac{3}{3} + \frac{13}{3} = b$$

$$\frac{28}{3} = b$$

$$y = mx + b$$

$$\boxed{y = \frac{-13}{3}x + \frac{28}{3}}$$

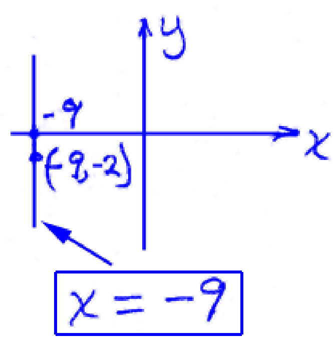
The equation of a vertical line passing through the point (a, c) is:

x = a Notice it uses the *a* part of (a, c).

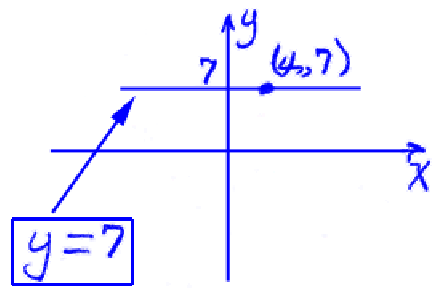
The equation of a horizontal line passing through (d, b) is:

y = b Notice it uses the *b* part of (d, b).

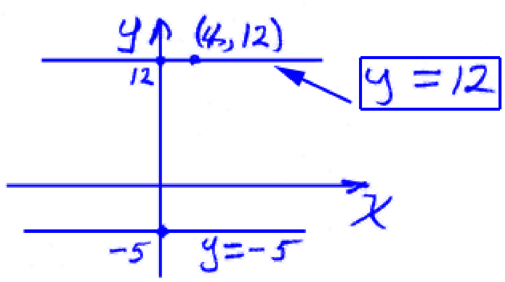
Example 2: Sketch the line parallel to the y-axis and passing through (-9, -2). Find its equation.



Example 3: Sketch the horizontal line that passes through (4, 7). Find its equation.



Example 4: Sketch the line passing through (4, 12) and parallel to the line given by y = -5, and then find its equation.



Assignment: Find the equation of all lines in slope-intercept form.

1. Find the equation of the line passing through (-9, 11) and (-1, 1).

2. What is the equation of the line connecting (4, 8) and (7,-2)?

3. What is the equation of the line that passes through the origin and whose x-intercept is 9?

4. Find the equation of the line that has an x-intercept of 18 and a y-intercept of -10.

5. Sketch the line parallel to the x-axis and passing through (-6, 8). Find its equation.

6. Sketch the vertical line that passes through (7, 0). Find its equation.

7. Sketch the line parallel to the y-axis and passing through (11, 22). Find its equation.

8. Sketch the line parallel to the x-axis and passing through (11, 22). Find its equation.

9. Sketch the line passing through (2, 3) and parallel to the line given by x = -2, and then find its equation.

10. Sketch the line passing through (-8, 8) and parallel to the line given by y = -1, and then find its equation.

11. Which of the following lines is a vertical line?

a. y = 3
b. x = 3
c. y = 3x

12. Which of the following lines is a horizontal line?

a. y = 3
b. x = 3
c. y = 3x

13. Which of the following lines passes through the origin?

a. y = 3
b. x = 3
c. y = 3x

14. Which of the following lines is not a function?

a. y = 3
b. x = 3
c. y = 3x

15. What is the equation of a line parallel to x = -8 and having an x-intercept of 4?

16. What is the equation of a line parallel to y = -2 and having an y-intercept of 36?

*17. Find the equation of the line connecting the point 4 units to the left of the origin on the x-axis with the point 5 units above the origin on the y-axis.

Unit 7: Lesson 03 — Perpendicular and parallel lines

An important relationship between two different lines relates to the **angle between the two lines**. In this lesson we will examine the relationship between parallel and perpendicular lines.

Parallel lines:

Slopes are equal. ($m_1 = m_2$)

Perpendicular lines:

Slopes are negative reciprocals of each other. ($m_1 = -1/m_2$)

Notice that if $m_1 = -1/m_2$ is cross-multiplied, the result is $\mathbf{m_1(m_2) = -1}$.

Example 1: Examine the two lines given by 2x – 8y = 7 and -x + 4y – 1 = 0 to determine if they are parallel, perpendicular or neither.

$2x - 8y = 7$
$-8y = -2x + 7$
$y = \frac{-2x}{-8} + \frac{17}{-8}$
$y = \frac{1}{4}x - \frac{17}{8}$
$m_1 = \frac{1}{4}$

$-x + 4y - 1 = 0$
$4y = x + 1$
$y = \frac{1}{4}x + \frac{1}{4}$
$m_2 = \frac{1}{4}$

$m_1 = m_2$ $\boxed{\text{parallel}}$

Example 2: Examine the two lines given by 3x – 2y = 7 and 6x + y = -8 to determine if they are parallel, perpendicular or neither.

$3x - 2y = 7$
$-2y = -3x + 7$
$y = \frac{-3x}{-2} + \frac{7}{-2}$
$y = \frac{3}{2}x - \frac{7}{2}$
$m_1 = 3/2$

$6x + y = -8$
$y = -6x - 8$
$m_2 = -6$

$\boxed{\text{Neither}}$

Example 3: Examine the two lines given by 4x – 12y = 2 and 6x +2y – 7 = 0 to determine if they are parallel, perpendicular or neither.

$4x - 12y = 2$
$-12y = -4x + 2$
$y = \frac{-4x}{-12} + \frac{2}{-12}$
$y = \frac{1}{3}x - \frac{1}{6}$
$m_1 = \frac{1}{3}$

$6x + 2y - 7 = 0$
$2y = -6x + 7$
$y = -\frac{6x}{2} + \frac{7}{2}$
$y = -3x + \frac{7}{2}$
$m_2 = -3$

$m_1 m_2 = \frac{1}{3}(-3) = -1 \rightarrow$ $\boxed{\text{perpendicular}}$

Example 4: Write the equation of a line that passes through (4, - 2) and is parallel to the line given by 4x + y = 11.

$4x + y = 11$
$y = -4x + 11$
$m_1 = -4$

$y = mx + b$
$y = -4x + b$ $(4, -2)$
$-2 = -4(4) + b$
$-2 + 16 = b$
$14 = b$

$y = mx + b$
$\boxed{y = -4x + 14}$

Example 5: Write the equation of a line that passes through (-1, 9) and is perpendicular to the line given by -x + 5y + 8 = 0.

$-x + 5y + 8 = 0$
$5y = x - 8$
$y = \frac{1}{5}x - \frac{8}{5}$
$m = \frac{1}{5}$
$m_\perp = -5$

$y = mx + b$
$y = -5x + b$ $(-1, 9)$
$9 = -5(-1) + b$
$9 = 5 + b$
$9 - 5 = b$
$4 = b$

$y = mx + b$
$\boxed{y = -5x + 4}$

See **Enrichment Topic C** for how to apply the equations of lines in two dimensions to the solutions of **two-dimensional inequalities**.

Assignment: In problems 1-5, determine if the two lines given by the pair of equations are parallel, perpendicular, or neither.

1. $x + y = 7$; $4x – 6 = 4y$

2. $3/4\ y = x – 6$; $x + 4/3y = 1$

3. $x = y$; $3 + 6y = 6x$

4. $y = 2x - 1$; $10x - 5y = 2$

5. $(1/5)x - y + 7 = 0$; $3y = -15x + 11$

6. Write the equation of a line that passes through (5, -2) and is parallel to the line given by $-3x + y = 10$.

7. Write the equation of a line that passes through (5, -2) and is perpendicular to the line given by $-x - y = 22$.

8. Write the equation of a line that has a y-intercept of 5 and is parallel to the line given by $-2x + 10y = 7$.

9. Write the equation of a line that has an x-intercept of -4 and is perpendicular to the line given by $x + 2y = 1$.

*10. Write the equation of a line that passes through (8, -2) and is parallel to the line connecting (4, 5) and (6, 15).

---

11. If line #1 has a slope of m_1 and line #2 has a slope of m_2, what relationship must exist between lines #1 and #2 if $m_1(m_2) = -1$?

Unit 7: Lesson 04 — Linear function word problems
Calculator tables

In the process of solving a word problem it is often useful to make a chart of a few specific instances of the independent and dependent variables. Typically, a **pattern** will be noticed from which the function can be written.

Example 1: Answer the following questions from this scenario:
Farmer Jones sells his prize winning apples in a special decorative box. He charges \$1.50 for the box plus 42 cents for each apple. Write a function that expresses the total cost for any given number of apples purchased.

Choose an independent variable. What is its meaning? *x, number of apples*

Choose a dependent variable. What is its meaning? *$y = f(x)$, total cost*

Make a chart for the two variables and compute the cost for 1, 2, 3, and 4 apples.

x, apples	*$y = f(x)$, total cost*
1	$y = (.42)1 + 1.50 = 1.92$
2	$y = (.42)2 + 1.50 = 2.34$
3	$y = (.42)3 + 1.50 = 2.76$
4	$y = (.42)4 + 1.50 = 3.18$

Write out the linear function: $y = f(x) = .42x + 1.50$

What is the slope and what does it represent? *.42, the cost of one apple.*

What is the y-intercept and what does it represent? *1.50, the cost of the box*

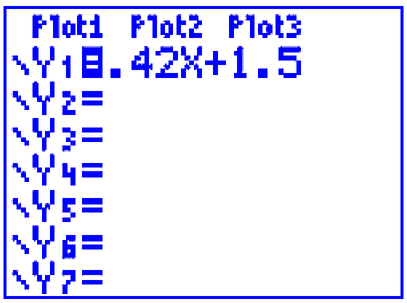
What would be the total cost of 8 apples? $f(8) = (.42)8 + 1.50 = \$4.86$

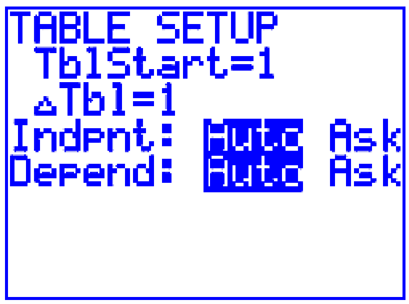
What would be the new function if the price of the box increased to \$2.00 and the price of each apple went up to 51 cents? $y = f(x) = .51x + 2.00$

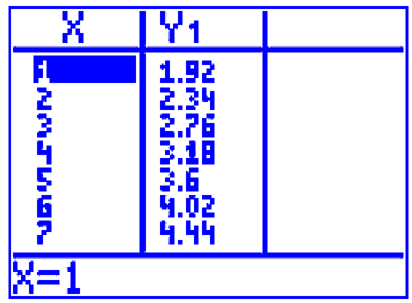
The chart of Example 1 can easily be created on a graphing calculator. On the calculator it is not called a chart, rather a **table**.

See **Calculator Appendix G** (and an associated video) for how to set-up a table and for how to actually produce the table for a given function.

Example 2: Create the table of example 1 on a graphing calculator. Write out the table settings as well as the table itself.







Assignment:

1. Billy Bob Matherstein gets into a lot of trouble in his Alg I class. In fact, he now has 132 minutes of detention assigned. Miss Informed, his math teacher, has made a deal with Billy Bob. For every problem he gets right on the next test, she will deduct 5 minutes of detention. Write a function that expresses the amount of detention he has in terms of the number of problems he gets right.

Choose an independent variable. What is its meaning?

Choose a dependent variable. What is its meaning?

Make a chart for the two variables and compute the number of minutes for 1, 2, 3, and 4 correct problems.

Write out the linear function indicated by this table:

What is the slope and what does it represent?

What is the y-intercept and what does it represent?

What would be his detention time if he gets 6 problems right?

What would be the new function if he initially had 200 min of detention but gets rewarded with 10 min off for each correct problem?

2. Mr. Appleton grows apples (what else?) and hires teenagers in the summer to harvest his apples. Each person receives $10 just for showing up for work plus 8 cents for each apple he or she picks. Write a function that expresses the amount of money a person earns in terms of the number of apples picked.

Choose an independent variable. What is its meaning?

Choose a dependent variable. What is its meaning?

Make a chart for the two variables and compute the money earned by a person picking 30, 50, 70, and 90 apples.

Write out the linear function indicated by this table:

What is the slope and what does it represent?

What is the y-intercept and what does it represent?

What would be the pay if a person picks 300 apples?

What would be the new function if only $5 is paid for showing up but the price is now 10 cents per apple?

3. Produce the chart of problem 1 on a graphing calculator and show the table set-up here as well as the table.

4. Produce the chart of problem 2 on a graphing calculator and show the table set-up here as well as the table.

Unit 7: Cumulative Review

1. Simplify 3(5 – 17)/2 – 1.

2. Simplify 3/8 – 5/11 + 2.

3. Simplify |5 – 8|(5 – 8)/(-2).

4. Evaluate (3x – 2y + 9)x when x = 5 and y = -1.

5. Solve 8(x + 1) – (2 + x) = 4x + 1.

6. Present the solution to $4x + 2 < 6x - 20$ both algebraically and graphically.

7. Use unit multipliers to convert 36 miles to meters.

8. 8 is what percent of 300?

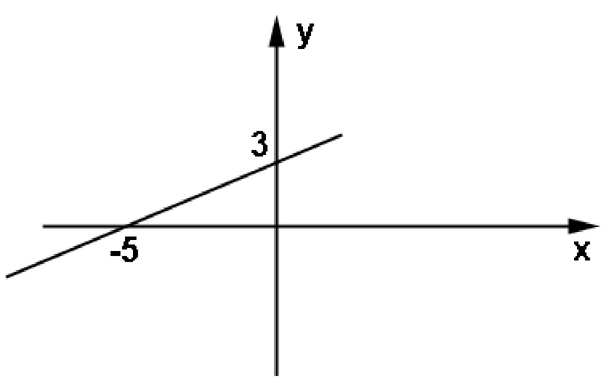
9. What is the slope of a line passing through (5, -8) and (-2, -18)?

10. If the base of a triangle is 8 more than either of the other two equal sides, how long is the base if the perimeter of the triangle is 98?

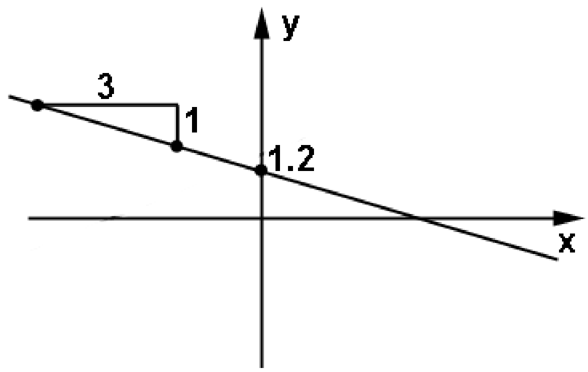
11. What is the range of $f(x) = 11x – 12$ when its domain is { -8, -2, 0, 6, 10 }?

12. What is the equation of a line parallel to the x-axis and passing through (5, 8)?	13. What is the equation of a line parallel to the y-axis and passing through (5, 8)?

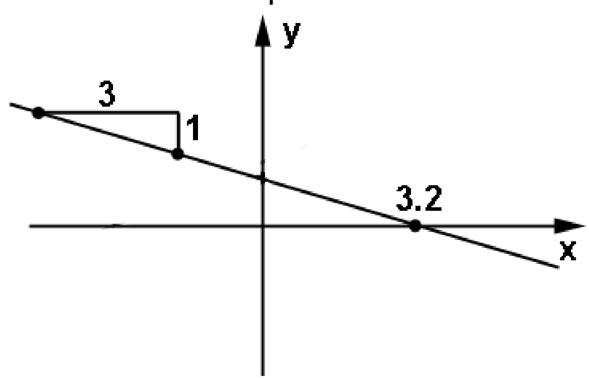
14. What is the equation of the line shown here?



15. What is the equation of the line shown here?



16. What is the equation of the line shown here?



17. Which of the following sets of ordered pairs are **not** functions?

A. { (4, 22), (9, -6), (-4, 22), (1, 1) }
B. { (-1, 6), (1, 6), (6, -6), (-1, 1) }
C. { (2, 7), (7, 7), (5, 7), (-2, 7), (8, 7) }
D. { (4, 5), (4, 6), (4, 7), (4, 8) }

Unit 7: Review

1. What is the slope of a line that is parallel to the line given by $-8x - 7y = 14$?

2. What is the slope of a line that is perpendicular to the y-axis?

3. What is the equation of the line that has slope 4 and passes through (2, -6)?

4. What is the equation of the line having a slope of -2/3 and passing through the y-axis at -5?

5. What is the equation of the line in which the ratio of the rise to the run is 4/5 and whose y-intercept is 2?

6. What is the equation of the line that passes through (-2, 2) and is parallel to the line given by $x = -7$?

7. What is the equation of the line passing through (4, 10) and (-4, 8)?

8. What is the equation of the line passing through the origin and perpendicular to the line joining the two points in problem 7?

9. Which of the following line(s) passes through the y-axis at y = 5?

a. $y + x + 5 = 0$
b. $y = 5x + 1 = 0$
c. $y – x – 5 = 0$
d. $5y + x + 1 = 0$
e. $2y + 8x – 10 = 0$

10. What is the equation of the line passing through (1, 2) and perpendicular to the y-axis?

11. Which of the following relations is not a function?

a. $y = 3$
b. $x = -9$
c. $y = x$
d. $y = -x$
e. $y/3 = x/2 + 7$

12. Examine the two lines given by $4y + 2x - 6 = 0$ and $-x - 2y + 11 = 0$ and determine if they are parallel, perpendicular or neither.

---

13. Examine the two lines given by $4y + 2x - 6 = 0$ and $2x - y + 11 = 0$ and determine if they are parallel, perpendicular or neither.

---

14. Write the equation of a line that passes through (4, 8) and is perpendicular to the line given by $12x - y = 19$.

15. The Old Tyme Ice Cream Parlor sells a bowl of their homemade ice cream for \$3.50. They have many different toppings, each priced at 25 cents. It is possible to get no toppings or as many as desired. Consider the function that gives the total price of a bowl of ice cream with toppings in terms of the number of toppings.

Choose an independent variable. What is its meaning?

Choose a dependent variable. What is its meaning?

Make a chart for the two variables and compute the cost for 0, 2, 4, and 6 toppings.

Write out the linear function indicated by this table:

What is the slope and what does it represent?

What is the y-intercept and what does it represent?

What would be the price for a bowl of ice cream with 5 toppings?

What would be the new function be if the price of a plain bowl is \$3.75 and only 20 cents per topping?

16. Using a graphing calculator, enter the function of problem 15 and then produce a table on the calcultor corresponding to the one described in problem 15. Show the table settings as well as the table the calculator produces.

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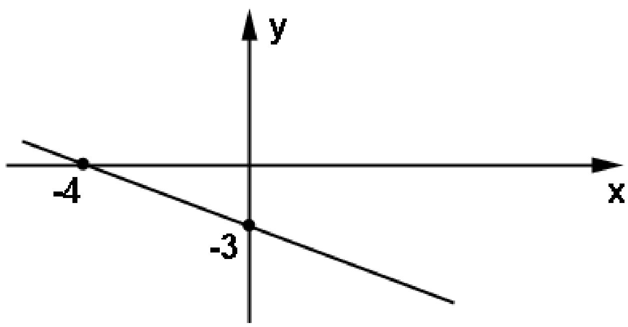
17. Find the equation of this line.



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18. Which quadrants are touched by the line in problem 17?

Alg 1, Unit 8

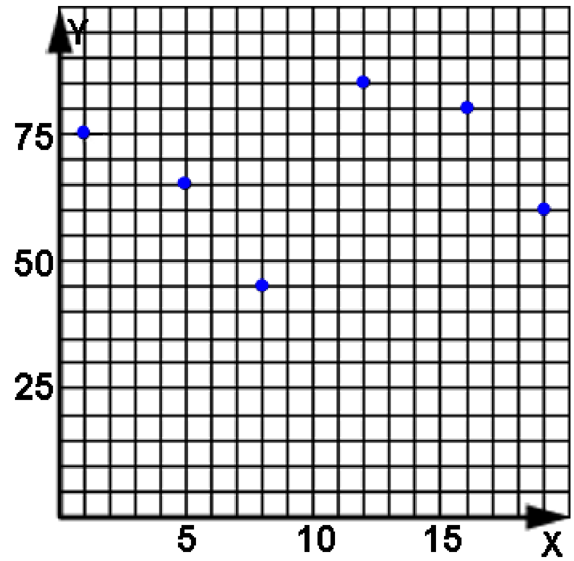
Lines of Best Fit, Correlation
Interpreting data

Unit 8: Lesson 01 Manual scatter plot, correlation

Example 1: Plot the data from the following table of randomly selected test grades. The *x* column is for the test number and the *y* column is the grade made on that test.

Comment on any trend or pattern you see in the data. If a trend is observed, draw a line of "best-fit."

x, Test #	y, grade
1	75
5	65
8	45
12	85
16	80
19	60

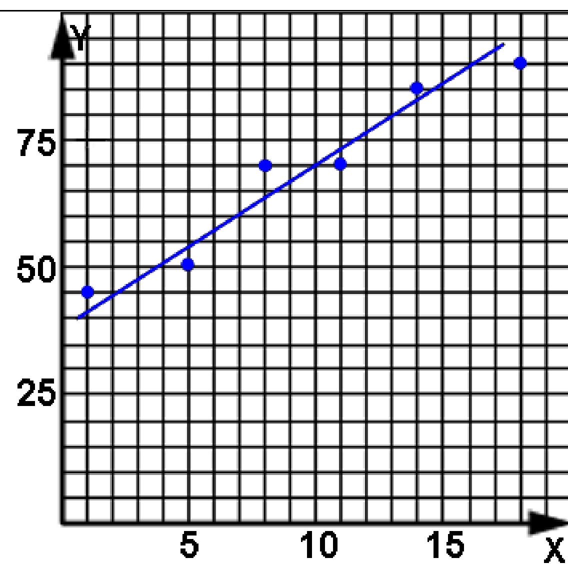


No pattern is observed. The grades are not steadily going up or down. Rather they seem to be randomly scattered.

Example 2: Plot the data from the following table of randomly selected test grades. The *x* column is for the test number and the *y* column is the grade made on that test.

Comment on any trend or pattern you see in the data. If a trend is observed, draw a "line of best-fit."

x, Test #	y, grade
1	45
5	50
8	70
11	70
14	85
18	90

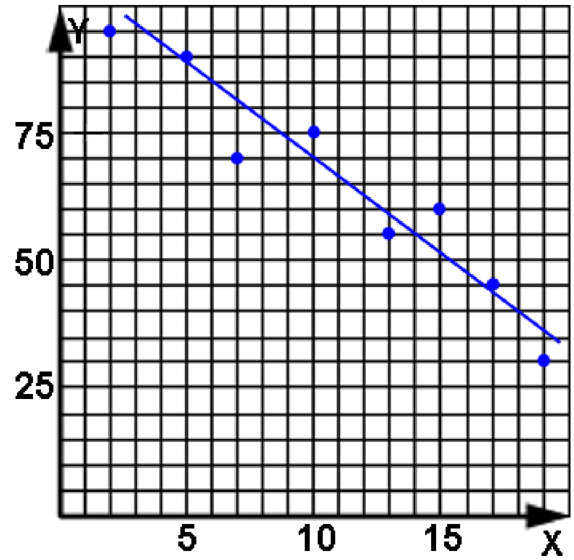


The grades seem to be steadily increasing with each new test.

Example 3: Plot the data from the following table of randomly selected test grades. The *x* column is for the test number and the *y* column is the grade made on that test.

Comment on any trend or pattern you see in the data. If a trend is observed, draw a "line of best-fit."

x, Test #	y, grade
2	95
5	90
7	70
10	75
13	55
15	60
17	45
19	30



The grades seem to be steadily decreasing with each new test.

Positive correlation:

When the "line of best-fit" has a **positive slope** as was seen in Example 2, it can be said that the data plot has **positive correlation** (also called a **positive trend**).

Negative correlation:

When the "line of best-fit" has a **negative slope** as was seen in Example 3, it can be said that the data plot has **negative correlation** (also called a **negative trend**).

No correlation:

When there is no line of best-fit and the data seems to be **randomly scattered** as was seen on Example 1, it can be said that the data plot has **no correlation**.

Example 4: Identify the data plots in the three previous examples as having either positive, negative, or no correlation.

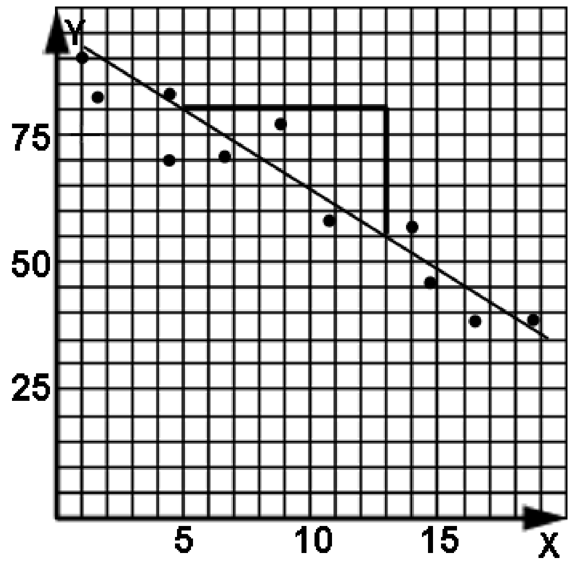
Example 1 _____*no correlation*___________

Example 2 _____*positive correlation* _____

Example 3 _____*negative correlation* _____

Example 5: From this scatter plot and line of best-fit, find the equation of the line of best-fit. (Use the rise and run indicated in the picture to find the slope.)

What is the correlation of this scatter plot?



negative correlation

$m = \frac{rise}{run} = \frac{-25}{8}$

(5, 80)

$y = mx + b$

$y = -\frac{25}{8}x + b$, (5, 80)

$80 = -\frac{25}{8}(5) + b$

$80 + \frac{125}{8} = b$

$\frac{80}{1} \cdot \frac{8}{8} + \frac{125}{8} = b$

$\frac{765}{8} = b$

$y = mx + b$

$\boxed{y = -\frac{25}{8}x + \frac{765}{8}}$

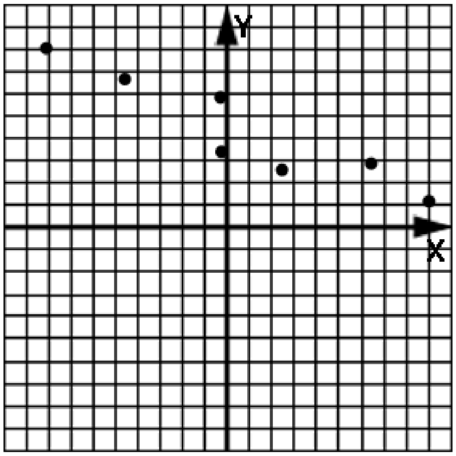
Example 6: From the line of best-fit developed in Example 5, what would be the expected grade on test 22?

$y = F(x) = \frac{-25}{8}x + \frac{765}{8}$

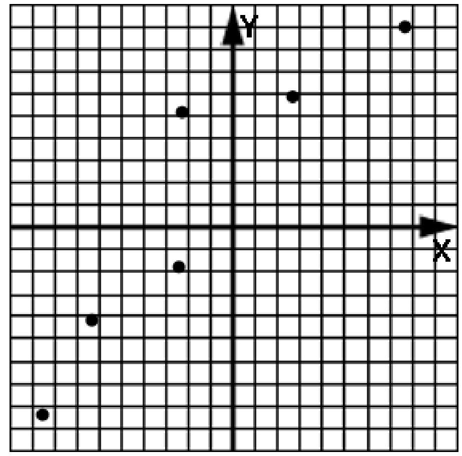
$F(22) = \frac{-25}{8}(22) + \frac{765}{8} = \frac{215}{8} = \boxed{26.875}$

Assignment: In problems 1-6 determine if there is positive, negative, or no correlation of the data. Draw a line of best-fit (if possible).

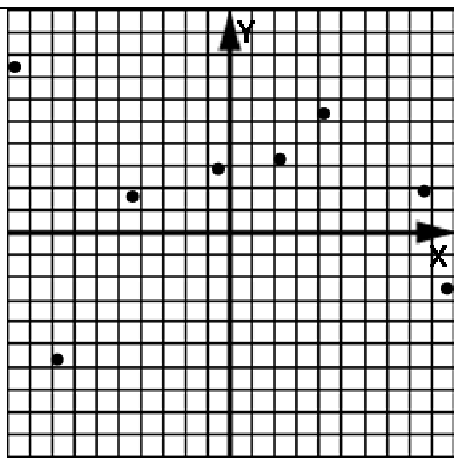
1.



2.



3.



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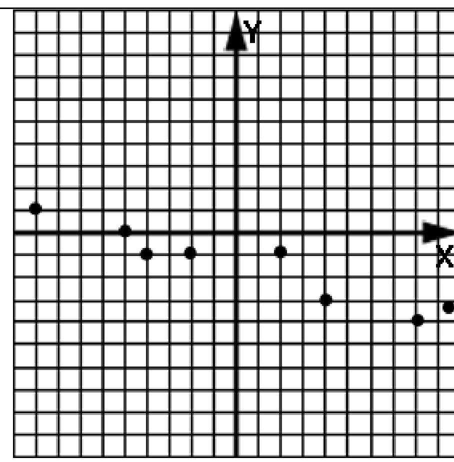
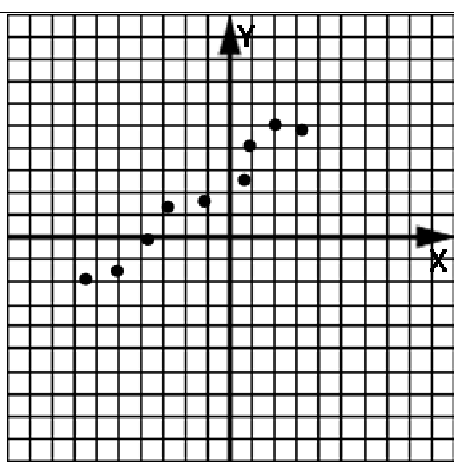
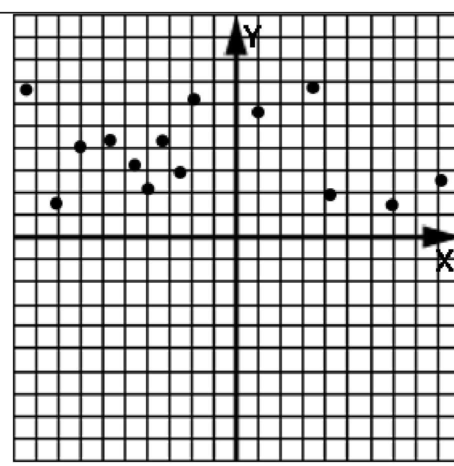


5.

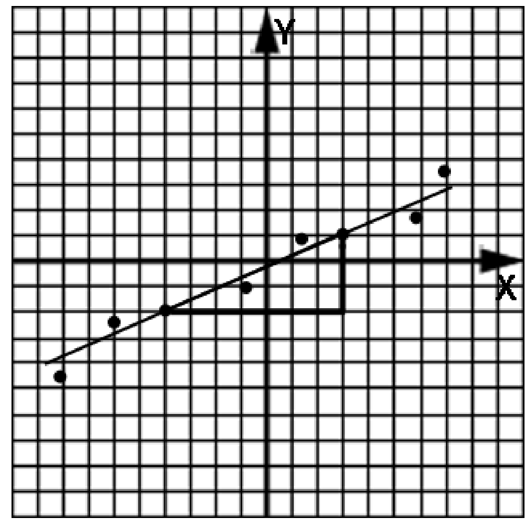


6.



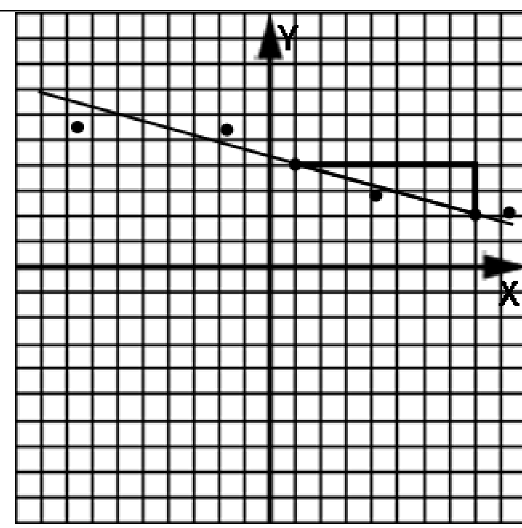
7. From this scatter plot and line of best-fit, find the equation of the line of best-fit. (Use the rise and run indicated in the picture to find the slope.)

What is the correlation of this scatter plot?

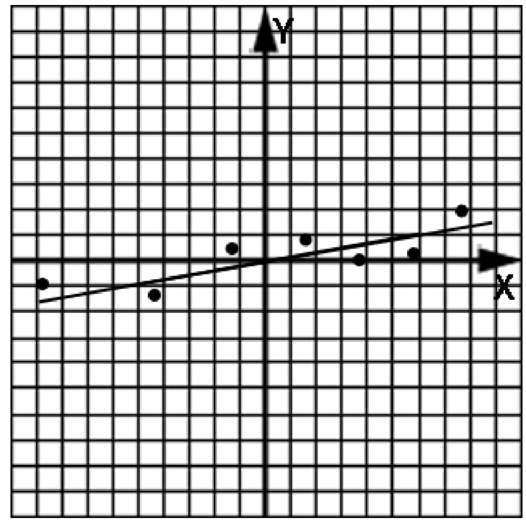


8. From this scatter plot and line of best-fit, find the equation of the line of best-fit. (Use the rise and run indicated in the picture to find the slope.)

What is the correlation of this scatter plot?

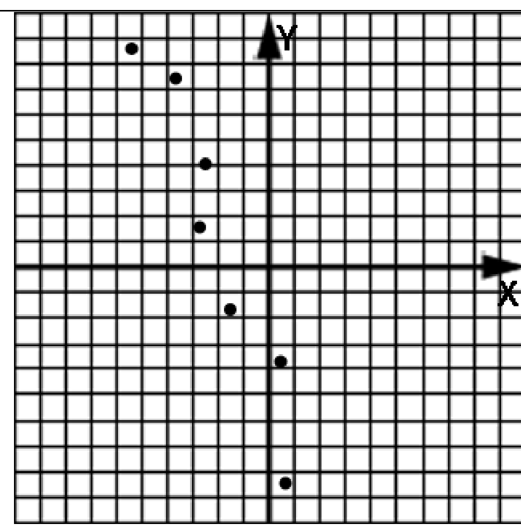


9. From this scatter plot and line of best-fit, find the equation of the line of best-fit. (Find two points through which the line passes and use the rise and run from those two points to find the slope.)



What is the correlation of this scatter plot?

10. Draw a line of best-fit for this scatter plot and then find the equation of the line.



What is the correlation of this scatter plot?

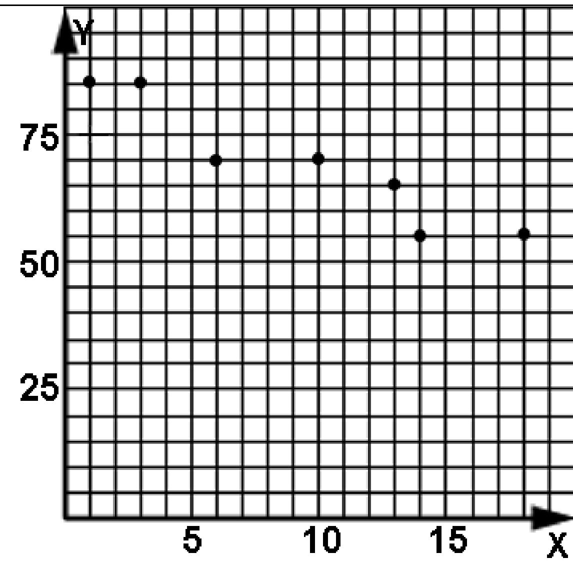
11. From the equation of the linear function in problem 9, find the value of f(12).

12. From the equation of the linear function in problem 10, find the value of f(-20).

13. Plot the data from the following table of randomly selected test grades. The *x* column is for the test number and the *y* column is the grade made on that test.

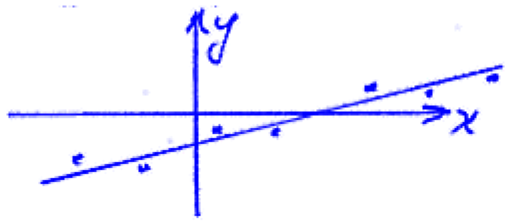
Comment on any trend or pattern you see in the data. If a trend is observed, draw a line of "best-fit" and then find the equation of the line.

x, Test #	**y, grade**
1	85
3	85
6	70
10	70
13	65
14	55
18	55



Unit 8: Lesson 02 Scatter plots and linear regression on the calculator

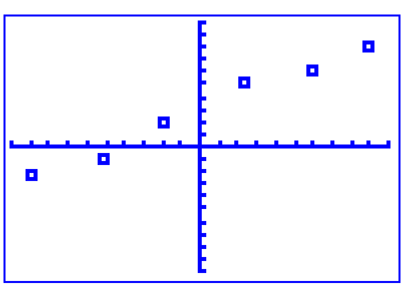
Data coming from experiments is typically not exact. Suppose we have the following (x, y) data points that don't exactly line up in a straight line. What line (y = mx + b) would best-fit these points?



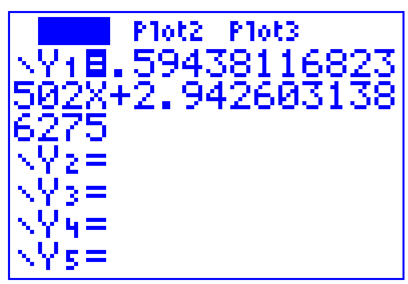
This very complex problem is easily and **exactly** solved on a graphing calculator. See Calculator **Appendix H** for how to enter the points and produce a scatter plot. **Appendix I** shows how to produce a linear regression (best-fit line) with these points.

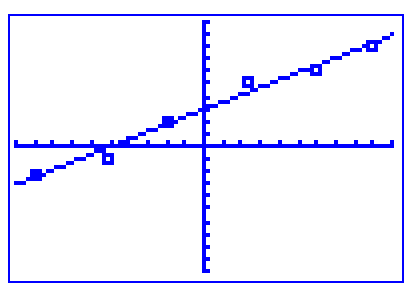
Example 1: Enter the following points into the calculator using **Stat | Edit,** enable a "scatter-plot" with **2nd | Stat Plot**, and then **Graph**.

x	y
–9.0	–2.1
–5.2	–1.0
–2.0	2.04
2.3	5.0
6.0	6.11
8.9	8.2



Example 2: Perform a linear regression on these same points and give the equation of the resulting line.





Assignment: In each of the following problems, perform a linear regression on the provided data and show the equation of the best-fit line along with a sketch of the line and scatter-plot.

1.

x	y
–4.0	9.0
–1.1	4.2
2.0	–0.5
1.9	–2.0
6.0	–5.0
5.8	–7.0

2.

x	y
–70	–40
–29	–52
28	–32
60	–30
89	–25

3.

x	**y**
–8.0	4.0
–1.0	5.2
2.5	6.4
7.0	7.3

4.

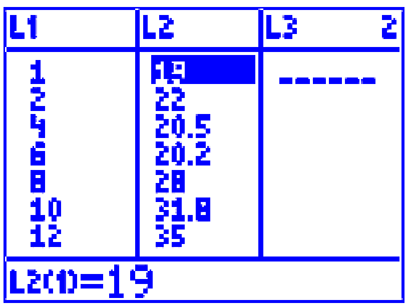
x	**y**
3	7
9	–7

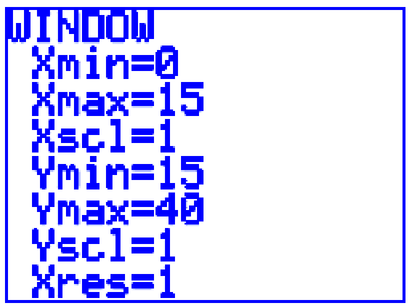
Unit 8: Lesson 03 Interpretation of linear data using a graphing calculator

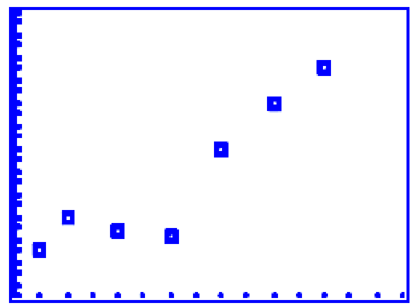
Example 1: Use the table to the right to produce a scatter plot on a graphing calculator (**Calculator Appendix H**). In the table, the left column (*x*) is for a particular year while the corresponding number in the right column (*y*) is the amount of rainfall (in inches) for that particular year.

x (year)	y (inches of rain)
1	19
2	22
4	20.5
6	20.2
8	28
10	31.8
12	35

Make a sketch of the scatter plot produced by the calculator.







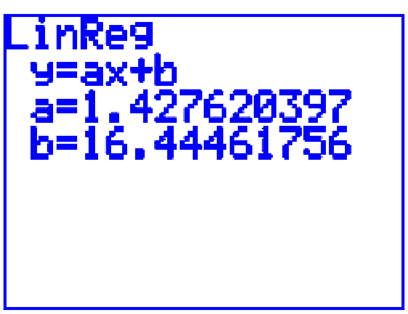
Example 2: What is the independent variable of Example 1?

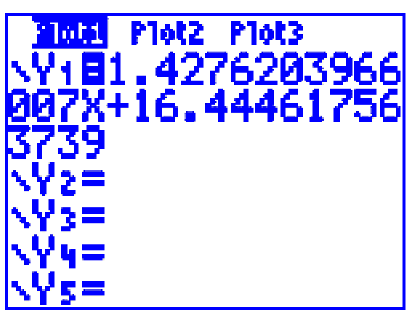
x (years)

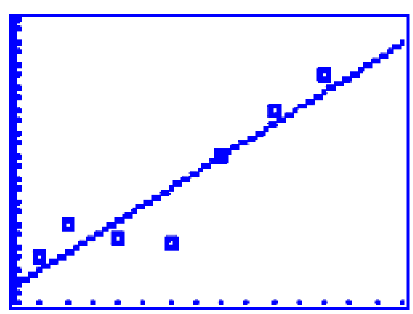
Example 3: What is the dependent variable of Example 1?

y (inches of rain)

Example 4: Perform a linear regression on the scatter plot of Example 1 (See **Calculator Appendix I**). Make a sketch of the calculator display.







Example 5: What is the equation of the line of best-fit?

$y = 1.427620397x + 16.44461756$

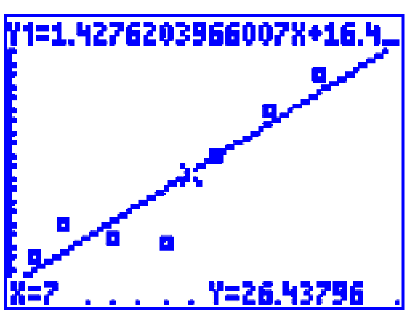
Example 6: What is the correlation of this data (positive, negative, or none)? Justify your answer.

positive correlation… because the line of best-fit has a positive slope.

Example 7: What is the slope of the line of best-fit? What is the y-intercept? Give both rounded to 2 decimal places.

slope = m = 1.43
y-intercept = b = 16.44

Example 8: Based on the line of best-fit, what would be a good estimate for the rainfall (rounded to the nearest whole number) that would be expected in year 7. Use the calculator to evaluate the linear function of best-fit at x = 7. (See **Calculator Appendix F** and an associated video for how to evaluate a function at a particular *x* value.)



26 inches of rain

Example 9: What is the **rate of increase** of rainfall and what are its units?

The rate of increase of rainfall is the slope = 1.43 inches/year

Example 10: Using a rainfall linear function built from the slope and intercept of Example 7, in what year would there be 33 inches of rain? (Do this problem algebraically, not graphically on the calculator.)

$$y = 1.43x + 16.44$$

$$33 = 1.43x + 16.44$$

$$33 - 16.44 = 1.43x$$

$$16.56 = 1.43x$$

$$\frac{16.56}{1.43} = x \; ; \; \boxed{x = 11.58 \text{ yr}}$$

See **Enrichment Topic I** for an extension of the ideas presented in the three lessons of this unit: statistics.

See **Calculator Appendix P** for how to do statistics on a graphing calculator.

Assignment:

1. Use the table to the right to produce a scatter plot on a graphing calculator. In the table, the left column (*x*) is for the time in minutes while the corresponding number in the right column (*y*) is the altitude in feet of an airplane at that time.

x (time, sec.)	**y (altitude, ft.)**
0	3,000
4	2,800
10	2,200
15	1,700
18	1,200
23	1,000
27	800

Make a sketch of the scatter plot produced by the calculator.

2. What is the dependent variable of problem 1?

3. What is the independent variable of problem 1?

4. Perform a linear regression on the scatter plot of problem 1. Make a sketch of the calculator display.

5. What is the equation of the line of best-fit in slope-intercept form?

6. What is the correlation of this data (positive, negative, or none)? Justify your answer.

7. What is the slope of the line of best-fit? What is the y-intercept? Give both rounded to 2 decimal places.

8. Based on the line of best-fit, what would be a good estimate for the altitude of the airplane (rounded to the nearest whole number) that would be expected at time x = 12 sec. Use the calculator to evaluate the linear function of best-fit at x = 12.

9. What is the rate of descent of the airplane? What are the units of the rate?

10. Using an altitude linear function built from the slope and intercept of problem 7, at what time would the altitude be 500 ft? (Do this problem algebraically, not graphically on the calcultor.)

When this lesson is finished, proceed to the cumulative review.

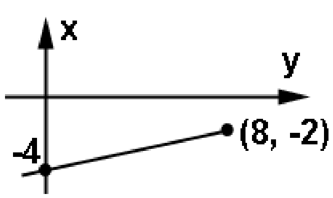
Unit 8: Cumulative Review

1. From the phrase, “8 less than 12 times the height”, define a variable and then write the expression algebraically.

2. Use unit multipliers to convert 8 inches into miles. (5280 ft = 1 mile)

3. Solve 3(x – 4) = 2(x + 1).

4. Find the equation of this line:



5. What is the equation of the line passing through (11,2) and (- 9, 6)?

6. What is the equation of the line passing through the origin and perpendicular to the line joining the two points of problem 5?

7. What is the equation of the line having slope = -2/3 and passing through the point (-6, 11)?

8. What is the slope of the line given by $8x - 7y + 4 = 0$?

9. Given the following ordered pairs a solutions to a linear function, find the function rule.

x	**y = f(x)**
0	-2
3	22
5	38
6	46

10. An orange tree currently has 30 pounds of ripe oranges. If another two pounds ripen every day, what function describes how many pounds of ripe oranges there will be several days later?

11. After how many days will there be 42 pounds of ripe oranges on the tree?	12. How many ripe oranges will there be after 10 more days?

13. A full can of soft drink straight out of a refrigerator (35 degrees) is tossed into a large swimming pool that maintains a constant temperature of 82 degrees. Sketch a graph of the temperature of the can as a function of time.

Which is the dependent variable?

Which is the independent variable?

The ________ is a function of ________
and the functional notation is ________

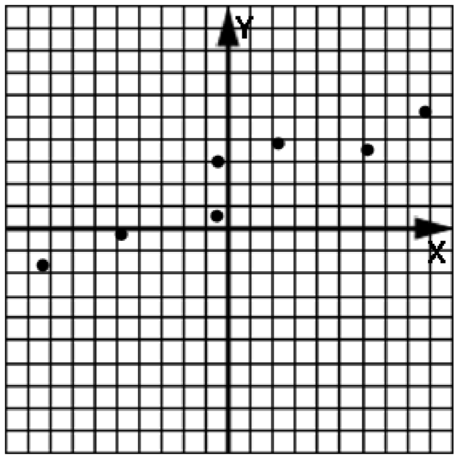
14. If $f(x) = 3x + 2$ and $g(x) = x^2 – 12$, find $f(2) + g(-3)$.

15. Solve the following inequality and show the answer both algebraically and graphically: $3x < 5(x – 4) + 8$

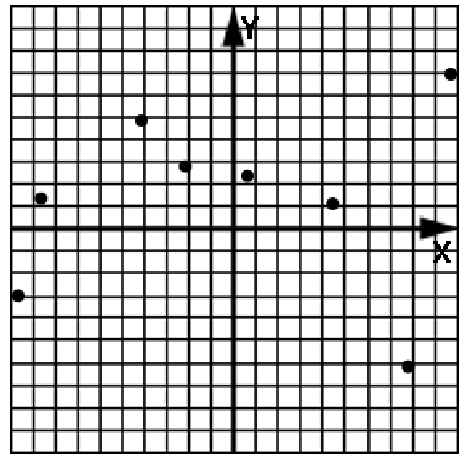
Unit 8: Review

In problems 1-4, draw a line of best-fit (if possible) and state the type of correlation exhibited by the data (positive, negative, or none).

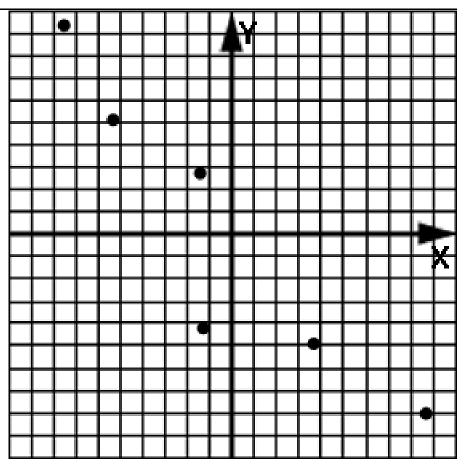
1.



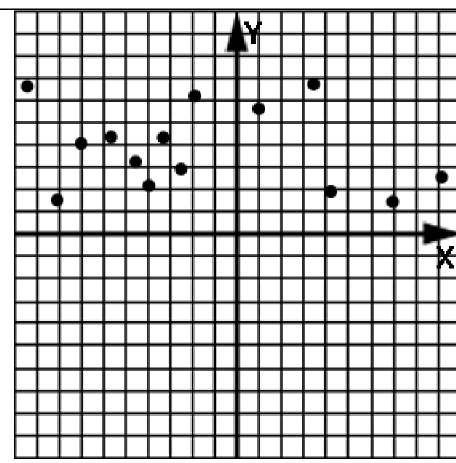
2.



3.



4.



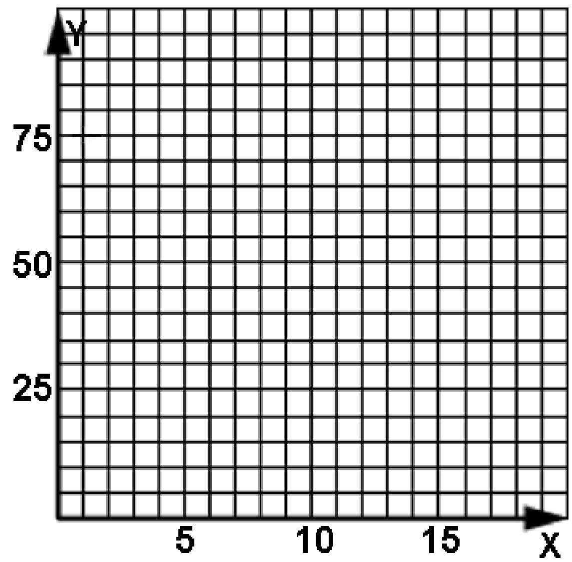
5. What is the meaning of positively correlated data?

What is the meaning of negatively correlated data?

6. Plot the data from the following table of randomly selected test grades. The *x* column is for the test number and the *y* column is the grade made on that test.

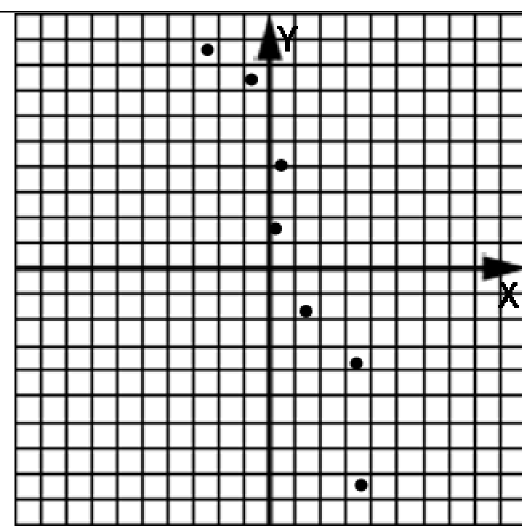
Comment on any trend or pattern you see in the data. If a trend is observed, draw a line of "best-fit" and then find its equation.

x, Test #	**y, grade**
1	50
3	50
6	65
10	65
13	70
14	80
18	80



7. Draw a line of best-fit for this scatter plot and then find the equation of the line.

What is the correlation of this scatterplot?



8. Use a graphing calculator to make a scatter plot of the data in this table. Use linear regression to produce a line of best-fit. Sketch the calculator display.

x	y
-8	120
-4	82
-1	67
2	36
6	15
8	-5

Problems 9-13 refer to the data and the line of best-fit in problem 8.

9. What type of correlation does the data of problem 8 exhibit?

10. What is the slope (to two decimal places) of the line of best-fit?

11. What is the y-intercept (to two decimal places) of the line of best-fit?

12. What is the equation of the line of best-fit in slope-intercept form?

13. Using the line of best-fit, algebraically determine the function value at x = 22.

14. The table shown here gives the depth, *y*, of a submarine below the surface in feet. Its depth at various times, *x*, in minutes is shown.

Without plotting the data either manually or with a calculator, what is the correlation of the data? Justify your answer.

x (time, min.)	**y (feet below surface)**
0	-100
5	-200
10	-282
22	-350
36	-448
47	-560

Problems 15-17 refer back to the data of problem 14.

15. What is the dependent variable and what are its units?

16. What is the independent variable and what are its units?

17. What are the units of the slope of the line of best-fit?

Alg 1, Unit 9

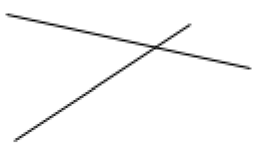
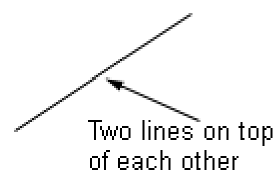
Systems of linear equations

Unit 9: Lesson 01 The meaning of a solution to a system of linear equations

Consider the following two equations:

$2x - 4y = 9$
$11x - 5y = -8$

What does it mean to **solve this system of equations**? Very simply, it means to **find all the points of intersection** of these two lines. In general there are three distinct possibilities as shown below:

<table>
<tr>
<td>The two lines intersect in a single point. The x and y values of that point are the solutions to the system.
</td>
<td>The two lines never intersect because the lines are parallel but separate.
</td>
<td>The two lines are directly on top of each resulting in an infinite number of intersection points.

</td>
</tr>
</table>

So, how can we tell by just looking at the equations of two lines which of the three pictures above represents their orientation?

If the **slopes of the two lines are different**, then it's the left picture above and we have only **one point** of intersection.

If the **slopes are the same** and the **y-intercepts are different**, then it's the middle picture above and we have **no points** of intersection.

If the **slopes are the same** and the **y-intercepts are the same** (they are actually the same line), then it's the right-hand picture above and there are **infinitely many points** of intersection.

In each example below, examine the slope and y-intercept. Then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

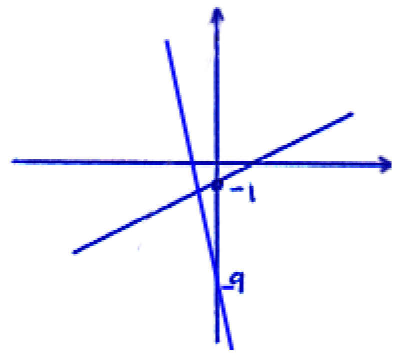
Example 1: $-4x = y + 9$ and $y = x - 1$

$-4x - 9 = y$

$m_1 = -4$ $\qquad m_2 = 1$

Slopes are different

(one intersection point)



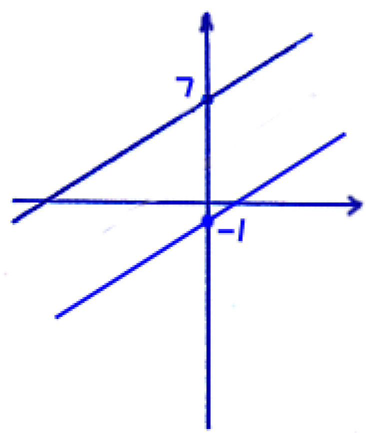
Example 2: $-x + y = 7$ and $y = x - 1$

$y = x + 7$ $\qquad m_2 = 1$

$m_1 = 1$

$m_1 = m_2$ so the lines are parallel. They have dif. b values so separate.

(No Solution)

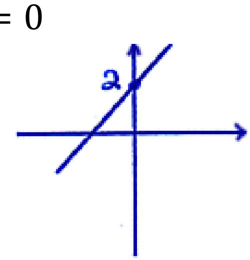


Example 3: $-x + y = 2$ and $6 + 3x - 3y = 0$

$y = x + 2$ $\qquad -3y = -3x - 6$

$m_1 = 1$ $\qquad y = x + 2$

$m_2 = 1$



Slope are the same; infact, the equations are the same. They sit on top of each other. So there are an infinite number of points as given by $y = x + 2$

$\boxed{\text{Infinitely many solutions}}$

A point (x, y) can be proven to be a solution to a system of two linear equations

if it "satisfies" **both** equations. (When the point is substituted into both equations, they are **both** still true.)

Example 4: Prove that (x, y) = (1, -2) is a solution to this system:

3x + y = 1
2x –y = 4

$(1,-2) \rightarrow$ $3x + y = 1$
$3 \cdot 1 - 2 = 1$
$3 - 2 = 1$
$1 \overset{\checkmark}{=} 1$

$(1,-2) \rightarrow$ $2x - y = 4$
$2 \cdot 1 - (-2) = 4$
$2 + 2 = 4$
$4 \overset{\checkmark}{=} 4$

Yes, it is a solution since it satisfies both equations.

Example 5: Determine if (-3, 1) is a solution to this system:

x + y = -2
3x – 2y = 1

$(-3, 1) \rightarrow$ $x + y = -2$
$-3 + 1 = -2$
$-2 \overset{\checkmark}{=} -2$

$(-3, 1) \rightarrow$ $3x - 2y = 1$
$3(-3) - 2 \cdot 1 = 1$
$-9 - 2 = 1$
$-11 \neq 1$

No, it is not a solution since it does not satisfy both equations.

Assignment: In the following problems, examine the slopes and y-intercepts of the two lines and then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

1. $x + 4 = y \ \ and \ \ y = 3x - 8$

---

2. $2x - 3y = -1$ and $8x - 12y = -4$

---

3. The line given by (1, –5) & (6, –10) and $3y = -3x + 11$

4. Line A has a slope of 3/5 and crosses the y-axis at –8. Line B has a slope of 3/5 and crosses the y-axis at 22.

---

5. Line A has a slope of 1 and crosses the y-axis at 3. Line B has a slope of 3 and crosses the y-axis at –1.

---

6. The product of the slope of the two lines is –1. Hint: This has something to do with the orientation of the line (parallel, perpendicular, etc).

---

7. $y = 4.1x + 2 \quad and \quad 2y = 4.1x + 2$

8. The line connecting (0, 0) & (1,1) and the line connecting (–2, –2) & (5, 5)

9. $y = 3$ and $x = 3$

10. $y = 4$ and $y = -5$

11. Determine if $(x, y) = (2, -3)$ is a solution of this system:

$$x - 2y = 8$$
$$2x + y = 1$$

12. Determine if $(x, y) = (3, 6)$ is a solution of this system:

$$4x - 2y = 0$$
$$x - 3y = 1$$

Unit 9: Lesson 02 Solving two linear equations by graphing

A system of equations is two or more equations. The two equations that follow comprise a **system** of equations:

4x – 3y = 0
5x + 7y = -3

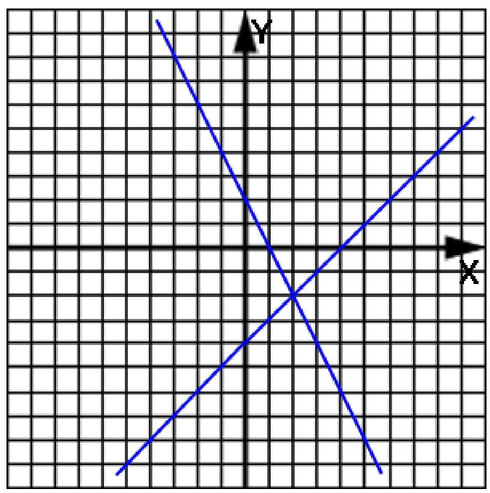
A system of two linear equations can be solved by graphing them and then observing where they intersect.

The (x, y) intersection point is the solution of the system.

Example 1: Solve this system by graphing:

y = -2x + 2
y = x – 4

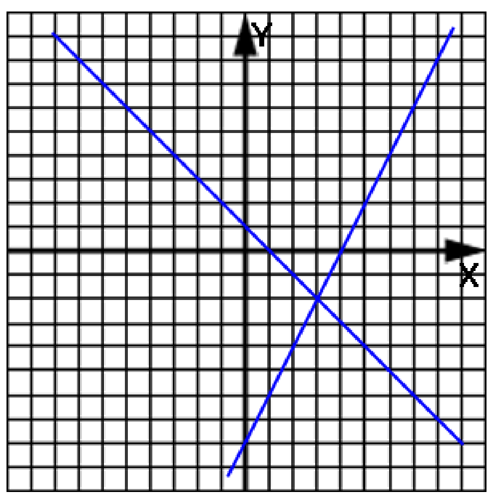
(x, y) = (2, -2)



Example 2: Graphically find the intersection point of this system:

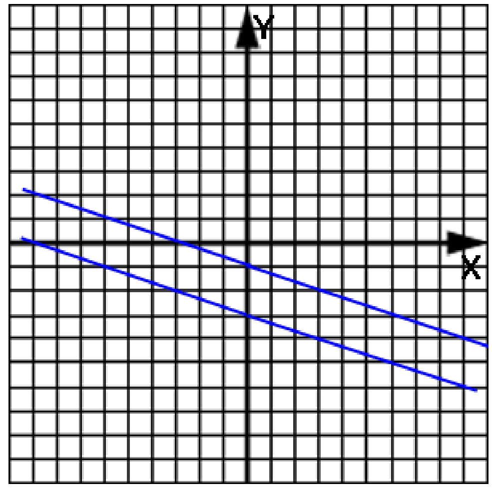
y = 2x – 8
y = -x + 1

(x, y) = (3, -2)



Example 3: Solve this system:

$-3y - x = 3$
$6y + 2x = -18$



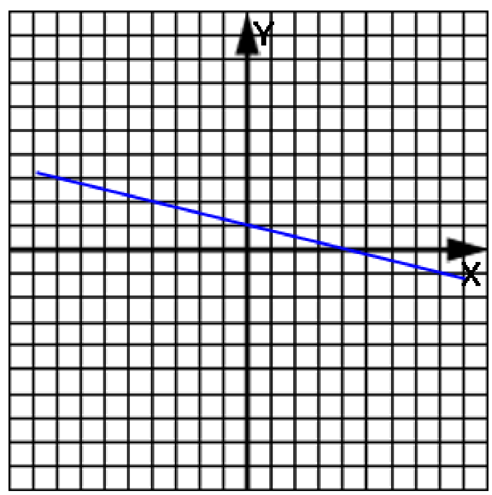
$-3y - x = 3$
$-3y = x + 3$
$y = \frac{1}{-3}x + \frac{3}{-3}$
$y = -\frac{1}{3}x - 1$
$m_1 = \frac{-1}{3}$

$6y + 2x = -18$
$6y = -2x - 18$
$y = \frac{-2}{6}x + \frac{-18}{6}$
$y = -\frac{1}{3}x - 3$
$m_2 = \frac{-1}{3}$

parallel lines, no solution

Example 4: Solve for *x* and *y* from:

$x + 4y = 4$; $-2x - 8y = -8$



$x + 4y = 4$
$4y = -x + 4$
$y = -\frac{1}{4}x + 1$

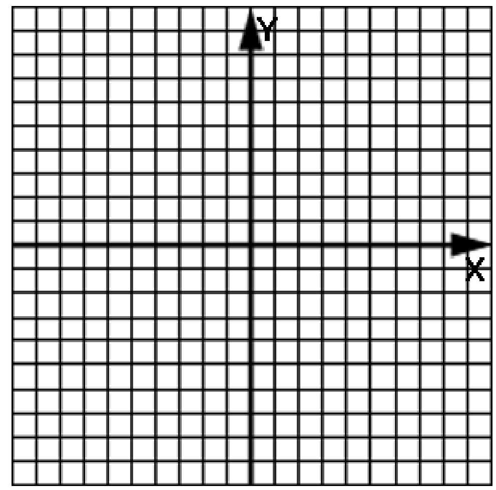
$-2x - 8y = -8$
$-8y = 2x - 8$
$y = \frac{2}{-8}x - \frac{8}{-8}$
$y = -\frac{1}{4}x + 1$

same

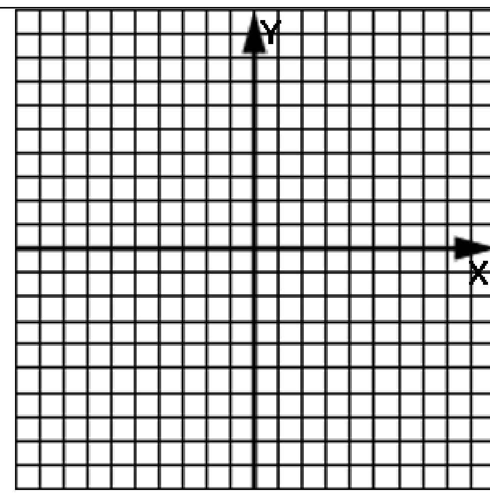
...same equation . . . so, there are two lines on top of each other. Infinitely many solutions along the line.

Assignment: Solve the following systems by graphing and finding the intersection point.

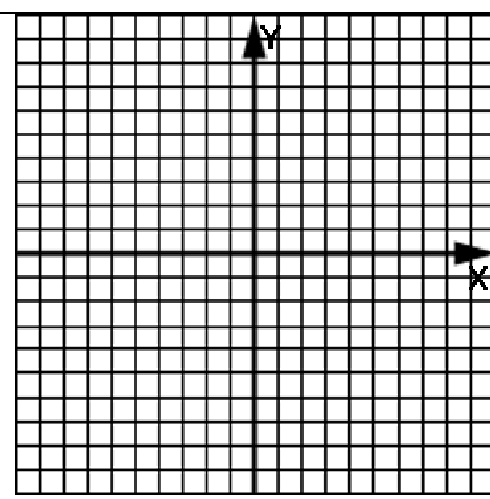
1. $y = 4x - 3$; $y = -2x + 9$



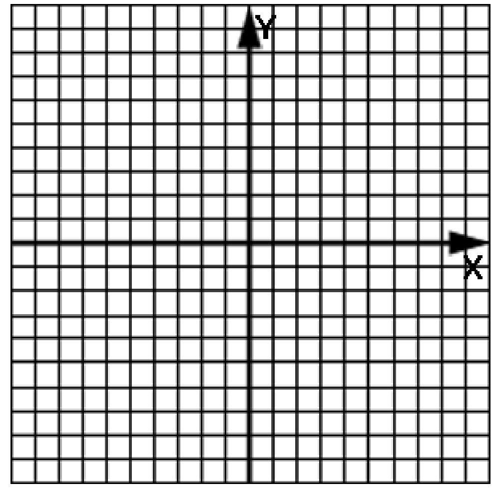
2. $2x - y = 1$; $4x - 2y = 2$



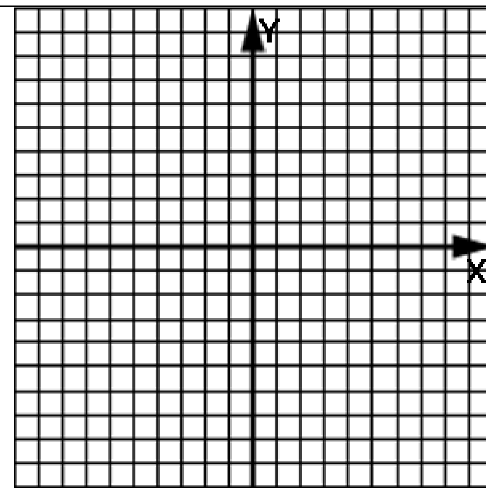
3. $y = (1/2)x - 1$; $y = (1/2)x - 4$



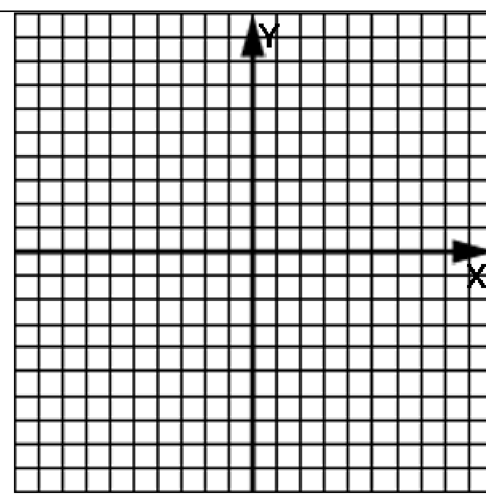
4. $y = 2x + 6$; $y = -x - 3$



5. $4x - 3y = -6$; $4x - 2y = 0$



6. $x - 2y = 8$; $2x + y = 1$



Unit 9: Lesson 03 Solving two linear equations by substitution

In the previous lesson we found the solution to a system of two linear equations by graphically **finding the intersection point** of the two lines.

Here, we will find that same (x, y) point by **strictly algebraic means**.

Example 1 (when *y* is isolated in one of the equations): Solve this system by substitution:

$y = 3x + 4$
$x – y = 2$

$y = 3x + 4$ $\quad$ $x - y = 2$

$y = 3(-3) + 4$ $\quad$ $x - (3x + 4) = 2$

$y = -9 + 4$ $\quad$ $x - 3x - 4 = 2$

$y = \boxed{-5}$ $\quad$ $-2x = 4 + 2$

$x = \frac{6}{-2} = \boxed{-3}$

or (– 3, – 5)

Example 2 (when *x* is isolated in one of the equations): Find the intersection point of these lines.

$x = y – 2$
$x + 2y – 1 = 0$

$x = y - 2$ $\quad$ $x + 2y - 1 = 0$

$x = 1 - 2$ $\quad$ $y - 2 + 2y - 1 = 0$

$x = \boxed{-1}$ $\quad$ $3y - 3 = 0$

$3y = 3$

$y = \frac{3}{3} = \boxed{1}$

or (–1, 1)

Example 3 (when neither variable is isolated):

x – 3y = 4
2x + 7y = -5

$x - 3y = 4$

$x = 3y + 4$

$2x + 7y = -5$

$2(3y+4) + 7y = -5$

$6y + 8 + 7y = -5$

$13y = -8 - 5$

$13y = -13$

$y = \frac{-13}{13} = \boxed{-1}$

$x = 3(-1) + 4$

$x = -3 + 4$

$x = \boxed{1}$

or (1, - 1)

Assignment: Solve the following systems using the substitution method.

1. $x + y = 8$; $y = 3x$

---

2. $y = 3x - 8$; $x + y = 4$

---

3. $3x - 5y = 11$; $x = 3y + 1$

---

4. $x + 4y = 1$; $2x + y = 9$

5. $2a + 7b = 3$; $a = 1 - 4b$

6. $p - 5q = 2$; $2p + q = 4$

7. $-4a + 5b = 17$; $5a - b = 5$

8. $y = 3x - 13$; $4x + 5y = 11$

9. $2x + 3y = 1$; $-3x + y = 15$

10. $8 = x - y$; $x + 3y = 12$

Unit 9: Lesson 04 — Solving two linear equations by elimination

Elimination method (sometimes called the addition method):

- A system of two equations in two variables can be solved by adding the two equations together so that one of the variables is eliminated.
- Solve the resulting equation for the remaining variable.
- Substitute this solved variable back into one of the original equations and solve for the other variable.

Example 1: Solve the system – 2x + 3y = 11 ; 2x + y = 1.

$$\cancel{-2x} + 3y = 11$$
$$\underline{\cancel{2x} + y = 1}$$
$$4y = 12$$
$$y = \frac{12}{4}$$
$$y = \boxed{3}$$

$$2x + y = 1$$
$$2x + 3 = 1$$
$$2x = 1 - 3$$
$$2x = -2$$
$$x = \frac{-2}{2} = \boxed{-1}$$

Example 2: Solve the system 6a + 7b = -15 ; 6a – 2b = 12.

$$6a + 7b = -15 \longrightarrow \cancel{6a} + 7b = -15$$
$$-1(6a - 2b) = 12(-1) \longrightarrow \underline{\cancel{-6a} + 2b = -12}$$
$$9b = -27$$
$$b = \frac{-27}{9} = \boxed{-3}$$

$$6a + 7b = -15$$
$$6a + 7(-3) = -15$$
$$6a = 21 - 15 = 6$$
$$a = \frac{6}{6} = \boxed{1}$$

Example 3: Solve the system 2x – 3y = 4 ; x + 4y = -9.

$$2x - 3y = 4 \longrightarrow \cancel{2x} - 3y = 4$$
$$-2(x + 4y) = -9(-2) \longrightarrow \cancel{-2x} - 8y = 18$$
$$-11y = 22$$
$$y = \frac{22}{-11} = \boxed{-2}$$

$$x + 4y = -9$$
$$x + 4(-2) = -9$$
$$x - 8 = -9$$
$$x = -9 + 8$$
$$x = \boxed{-1}$$

Example 4: Solve the system 3x – 8y = 13 ; 4x – 5y = 6.

$$4(3x - 8y) = 13(4) \longrightarrow \cancel{12x} - 32y = 52$$
$$-3(4x - 5y) = 6(-3) \longrightarrow \cancel{-12x} + 15y = -18$$
$$-17y = 34$$
$$y = \frac{34}{-17} = \boxed{-2}$$

$$4x - 5y = 6$$
$$4x - 5(-2) = 6$$
$$4x + 10 = 6$$
$$4x = 6 - 10$$
$$4x = -4$$
$$x = \frac{-4}{4} = \boxed{-1}$$

Assignment: Solve the following systems using the elimination method.

1. $4x – 3y = -2$; $2x + 3y = 26$

2. $a – b = 4$; $a + b = 8$

3. $2x – 5y = -6$; $2x – 7y = -14$

4. $3x + y = 4$; $5x – y = 12$

5. $5p + 2q = 6$; $9p + 2q = 22$

6. $5x + 12y = -1$; $8x + 12y = 20$

7. $3h - 5g = -35$; $2h - 5g = -30$

8. $4a - 5b = 23$; $3a + 10b = 31$

9. $2x + 7y = 4$; $3x - 7y = 6$

10. $m + 5n = 4$; $3m - 7n = -10$

11. $5x + 9y = 1$; $3x + 4y = 2$

12. $3x - 4y = 8$; $4x + 3y = 19$

Unit 9: Lesson 05 Graphing calculator solutions of linear systems

Finding the intersection point of two lines:

Using the **Y=** button, enter the two linear equations as Y1 and Y2. Press the **Graph** button to display the two lines simultaneously.

If the intersection point of the lines is not visible, use the **Zoom** button and then zoom **In** or **Out**. The **ZStandard** zoom will often be the most useful. If none of these show the intersection point, make an estimate of where it is and use the **Window** button to adjust the max and min values accordingly.

With the intersection point displayed, access **2nd Calc | 5.intersect**. You will then be asked to identify the "first curve." Move the blinker with the left and right arrows until it is clearly on one of the lines. Press Enter. You will then be asked to similarly identify the "2nd curve." Finally, you are asked to "guess" the intersection. Move the blinker until it is very close to the intersection point and press Enter. The *x* and *y* values of the intersection point will be given at the bottom of the display.

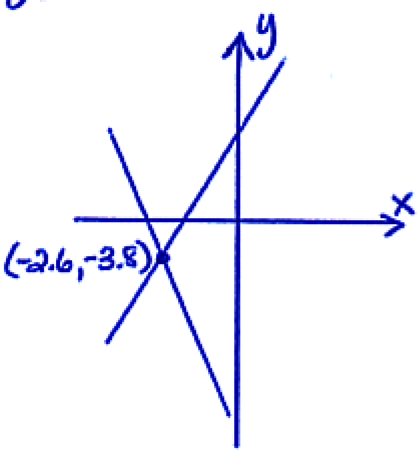
See **Calculator Appendix C** and a related video for more details on finding the intersection point of two lines.

Using the techniques described above, find the intersection point of the following systems of linear equations. Make a rough sketch of the calculator display.

Example 1:

y= 3x + 4 and y= –2x – 9

$y_1 = 3x+4$
$y_2 = -2x-9$

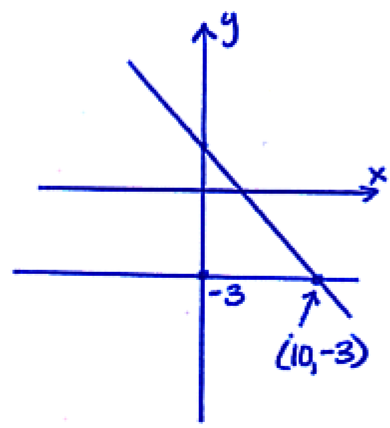


Example 2:

y = -3 and 2y – 4 + x = 0

$2y = -x+4$
$y = -\frac{1}{2}x+2$

$y_1 = -3$
$y_2 = -\frac{1}{2}x+2$



Assignment:

Using a graphing calculator, find the intersection point of the following systems of linear equations. Make a rough sketch of the calculator display.

1. $x + 4 = y$ and $y = 3x - 8$

2. $2x - 3y = -1$ and $8x - 12y = -4$

3. $y = 4.1x + 2$ and $2y = 4.1x + 2$

4. y = – 6x + 110 and y = x + 55

5. $\left(\frac{3}{5}\right)x - y = 1 \quad and \quad y = x$

6. y = x + 99   and   y – x = -3

7. .1 x + .2 = .3y   and   y = x + 1

8. 1002y = –5x – 55   and   y= 4x – 17

Unit 9: Lesson 06 Solving for two variables in word problems

When working word problems involving two unknowns,

- draw a picture if possible,
- create two variables and define them (or label them in the picture),
- create two equations using the two variables, and
- solve for the variables using either substitution or elimination.

Example 1: A pile of coins consists of dimes and quarters. If the total number of coins is 40 and their total value is $4.75, how many of each type coin is there?

d = number of dimes; q = number of quarters

$d + q = 40 \longrightarrow -10(d+q) = 40(-10) \longrightarrow -10d - 10q = -400$

$100(.10d + .25q) = 4.75(100) \rightarrow 10d + 25q = 475 \longrightarrow 10d + 25q = 475$

$15q = 75$

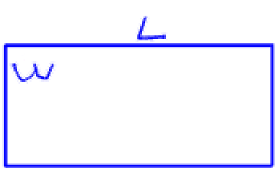
$q = \boxed{5 \text{ quarters}}$

$d + q = 40$

$d + 5 = 40$

$d = 40 - 5 = \boxed{35 \text{ dimes}}$

Example 2: A rectangular garden has a perimeter of 76 meters. If the width is 12 meters less than the length, what are the dimensions of the garden?



per $2L + 2W = 76 \qquad W = L - 12$

$2L + 2(L - 12) = 76$

$2L + 2L - 24 = 76$

$4L - 24 = 76$

$4L = 76 + 24 = 100$

$L = \frac{100}{4} = \boxed{25\,m}$

$W = L - 12$

$W = 25 - 12$

$W = \boxed{13\,m}$

Example 3: One number is 8 more than 11 times another number. When added, their total is 92. What are the numbers?

x = the first number ; y = the second number

$x = 11y + 8$ $\quad x + y = 92$

$(11y + 8) + y = 92$

$12y + 8 = 92$

$12y = 92 - 8 = 84$

$y = \frac{84}{12} = \boxed{7}$

$x = 11y + 8$

$x = 11 \cdot 7 + 8$

$x = 77 + 8$

$x = \boxed{85}$

Assignment: Work these problems by creating two equations with two variables.

1. Tickets for a small private concert sell for $30 for seats near the stage. All other tickets sell for $25. If a total of 52 tickets are sold totaling $1360, how many tickets were sold for seats near the stage?

2. If the width of a rectangle is 4 ft less than its length and the perimeter is 68 ft, what are the dimensions of the rectangle?

3. Bill has coins worth $5.15 in his pocket. If he has only dimes and quarters and there are 8 more quarters than dimes, how many quarters does Bill have?

4. On an archeological field trip, Lucy and Chuck found a total of 92 arrowheads. If Lucy found 6 more than Chuck, how many did Chuck find?

---

5. A certain rectangle having a perimeter of 84 inches has a length that is 2 inches more than its width. What is the length of the rectangle?

---

6. If a triangle's base is 10 ft and its height is 8 less than the area, what is the area of the triangle?

7. A total of 50 coins is worth \$4.15. If only nickels and dimes are present, how many of each is there?

8. The Imperial Fruit Stand only sells very high quality, expensive fruit. Three pears and two apples cost \$8.25 while two pears and three apples would cost \$8.00. How much would 4 pears and one apple cost?

9. The length of a rectangle of perimeter 130 meters is 5 meters more than twice its width. What are the dimensions of the rectangle?

Unit 9: Cumulative Review

1. Use a graphing calculator to make a scatter plot of the data in this table. Use a linear regression to produce a line of best-fit. Sketch the calculator display.

x	y
10	300
20	325
30	401
40	444
50	512

2. What type of correlation does the data of problem 1 exhibit?

3. What is the slope of the line of best-fit in problem 1?

4. What is the y-intercept of the line of best-fit in problem 1?

5. What is the equation of the line of best-fit (in problem 1) in slope-intercept form?

6. Using the equation for the line of best-fit in problem1 , algebraically determine the function value at x = 37.

7. Write an inequality that specifies all the numbers between -7(inclusive) and 13(exclusive).

8. Solve for *b* from 3(2 – b) – b = 19 + b.

9. A triangle's base is 12 inches and its height is 3 inches less than its area. What is its area? (Use two equations with two variables to solve this problem.)

10. 36 is what percent of 79?

11. 22.5% of what is 234?

12. What is the equation of a line that has a slope of 3 and passes through (10, -1)?

13. What is the equation of a line that passes through (-8, 4) and is parallel to the line given by x = 19?

14. What is the equation of the line passing through (4, 10) and (-4, 8)?

15. What is the equation of the line passing through the y-axis one unit above the origin and perpendicular to the line given by $8x - 2y = 9$?

---

16. If $f(x) = 4x - x - 9$ and $g(x) = x^2 + 5x$, find $3f(-2) + 2g(3)$.

---

17. Find the domain and range represented by this set of ordered pairs. Is it a function? Why?

$\{ (-4, 1), (7, 2), (8, 3), (7, -11), (10, 5) \}$

Unit 9: Review

1. Determine if (x, y) = (3, -2) is a solution to this system:
   $y = 2x - 8$; $y = -x + 1$

In problems 2 – 4, examine the slopes and y-intercepts to determine how many points are in the solution set to the given systems. Justify your answers.

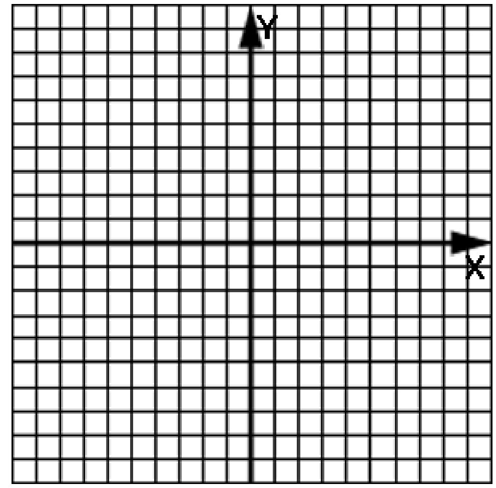
2. $y = 2x + 6$; $y = -x - 3$

3. $8x - 2y = 8$; $4x - y = 4$

4. $-3y + x = 3$; $-6y + 2x = 18$

5. Graph these two lines to find their intersection point.

$x - 2y = 8$; $2x + y = 1$



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6. Use the substitution method to find the intersection point of these lines:

$x = 4y - 9$; $2x + y = 9$

---

7. Use the substitution method to solve this system of equations:

$7x - 2y = -6$; $x - y = 2$

8. Use the substitution method to solve this system:

   $3a - 5b = -35$; $2a - 5b = -30$

---

9. Use the elimination method to find the intersection point of these two lines:

   $-7y - 2x = 5$; $x - 3y = 4$

---

10. Use the elimination method to solve this system of equations:

   $4h - 7g = 10$; $3h + 2g = -7$

11. Use the elimination method to solve this system of equations:

$2x - 5y = 27$; $8x - 3y = -11$

12. The number of boys in Mrs. Assignmore's algebra class is 5 more than the number of girls. If there are 31 students in the class, how many girls are in the class?

13. A triangle with a perimeter of 27 meters has a base that is 3 meters more than the other two equal sides. How long is each side of the triangle?

14. Use a graphing calculator to find the intersection point of these two lines. Make a sketch of the graphed lines in the calculator display.

$y = .35x + 2.01$; $y = -1.056x – 2.2$

Alg 1, Unit 10

Direct and Indirect Variation

Unit 10: Lesson 01 Direct variation

Consider **y = mx**.

There are several ways to describe the relationship between *x* and *y*:

- *y* varies directly as *x*
- *y* varies as *x*
- *y* is directly proportional to *x*
- *y* is proportional to *x*
- *y* varies linearly with *x*

When working with lines we call ***m*** the slope; however, in the context here we call it the **constant of proportionality**. In fact, most of the time we call the constant *k* and write:

$$y = kx$$

Solving for *k* we get:

$$\frac{y}{x} = k$$

Think of two ordered pairs that satisfy this equation, (x_1, y_1) and (x_2, y_2). Substituting these in we get two equations:

$$\frac{y_1}{x_1} = k \quad \text{and} \quad \frac{y_2}{x_2} = k$$

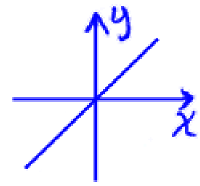
Since both equal *k*, they equal each other:

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

Hence, ***y = kx*** implies $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

Notice that when *y* is directly proportional to *x*:

- the function *y* written as a function of *x* is a **linear function**,
- the constant of proportionality is the **slope** of the line,
- and the **y-intercept is 0**.



Example 1: *y* varies directly as *x*. When *x* is 3, *y* is 14. What is *x* when *y* is 11?

$$y = kx \rightarrow \frac{y_1}{x_1} = \frac{y_2}{x_2}$$

$$\frac{14}{3} = \frac{11}{x_2}$$

$$14x_2 = 33$$

$$x_2 = \boxed{\frac{33}{14}}$$

Example 2: If *b* is directly proportional to *x* then b = 7 when x = 9. What is *x* when b = 11?

$$b = kx \rightarrow \frac{b_1}{x_1} = \frac{b_2}{x_2}$$

$$\frac{7}{9} = \frac{11}{x}$$

$$7x = 99$$

$$x = \boxed{\frac{99}{7}}$$

Example 3: The profit *p* from the crop varies linearly with the amount of rainfall *r*. If p = $62,000 when the rainfall is 26 inches, what is the expected profit when the rainfall is 32 inches?

$$p = kr \rightarrow \frac{p_1}{r_1} = \frac{p_2}{r_2}$$

$$\frac{62{,}000}{26} = \frac{p}{32}$$

$$26p = (62{,}000)32$$

$$p = \frac{1{,}984{,}000}{26} = \boxed{\$76{,}307.69}$$

Example 4: *p* is directly proportional to *q*. When *p* is 10, *q* is 200. What is the constant of proportionality?

$$p = kq$$

$$10 = k200$$

$$\frac{10}{200} = k$$

$$\boxed{\frac{1}{20}} = k$$

Assignment:

1. c is proportional to j. When j is 2, c is 5. What is the value of j when c is 11?

2. When x is 9, p is 11. If x varies linearly with p, what is x when p is 10?

3. What is the constant of proportionality in problem 2?

4. Write the function describing the relationship between w and r when r varies as w.

5. The number of failures f was proportional to level of difficulty d of the material. The teacher found that 3 students failed when the difficulty level was 18. How many would fail if the difficulty level was raised to 36?

6. The reds vary linearly with the blues. If there are 14 reds when there are 4 blues, how many reds can be expected when there are only 2 blues?

7. z varies as m. Find the missing value in the table below.

m	***z***
2	6
14.1	?

8. If m varies as n with a constant of proportionality of 12, what will be the value of n when m = 104?

9. Sketch a graph of y as a function of x when it is known that y varies as x with a constant of proportionality of -2.

10. If y varies directly as x, what is the y-intercept when y is written as function of x and then graphed?

11. The population of a particular ancient civilization was always proportional to the amount of yearly rainfall. One year the population was 72,000 when the rain totaled 36 inches. What would be the population when the rain was only 20 inches?

12. Does the data in the table below indicate that there is a direct variation relationship between the two variables? (justify your answer.) If so, write the function rule.

x	y
3	9
5	15
8	24
11	33
15	45
32	96

13. Does the data in the table below indicate that there is a direct variation relationship between the two variables? (justify your answer.) If so, write the function rule.

x	y
2	4
6	12
7	14
12	36
20	60
30	120

14. Write the equation that indicates that *j* is proportional to *d*.

15. The constant of proportionality involved in the direct variation of *b* with *c*, is 1.2. What is the slope of the line resulting from graphing *b* as a function of *c*?

16. Consider the function relating Celsius temperature and Fahrenheit.

$C = (5/9)(F - 32)$

Does *C* vary directly as *F*? Justify your answer.

17. Which of the following equations illustrate direct variation?

a. y = 4x
b. m + n = 0
c. u =3v + 2
d. g = 4h + 0
e. 3y = 12x
f. 2y + 4x + 1 = 0

18. If the number of healthy fish *f* varies as the amount *a* of food available, how many fish would be present when 6 lbs. of food are available? (Assume that 120 fish corresponds to 8 lbs. of food.)

19. What is the constant of variation if *y* varies directly as *x* and y = 5 when x = 20?

20. If *g* is proportional to *h* and the constant of variation is 22, what is the value of *g* when h = 11?

Unit 10: Lesson 02 Indirect variation

Consider **y = k/x**.

There are two ways to describe the relationship between x and y:

- *y* varies inversely as *x*
- *y* is inversely proportional to *x*

Solve for *k*, the **constant of proportionality** and get:

$\frac{y}{1} = \frac{k}{x}$ $xy = k$

Substituting in two ordered pairs, (x_1, y_1) and (x_2, y_2), that satisfy this equation, we get two equations:

$x_1 y_1 = k$ and $x_2 y_2 = k$

So $x_1 y_1 = x_2 y_2$

Hence ***y = k/x*** implies $\boldsymbol{y_1 x_1 = y_2 x_2}$

Example 1: *a* is inversely proportional to *b*. When *a* is 9, *b* is 12. What is *a* when *b* has a value of 2?

$a = \frac{k}{b} \longrightarrow a_1 b_1 = a_2 b_2$

$9 \cdot 12 = a \cdot 2$

$\frac{9 \cdot \cancel{12}\,6}{\cancel{2}} = a$

$a = \boxed{54}$

Example 2: The variable *z* varies inversely with *h*. What is the constant of proportionality when the pair (*h, z*) = (5, –1)?

$z = \frac{k}{h}$

$-1 = \frac{k}{5}$

$\frac{-1}{1} = \frac{k}{5}$

$k = \boxed{-5}$

Example 3: The average of the grades on a test is inversely proportional to the square of the amount of time spent in detention. What is the constant of proportionality when the test average is 75 and the detention time is 5 hours?

$$A = \frac{k}{T^2}$$

$$75 = \frac{k}{5^2}$$

$$75 = \frac{k}{25}$$

$$75(25) = k$$

$$\boxed{1875 = k}$$

Example 4: If *y* is inversely proportional to *x* and the constant of proportionality is 900, what is the value of *x* when *y* is 45?

$$y = \frac{900}{x}$$

$$45 = \frac{900}{x}$$

$$45x = 900$$

$$x = \boxed{20}$$

See Enrichment Topic D for how to solve problems that simultaneously combine direct and inverse (indirect) variation.

Assignment:

1. What is the constant of proportionality when *y* is inversely proportional to *x*, and y = 4 when x = 10?

2. Write the function describing the relationship between *w* and *r* when *r* varies inversely as *w*.

3. Write the function describing the relationship between *w* and *r* when *r* varies as *w*.

4. How many rabbits are there when there are 20 coyotes? The number of rabbits is inversely proportional to the number of coyotes where the constant of proportionality is 100.

5. *z* varies inversely as *c*. Find the missing value in the table below.

z	***c***
2	6
?	11

6. *z* varies inversely as *m*. Find the missing value in the table below.

m	***z***
2	6
14.1	?

7. *T* is inversely proportional to *P*. Find the missing data in the table below.

P	***T***
4	?
19	16

8. The number of free-throws *f* made was inversely proportional to the hostility *h* factor of the crowd. If 7 free throws were made when the hostility factor was 12, how many free-throws would be made when the hostility factor is only 4?

9. If *m* is directly proportional to *n* and m = 12 when n = 2.6, what would be the value of *n* when *m* is 21?

10. If *j* varies inversely with *p*, and the constant of proportionately is 7, what would be the value of *p* when j = 5?

11. Write the equation that indicates that A is inversely proportional to B.

12. If g/h = 22, is *g* directly proportional or inversely proportional to *h*?

13. If the number *n* of a fish within a particular species varies inversely as the salinity of the water, and 200 fish are present when the salinity factor *s* is 15, how many fish will be present when the salinity is 20?

14. Does the data below indicate that there is an inverse variation relationship between the two variables? (Justify your answer.) If so, write the function rule.

x	y
4	25
5	20
10	11
20	6
25	5

15. Does the data below indicate that there is an inverse variation relationship between the two variables? (Justify your answer.) If so, write the function rule.

x	y
2	40
4	20
5	16
8	10
10	8

16. If the greens are inversely proportional to the reds, what is the constant of proportionality when there are 16 greens and 4 reds?

17. Using the information in problem 16, how many reds would there be if there are 48 greens?

18. Which of the following equations represent an inverse relationship between x and y?

a. $x + y = 1$ b. $3x - y = 0$ c. $x + 1/y = 0$ d. $y = 14/x$ e. $2/x = y$

19. Which of the choices of problem 18 represent a direct variation between the variables *x* and *y*?

Sem 1: Review

Comprehensive Review

1. Simplify $2x - 9y + 4x + 11y$ by combining like terms and then evaluate at $x = -2$ and $y = 4$.

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2. Simplify $2/6 - 1/7 + 3$

---

3. Simplify $1 - 4(3z - 2) - 2(7 - z)$ and then evaluate at $z = -5$.

---

4. Solve for y: $11(y - 2) + 2[y - 3(y + 1)] = 0$

5. Solve for p: $-p-(5-6p)+(p+8)=15$

6. Solve for h: $3(-h-3)=-3(h+1)+2$

7. Solve this inequality and express the answer both symbolically and as a graph on a number line: $3x-11\le 4$

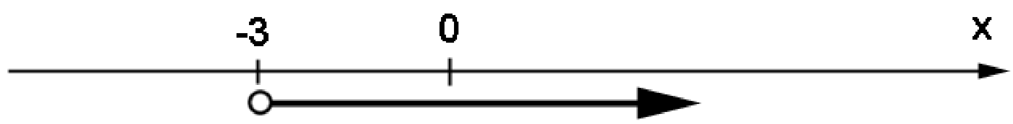
8. Solve this inequality and express the answer both symbolically and as a graph on a number line: $4(2+x)<6x+12$

9. Rewrite the inequality x ≤ y after multiplying both sides by – 6 .

10. Define a variable and then write this expression algebraically:

"47 decreased by the width"

11. Write the inequality that describes this graph:



12. Which of the following is a solution to the inequality shown in problem 11?

{ -5, -4, - 3, -2, 11.304 }

13. 26 is what percent of 79.2?

14. The length of a rectangle is 5 more than its width. What are the dimensions of the rectangle if its perimeter is 26?

15. What is the rate of commission on the sale of a car if the salesman makes \$300 on the sale of a \$4,700 used car?

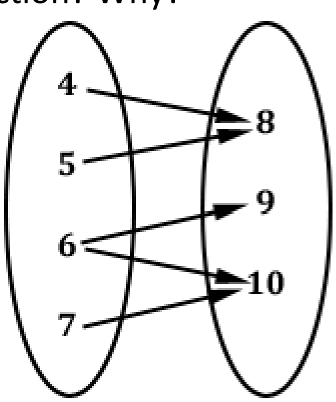
16. 29% of what is 172?

17. Convert .0289% to a decimal fraction.

18. Plot the point P(-8, 6) on a coordinate plane and then show a new point Q that is the reflection of P across the x-axis. In which quadrant is Q?

19. What is the equation of a vertical line passing through (5, – 6)?

20. Find the domain and range of the relation represented by this mapping. Is it a function? Why?



21. Find the domain and range of the relation represented by this table. Is it a function? Why?

x	y
2	6
9	9
13	13
22	30

22. If f(x) = –x + 16x – 2 and g(x) = 3x –7, find the value of 2g(3) – 5f(–1).

23. A left-over dish is taken from a refrigerator at 34^{o}F and placed in a 250^{o}F oven. Sketch a graph of the temperature of the dish over the next few hours.

Which is the dependent variable?

Which is the independent variable?

The _____ is a function of _______ and the functional notation is _________

24. What is the equation of the line having a slope of –3/5 and passing through the y-axis at y = 22?

25. What is the equation of the line that passes through the x-axis just two units to the left of the origin and the point (–5, 12)?

26. What is the equation of the line passing through the origin and perpendicular to the line given by $3y + 2x = 7$?

27. What is the equation of the line that passes through (4, –8) and is parallel to the line given by $y + 6 = 0$?

28. Examine these two linear equations and determine if their two-dimensional graphs are perpendicular, parallel, or neither. How many points would be in the solution to this system of equations? $4y + 3x = 12$ and $y - x + 234 = 1$

29. Examine these two linear equations and determine if their graphs are perpendicular, parallel, or neither. How many points would be in the solution to this system of equations? $x = 4$ and $17 - x = 0$

30. Examine these two linear equations and determine if their graphs are perpendicular, parallel, or neither. How many points would be in the solution to this system of equations? $8x + 3 = –5y$ and $16y – 10x + 1 = 0$

---

31. Draw a line that has an x-intercept of – 6 and a y-intercept of –10. What is the equation of this line? Which quadrants are touched by this line?

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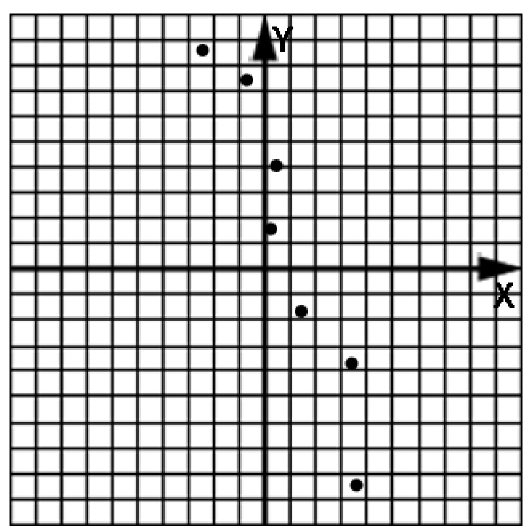
32. Draw a line of best-fit for this scatter plot and then find the equation of the line.

What is the correlation of this scatterplot?



33. Use a graphing calculator to make a scatter plot of the data in this table. Use linear regression to produce a line of best-fit. Sketch the calculator display.

x	**y**
-8	120
-4	82
-1	67
2	36
6	15
8	-5

34. What type of correlation does the data of problem 33 exhibit?

35. What is the slope (to two decimal places) of the line of best-fit?

36. What is the y-intercept (to two decimal places) of the line of best-fit?

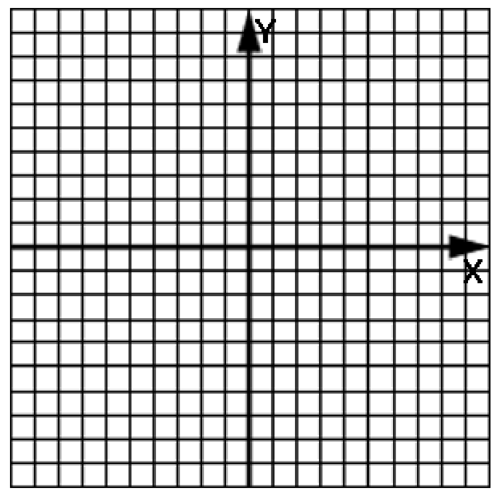
37. What is the equation of the line of best-fit in slope-intercept form?

38. Determine if (4, –5) is a solution to this system:

$y = 8x - 37$; $4x + 6y - 1 = 0$

39. Graph these two lines to find their intersection point.

$x - 2y = 8$; $2x + y = 1$



40. Use the substitution method to find the intersection point of these lines:

$x = 4y - 9$; $2x + y = 9$

41. Use the elimination method to find the solution to this system:

$4x - 6y + 2 = 0$; $2x - 5y = 11$

---

42. Use the elimination method to find the intersection point of these two lines:

$-7y - 2x = 5$; $x - 3y = 4$

---

43. Use the substitution method to find the intersection point of these two lines:

$2x - 5y = 2$; $3x + y = 0$

44. Use a graphing calculator find the intersection of these two lines. Make a sketch of the calculator display:

$y = .37x + 2.01$; $y = -1.156x – 2.4$

45. The number of boys in the class is 5 more than the number of girls. If there are 31 students in the class, how many girls are in the class?

46. The number of green birds is inversely proportional to the number of red birds. In one instance there were 33 green birds when there were 2 red birds. How many red birds can be expected when there are 22 green birds?

47. The profit *p* per unit varies as number of features *f* in the product. What is the constant of proportionality when 11 features yields a profit of $23?

48. If *g* is directly proportional to *h* and g = 2 when h = 3.4, what would be the value of *h* when g = 11?