# Blue Pelican

# Alg I Enrichment Topics

Teacher Version 1.01



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# Table of contents – Alg 1 Enrichment Topics

# Enrichment Topic A

**Commutative, Distributive, and Associative properties**

**Commutative property:**

For multiplication: $x \cdot y = y \cdot x$

**Example 1:**

$7 \cdot 9 = 9 \cdot 7$

For addition: $x + y = y + x$

**Example 2:**

$8 + 2 = 2 + 8$

Notice that numbers **don't commute** under the operation of **subtraction**:

$4 - 3 \neq 3 - 4$

**Distributive property:** The product of a number and a sum is equal to the sum of the individual products of addends and the number.

**Example 3:**

$3(5 + 11) = 3 \cdot 5 + 3 \cdot 11$

**Example 4:**

$a(b + c) = ab + ac$

**Associative property:** The addition or multiplication of a several numbers is the same regardless of how the numbers are grouped. The associative property will always involve 3 or more numbers. The parenthesis groups the terms that are considered one unit.

Associative property of **addition**:

**Example 5:**

$$5+(7+3)=(5+7)+3$$
$$(x+y)+z=x+(y+z)$$

Associative property of **multiplication**:

**Example 6:**

$$(4\cdot 7)\cdot 3=4\cdot(7\cdot 3)$$
$$a\cdot(b\cdot c)=(a\cdot b)\cdot c$$

**Example 7:** Name the properties illustrated by these equations:

| | |
|---|---|
| $8x + 2y = 2y + 8x$ | *Commutative property of addition* |
| $7 + (5 + 9) = (7 + 5) + 9$ | *Associative property of addition* |
| $4 + 19 = 19 + 4$ | *Commutative property of addition* |
| $5(8+3) = 5\cdot 8 + 5\cdot 3$ | *Distributive property* |
| $(x+y)+z = x+(y+z)$ | *Associative property of addition* |
| $(a+b)x = ax + bx$ | *Distributive property* |
| $3x(2y) = (2y)3x$ | *Commutative property of multiplication* |
| $5\cdot(9\cdot 3) = (5\cdot 9)\cdot 3$ | *Associative property of multiplication* |

**Assignment:** Name the properties illustrated by these equations:

1. $11 \cdot 4 = 4 \cdot 11$

   *Commutative property of addition*

---

2. $127(x + y + z) = 127x + 127y + 127z$

   *Distributive property*

---

3. 1 + (2 + 3 + 4) = (1 + 2 + 3) + 4

   *Associative property of addition*

---

4. $3 \cdot 5 + 8 \cdot 5 + 4 \cdot 5 = (3 + 8 + 4)5$

   *Distributive property*

---

5. $\mathrm{f} + \mathrm{g} = \mathrm{g} + \mathrm{f}$

   *Commutative property of addition*

---

6. $\mathrm{p} \cdot \mathrm{q} = \mathrm{q} \cdot \mathrm{p}$

   *Commutative property of multiplication*

7. $m \cdot (n \cdot p) \cdot q = m \cdot n \cdot (p \cdot q)$

*Associative property of multiplication*

---

8. $a \cdot b \cdot c = b \cdot a \cdot c$

*Commutative property of multiplication*

---

9. $115 \cdot (59 \cdot 19) = (115 \cdot 59) \cdot 19$

*Associative property of multiplication*

---

10. $(47 - 11)x = 47x - 11x$

*Distributive property*

---

*11. $(76 - x) \cdot (a + b) = (a + b) \cdot (76 - x)$

*Commutative property of multiplication*

---

*12. $(76 - x) + (a + b) = (a + b) + (76 - x)$

*Commutative property of addition*

# Enrichment Topic B

## Inequality conjunctions and disjunctions

Consider the inequality **conjunction**:

$-5 < x \leq 7$

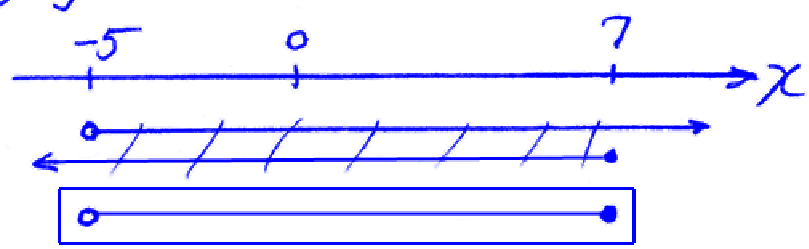
This is equivalent to

$-5 < x$ **and** $x \leq 7$ where the "and" implies an **intersection** (overlap) of the answers from each part.

**Example 1:** Draw the values of *x* given by $-5 < x \leq 7$ on a number line.

$-5 < x$ and $x \leq 7$

$x > -5$




In a similar way

$3x - 5 \leq 2 < x + 1$

is an inequality conjunction that can be separated into two parts:

$3x - 5 \leq 2$ **and** $2 < x + 1$ where, again, the "and" is implied.

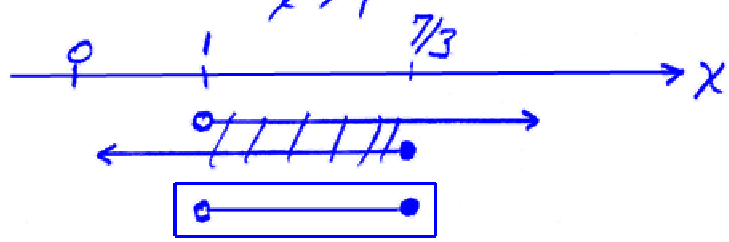
**Example 2:** Solve 3x – 5 ≤ 2 < x + 1

$3x - 5 \le 2$ and $2 < x + 1$

$3x \le 2 + 5$ $\qquad$ $x + 1 > 2$

$3x \le 7$ $\qquad$ $x > 2 - 1$

$x \le \frac{7}{3}$ $\qquad$ $x > 1$




An inequality **disjunction** is always written with an **explicit** "**or**" (with a conjunction, the "and" is often implied) and typically looks like this:

(Inequality statement #1) or (Inequality statement # 2)

The "or" indicates that the **union** is to be taken of the answers from both parts. The union, in turn, means to "take everything".

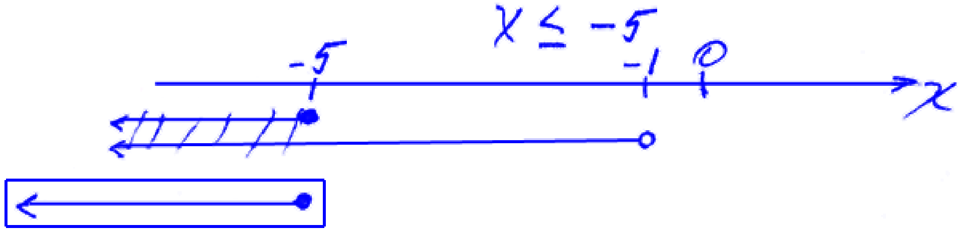
**Example 3:** Find the solution to x + 2 < 1 or 2x + 6 ≤ –4

$x + 2 < 1$ or $2x + 6 \le -4$

$x < -2 + 1$ $\qquad$ $2x \le -6 - 4$

$x < -1$ $\qquad$ $2x \le -10$

$x \le -5$



**Assignment:**

1. Separate $-5 < x \le -17$ into two different inequalities.

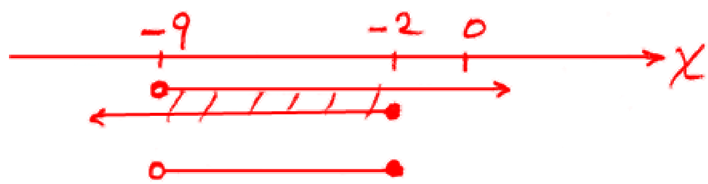
$-5 < x$ and $x \le -17$

---

2. Separate $-9 < x \le -2$ into two different inequalities and then graph the indicated values of *x* on a number line.

$-9 < x$ and $x \le -2$

$x > -9$

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3. Separate $-1 \le x + 3 < 8$ into two different inequalities and then graph the indicated values of *x* on a number line.

$-1 \le x+3$ and $x+3 < 8$

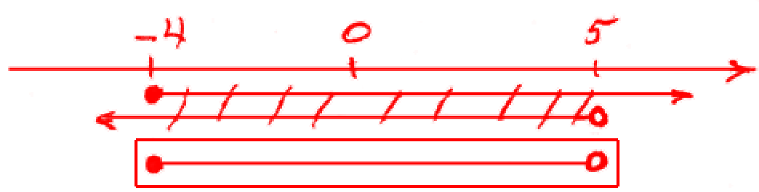
$x+3 \ge -1$

$x \ge -3 - 1$

$x \ge -4$

$x < 8-3$

$x < 5$

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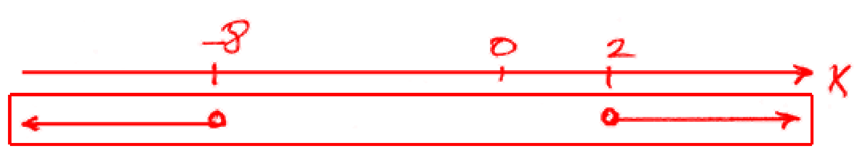
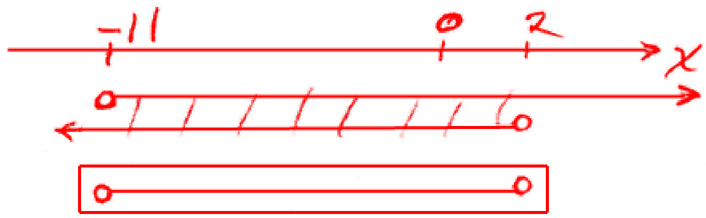
4. Graph the indicated values of *x* on a number line for this inequality disjunction:
$x > 2$ or $x < -8$



5. Graph the indicated values of *x* on a number line for this inequality conjunction:
   $x > -11$ and $x < 2$

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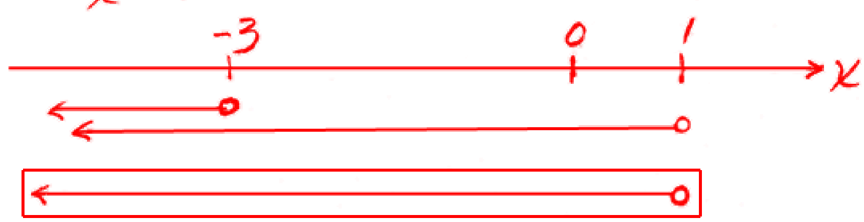
6. On a number line graph the values of *x* indicated by these inequalities:
   $-2x + 1 > 7$ or $x + 4 < 5$

$-2x+1>7$ or $x+4<5$
$-2x>6$ $\quad$ $x<5-4$
$x<-3$ $\quad$ $x<1$

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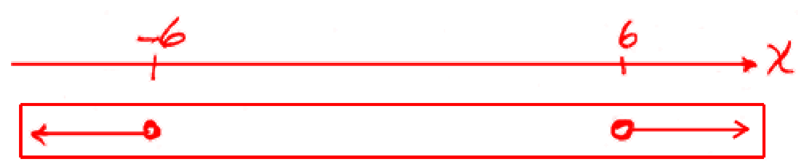
7. On a number line graph the values of *x* indicated by these inequalities:
   $x + 3 > 9$ or $x + 4 < -2$

$x+3>9$ or $x+4<-2$
$x>9-3$ $\quad$ $x<-4-2$
$x>6$ $\quad$ $x<-6$

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8. "and" is associated with

   A. conjunction
   B. disjunction
   C. neither

   *A*

9. "or" is associated with

   A. conjunction
   B. disjunction
   C. neither

   *B*

# Enrichment Topic C

## Two dimensional inequalities

To graph an inequality like y < 3x – 5 , we first **draw the line** y = 3x – 5. Then do the following:

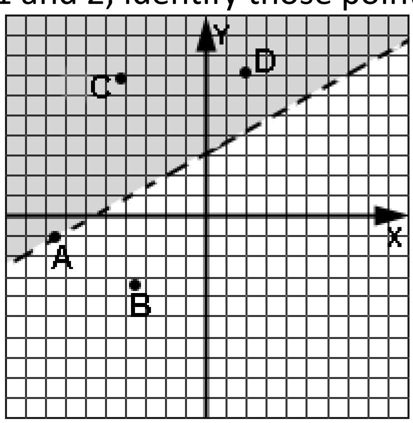
- If the inequality is **≥ or ≤** make the line **solid**. If the inequality is **< or >** make it **dotted**.
- If the inequality is **≤ or <**, shade **below** the line. If it is **≥ or >,** shade **above** the line.
- If the line is vertical then **≤ or <** dictates that we shade to the left. Shade to the right if **≥ or >.**

All the shaded points and/or a solid line are the solutions to the inequality.

In examples 1 and 2, identify those points that are solutions to the inequality.

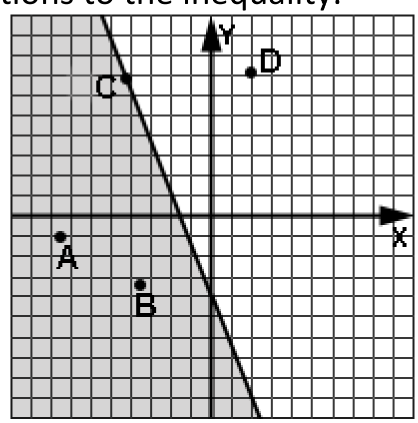
**Example 1:**

*C, D*




**Example 2:**

*A, B, C*




In Examples 3 and 4, determine algebraically if the point is part of the solution.

**Example 3:** $3x - 7y \le -2$ (–4, 10)

$3(-4)-7(10) \le -2$

$-12 - 70 \le -2$

$-82 \le -2$ ✓

True, so the point (-4,10) is part of the Solution. Yes!

**Example 4:** $x < 2y - 17$ (–8, 1)

$-8 < 2(1) - 17$

$-8 \not< -15$

False!

(-8,1) is not part of the Solution. No!

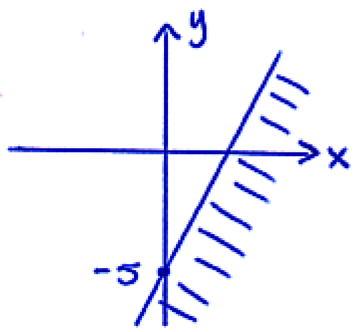
In examples 5 - 8, graph the inequality. Remember when dividing or multiplying by a negative number to reverse the inequality.

**Example 5:** 2x – y ≥ 5

$-y \geq -2x + 5$

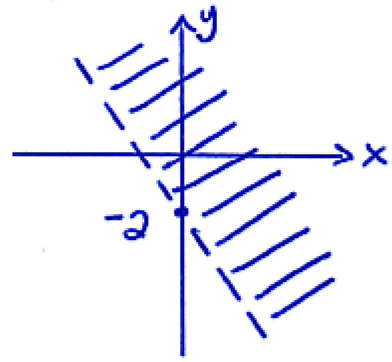
$y \leq 2x - 5$

Note reversal




**Example 6:** x + y > –2

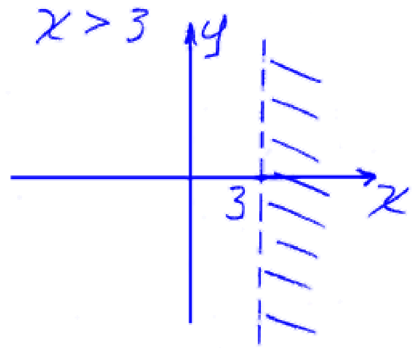
$y > -x - 2$



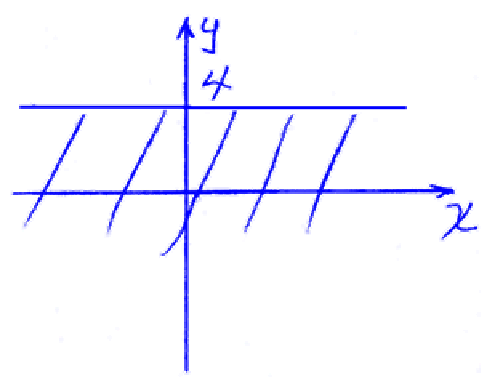

**Example 7:** –x < –3

$-x < -3$

$x > 3$



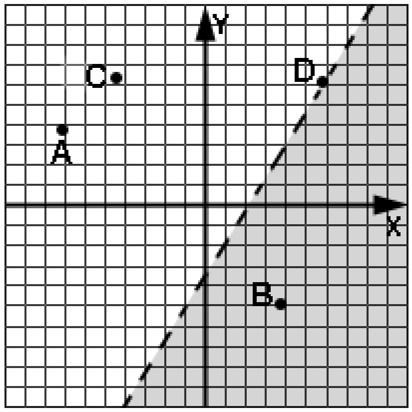

**Example 8:** y ≤ 4



**Assignment:**

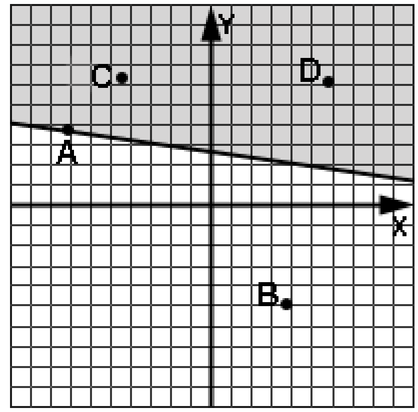
In problems 1 and 2, identify those points that are solutions to the inequality.

**1.**




*B*

**2.**




*A, C, D*

In problems 3 and 4, determine algebraically if the point is part of the solution to the inequality.

**3.** $77x - y < 2x - 1$ (0, 0)

$77(0) - 0 < 2(0) - 1$

$0 \not< -1$

False

(0,0) is not a solution.

No!

**4.** $10 \geq 4x - 7y$ (–1, –2)

$10 \geq 4(-1) - 7(-2)$

$10 \geq -4 + 14$

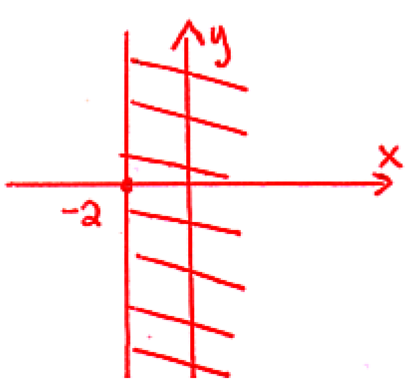
$10 \geq 10$

True!

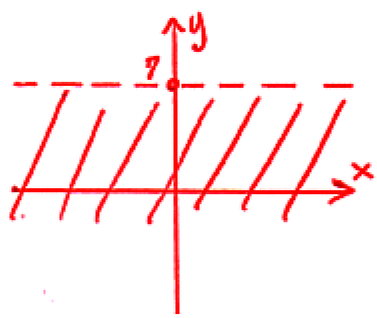
yes, (–1,–2) is a solution.

In problems 5 – 12 graph the inequality.

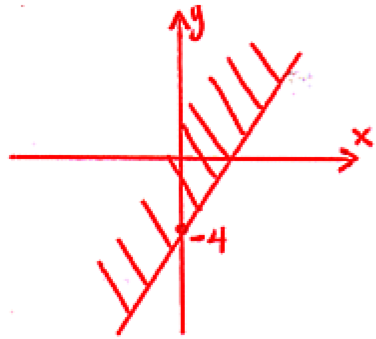
**5.** $x \geq -2$




**6.** $y < 7$



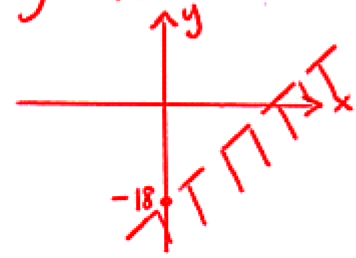
**7.** $y \geq 3x - 4$



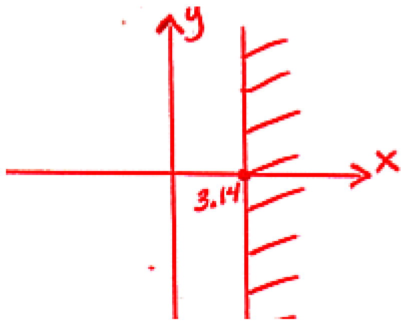

**8.** $x - y > 18$

$-y > -x + 18$

$y < x - 18$

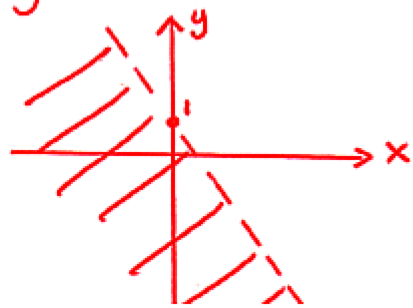



**9.** $x \geq \pi$



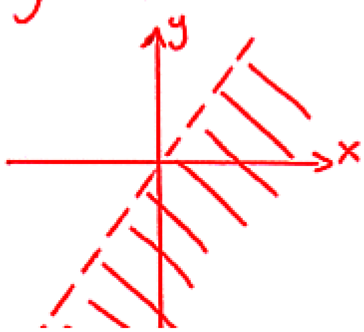

**10.** $x < -y + 1$

$y < -x + 1$

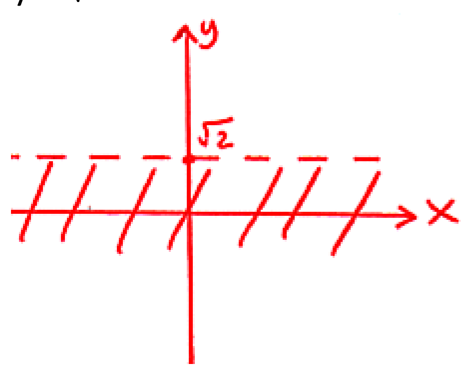



**11.** $3y < 12x$

$y < 4x$

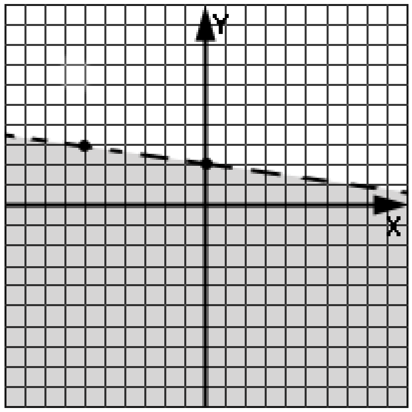


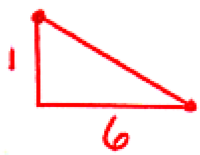

**12.** $y < \sqrt{2}$



In problems 13 and 14, state the inequality represented by the graph.

**13.**



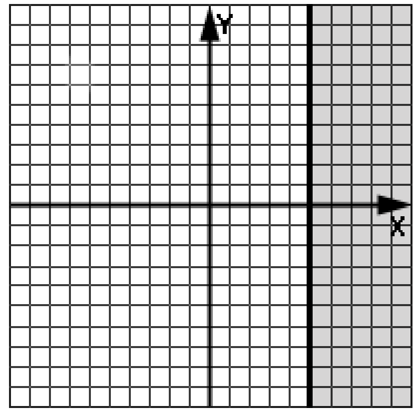

$m = -\frac{1}{6}$

$b = 2$

$y = mx + b$

$\boxed{y < -\frac{1}{6}x + 2}$

**14.**




$\boxed{x \geq 5}$

# Enrichment Topic D

## Combining direct and indirect variations

It is possible for a variable to be **proportional** to another variable while **simultaneously** being **inversely (indirectly) proportional** to yet another. The following example demonstrates this.

**Example 1:** The reds vary directly as the blues and inversely as the yellows. In a given case there are 12 reds, 3 blues, and 2 yellows. How many blues will there be when there are 14 reds and only 3 yellows?

$$R = \frac{kB}{Y}$$

$$RY = kB$$

$$\frac{RY}{B} = k$$

$$\frac{R_1 Y_1}{B_1} = k$$

$$\frac{R_2 Y_2}{B_2} = k$$

So, $\dfrac{R_1 Y_1}{B_1} = \dfrac{R_2 Y_2}{B_2}$

$$\frac{R_1 Y_1}{B} = \frac{R_2 Y_2}{B_2}$$

$$\frac{12 \cdot 2}{3} = \frac{14 \cdot 3}{B_2}$$

$$24 B_2 = 14 \cdot 9$$

$$B_2 = \frac{\cancel{14}^{7} \cdot 9}{\cancel{24}_{12}} = \frac{63}{12} = \boxed{\frac{21}{4}}$$

**Assignment:**

1. The number of yellows is directly proportional to the number of greens and inversely proportional to the number of whites squared. If the constant of proportionality is 14, write the yellows as a function of the greens and whites.

$$y = \frac{14G}{w^2}$$

2. Using the function developed in problem 1, how many whites would there be if there are 4 yellows and 6 greens?

$$y = \frac{14G}{w^2}$$

$$4 = \frac{14(6)}{w^2}$$

$$4w^2 = 84$$

$$w^2 = 21$$

$$w = \boxed{\pm\sqrt{21}}$$

3. The success of a particular business *S* is proportional to both the number of customers *N* and the quality of its products *Q*. Success is also inversely proportional to number of lazy workers *L*. Last year the success of the business was rated at 15. 1 while the number of customers was 1502, the quality of the products was rated at 5, and there were 23 lazy workers. What must be the quality number this year if the success number is to be 22 with only 1500 customers and 11 lazy workers?

$$S = \frac{kN \cdot Q}{L} \rightsquigarrow \frac{S_1 L_1}{N_1 Q_1} = \frac{S_2 L_2}{N_2 Q_2}$$

$$\frac{15.1(23)}{1502(5)} = \frac{22(11)}{1500\,Q_2}$$

$$15.1(23)(1500)Q_2 = 22(11)(1502)5$$

$$520950\,Q_2 = 1817420$$

$$Q_2 = \frac{1817420}{520950} = \boxed{3.489}$$

4. The relationship between *x* and *y* is $y = \frac{kx^3}{z}$. State this relationship in words using the words "directly", "inversely", etc.

y is directly proportional to the cube of x and inversely proportional to z.

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5. If *p* varies directly as *m*, directly as *q* squared, and inversely as *j*, what will be the new value of *m* when p = 6, q = 16, and j = 5? Formerly, p = 11, m= 10, q = 100, and j = 2.

$$p = \frac{kmq^2}{j} \rightarrow \frac{p_1 j_1}{m_1 q_1^2} = \frac{p_2 j_2}{m_2 q_2^2}$$

$$\frac{11 \cdot 2}{10 \cdot 100^2} = \frac{6 \cdot 5}{m_2 \cdot 16^2}$$

$$m_2\, 22 \cdot 16^2 = 30 \cdot 10 \cdot 100^2$$

$$m_2\, 5632 = 3{,}000{,}000$$

$$m_2 = \boxed{532.67}$$

# Enrichment Topic E

## (Scientific Notation)

In scientific notation, numbers are represented by:

- a mantissa…a number whose absolute value is between 1(inclusive) and 10(exclusive) and
- a characteristic… an exponent of 10.

$3.79 \times 10^4$

mantissa    characteristic

It is often necessary to convert numbers to scientific notation:

- Mantissa: Move the decimal place in the number so that the number's absolute value is between 1 (inclusive) and 10 (exclusive).
- Characteristic: If the decimal place was moved to the left *n* places, the characteristic will be *n*.

  If the decimal place was moved to the right *n* places, the characteristic will be -*n*.

**Example 1:** Convert 345 to scientific notation.

$345 = \boxed{3.45 \times 10^2}$

2

**Example 2:** Convert .00345 to scientific notation.

$.00345 = \boxed{3.45 \times 10^{-3}}$

3

Similarly, it is often necessary to convert numbers in scientific notation back to "normal" form:

> If the characteristic is positive, then move the decimal place in the mantissa that many places to the right.
>
> If the characteristic is negative, then move the decimal place in the mantissa that many places to the left.

**Example 3:** Convert 3.02 X $10^5$ to normal form.

$3.02 \times 10^5 = \boxed{\underbrace{302000}_{5}}$

**Example 4:** Convert 3.02 X $10^{-5}$ to normal form.

$3.02 \times 10^{-5} = \boxed{\underbrace{.0000302}_{5}}$

Occasionally, it is necessary to convert scientific notation into a form in which the characteristic (the exponent) is a specific number. This is easy using the following principal:

> If the characteristic is **decreased** by some number, then the mantissa must be **increased** by moving the decimal point by the same number of places.
>
> Similarly, if the characteristic is **increased** by some number, then the mantissa must be **decreased** by moving the decimal point by the same number of places.

**Example 5:** Rewrite 8.793 X $10^{-4}$ where the characteristic is 2.

$2-(-4)=6$

$8.793 \times 10^{-4}$

$= \boxed{.000008793 \times 10^{2}}$

6

This got smaller by 6 places

This got bigger by 6

**Example 6:** Rewrite 8.793 X $10^{-4}$ where the characteristic is –6.

$8.793 \times 10^{-4}$

$= \boxed{879.3 \times 10^{-6}}$

This got bigger by 2 places

This got smaller by 2

To multiply or divide numbers in scientific notation, follow the normal rules of combining exponents.

**Example 7:** (4.2 X $10^{5}$) (2 X $10^{-2}$)

$4.2 \times 10^{5}(2 \times 10^{-2})$

$= 8.4 \times 10^{5-2}$

$= \boxed{8.4 \times 10^{3}}$

**Example 8:** $\dfrac{4.2 \text{ X } 10^{5}}{2 \text{ X } 10^{-2}}$

$\dfrac{4.2 \times 10^{5}}{2 \times 10^{-2}} = 2.1 \times 10^{5-(-2)}$

$= 2.1 \times 10^{5+2}$

$= \boxed{2.1 \times 10^{7}}$

To add or subtract two numbers in scientific notation, adjust the characteristic of either (or both) number(s) so they are the same.

**Example 9:** 4.7 X $10^{3}$ + 2.1 X $10^{7}$

$4.7 \times 10^{3} + 2.1 \times 10^{7} = (4.7 \times 10^{3} + 21000. \times 10^{3})$

$= (4.7 + 21{,}000)10^{3} = 21{,}004.7 \times 10^{3}$

$= \boxed{2.10047 \times 10^{7}}$

**"Pure" scientific notation** is when the mantissa *n* satisfies $1 \le |n| < 10$.

Even when the mantissa **does not** satisfy this compound inequality (such as with $857.1 \times 10^3$) this form is still loosely referred to as "scientific notation".

See **Calculator Appendix L** for how to use scientific notation on the graphing calculator.

**Assignment:**

1. Convert 2,346 to scientific notation.

$2{,}346 = \boxed{2.346 \times 10^3}$

3

2. Convert .00781 to scientific notation.

$.00781 = \boxed{7.81 \times 10^{-3}}$

3

3. Convert 4.703 X $10^3$ to normal form.

$4.703 \times 10^3 = \boxed{4703.}$

3

4. Convert 2.6 X $10^{-4}$ to normal form.

$2.6 \times 10^{-4} = \boxed{.00026}$

4

5. Convert 10.36 to scientific notation.

$10.36 = \boxed{1.036 \times 10^1}$

1

6. Convert 1.26 X $10^{-2}$ to normal form

$1.26 \times 10^{-2} = \boxed{.0126}$

2

7. Write the equivalent of 4.73 X $10^3$ in which the characteristic is 6.

$4.73 \times 10^3 = \boxed{.00473 \times 10^6}$

3 smaller 3 larger

8. Write the equivalent of 20038 X $10^{-2}$ in which the characteristic is – 4.

$20038 \times 10^{-2} = \boxed{2003800 \times 10^{-4}}$

2 larger 2 smaller

9. Perform this addition and express the answer in pure scientific notation:
4 X $10^6$ + 5 X $10^7$

$4 \times 10^6 + 5 \times 10^7 = (.4 \times 10^7 + 5 \times 10^7) = (.4 + 5) \times 10^7$

$= \boxed{5.4 \times 10^7}$

10. Perform this addition and express the answer in pure scientific notation:
$2.78 \text{ X } 10^{3} + 5 \text{ X } 10^{-1}$

$2.78 \times 10^{3} + 5 \times 10^{-1}$
$= (2.78 \times 10^{3} + .0005 \times 10^{3}) = (2.78 + .0005) \times 10^{3}$
$= \boxed{2.7805 \times 10^{3}}$

11. Perform this subtraction and express the answer in pure scientific notation:
$4.08 \text{ X } 10^{-4} - 5 \text{ X } 10^{-1}$

$4.08 \times 10^{-4} - 5 \times 10^{-1}$
$= (.00408 \times 10^{-1} - 5 \times 10^{-1}) = \boxed{-4.99592 \times 10^{-1}}$

In the following problems, simplify and express in pure scientific notation:

12. $3 \text{ X } 10^{-1} (5.02 \quad 10^{4})$

$3 \times 10^{-1}(5.02 \times 10^{4})$
$= 15.06 \times 10^{4-1}$
$= 15.06 \times 10^{3}$
$= \boxed{1.506 \times 10^{4}}$

13. $4.45 \text{ X } 10^{-2} (126 \text{ X } 10^{-7})$

$4.45 \times 10^{-2}(126 \times 10^{-7})$
$= 560.7 \times 10^{-7-2}$
$= 560.7 \times 10^{-9}$
$= \boxed{5.607 \times 10^{-7}}$

14. $\dfrac{4 \text{ X } 10^{-3}}{6.72 \text{ X } 10^{5}}$

$\dfrac{4 \times 10^{-3}}{6.72 \times 10^{5}} = .5952 \times 10^{-3-5}$
$= .5952 \times 10^{-8}$
$= \boxed{5.952 \times 10^{-9}}$

15. $\dfrac{6.773 \text{ X } 10^{4}}{1.24 \text{ X } 10^{5}}$

$\dfrac{6.773 \times 10^{4}}{1.24 \times 10^{5}} = 5.4621 \times 10^{4-5}$
$= \boxed{5.4621 \times 10^{-1}}$

# Enrichment Topic F

## Greatest Common Factor (GCF)
## Least Common Multiple (LCM)

The greatest common factor (GCF) of several integers is the **largest integer that divides evenly** into them all.

To produce the GCF of a set of integers, produce the prime factors of each integer and then find the **intersection** set of those factors. The GCF is the **product** of those integers in the intersection set.

**Example 1:** Find the GCF of 16, 24, and 36.

*Prime factors of 16 = { 2, 2, 2, 2 }*
*Prime factors of 24 = { 2, 2, 2, 3 }*
*Prime factors of 36 = { 2, 2, 3, 3 }*

*The intersection of these three sets of prime factors is { 2, 2 }. Their product, the GCF, is 2(2) =* $\boxed{4}$.

The least common multiple (LCM) of several integers is the **smallest integer into which all the integers divide evenly**.

To produce the LCM of a set of integers, produce the prime factors of each integer and then find the **union** set of those factors. The LCM is the **product** of those integers in the union set.

**Example 2:** Find the LCM of 16, 24, and 36.

*Prime factors of 16 = { 2, 2, 2, 2 }*
*Prime factors of 24 = { 2, 2, 2, 3 }*
*Prime factors of 36 = { 2, 2, 3, 3 }*

*The union of these three sets of prime factors is { 2, 2, 2, 2, 3, 3 }. Their product, the LCM is =* $\boxed{144}$.

**Assignment:** For each of the given set of integers, find the LCM and GCF.

1. 6, 12, 18

*Prime factors of 6 = { 2, 3 }*
*Prime factors of 12 = { 2, 2, 3 }*
*Prime factors of 18 = { 2, 3, 3 }*

*The intersection of these three sets of prime factors is { 2, 3}. Their product, the, GCF, is 2(3) =* $\boxed{6}$ *.*

*The union of these three sets of prime factors is { 2, 2, 3, 3}. Their product, the, LCM, is 2(2)(3)(3) =* $\boxed{36}$ *.*

---

2. 22, 42, 68

*Prime factors of 22 = { 2, 11 }*
*Prime factors of 42 = { 2, 3, 7 }*
*Prime factors of 68 = { 2, 2, 17 }*

*The intersection of these three sets of prime factors is { 2}. The GCF is* $\boxed{2}$ *.*

*The union of these three sets of prime factors is { 2, 2, 3, 7, 11, 17}. Their product, the, LCM, is=* $\boxed{15{,}708}$ *.*

3. 126, 238, 476

*Prime factors of 126 = { 2, 3, 3, 7 }*
*Prime factors of 238 = { 2, 7, 17}*
*Prime factors of 476 = { 2, 2, 7, 17 }*

*The intersection of these three set of prime factors is { 2, 7}. The GCF is* $\boxed{14}$ .

*The union of these three sets of prime factors is { 2, 2, 3, 3, 7, 17}. Their product, the, LCM, is=* $\boxed{4{,}284}$ .

---

4. 8, 20, 900

*Prime factors of 8 = { 2, 2, 2 }*
*Prime factors of 20 = { 2, 2, 5}*
*Prime factors of 900 = { 2, 2, 3, 3, 5, 5 }*

*The intersection of these three set of prime factors is { 2, 2}. The GCF is* $\boxed{4}$ .

*The union of these three sets of prime factors is { 2, 2, 2, 3, 3, 5, 5 }. Their product, the, LCM, is=* $\boxed{1{,}800}$ .

# Enrichment Topic G

**(Derivation of the Quadratic Formula)**

Begin with the **general form of the quadratic equation**, $ax^2 + bx + c = 0$ and show that the two solutions are given by the **quadratic formula**:

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

solve by "completing the square"

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$\left(\frac{1}{2}\frac{b}{a}\right)^2=\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}\frac{4a}{4a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

The development above uses the technique of "Completing the Square". For more on this subject, see **Enrichment Topic H**.

# Enrichment Topic H

**(Completing the Square)**

Consider the following perfect squares. Notice the pattern:

$(x + 3)^2 = x^2 + 6x + 9 \rightarrow$ (half of 6)$^2$ is 9

$(x - 4)^2 = x^2 - 8x + 16 \rightarrow$ (half of –8)$^2$ is 16

$(x + 5)^2 = x^2 + 10x + 25 \rightarrow$ (half of 10)$^2$ is 25

**Example 1:** What would we need to add to $x^2 + 8x$ in order to make it a perfect square?

$$\left(\frac{8}{2}\right)^2 = (4)^2 = \boxed{16}$$

**Example 2:** What would we need to add to $x^2 + 9x$ in order to make it a perfect square?

$$\left(\frac{9}{2}\right)^2 = \boxed{\frac{81}{4}}$$

Quadratics can be solved by adding the appropriate quantity to both sides so as to complete the square:

$$x^2 + 6x = 2$$
$$x^2 + 6x + 9 = 2 + 9$$
$$\boxed{(x+3)^2 = 11}$$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

So add 9 to both sides

Now take the square root of both sides and solve for x. Be sure to **put a ± in front** of the numerical square root.

$$\sqrt{(x+3)^2} = \pm\sqrt{11}$$
$$x + 3 = \pm\sqrt{11}$$
$$x = \boxed{-3 \pm \sqrt{11}}$$

**Example 3:** Solve by completing the square. $x^2 + 12x = 13$

$$\left(\frac{12}{2}\right)^2 = 36$$

$$x^2 + 12x + 36 = 13 + 36$$

$$(x+6)^2 = 49$$

$$\sqrt{(x+6)^2} = \pm\sqrt{49}$$

$$x + 6 = \pm 7$$

$$x = -6 \pm 7$$

$$x = -6 + 7 = \boxed{1}$$

$$x = -6 - 7 = \boxed{-13}$$

**Example 4:** Solve by completing the square. $x^2 = 11x – 2$

$$x^2 - 11x = -2 \qquad \left(\frac{-11}{2}\right)^2 = \frac{121}{4}$$

$$x^2 - 11x + \frac{121}{4} = -2 + \frac{121}{4}$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{-2}{1}\cdot\frac{4}{4} + \frac{121}{4}$$

$$\sqrt{\left(x - \frac{11}{2}\right)^2} = \pm\sqrt{\frac{113}{4}}$$

$$x - \frac{11}{2} = \pm\frac{\sqrt{113}}{2}$$

$$x = \frac{11}{2} \pm \frac{\sqrt{113}}{2}$$

$$x = \boxed{\frac{11 \pm \sqrt{113}}{2}}$$

The “completing the square” rule of taking adding half of the linear coefficient and squaring **only works** when the **coefficient of the squared term is 1**. If that coefficient is other than one, then begin by **dividing by that coefficient**:

**Example 5:** Solve $2x^2 + 12x = 2$

$$\frac{2x^2}{2} + \frac{12x}{2} = \frac{2}{2} \qquad \left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

$$x^2 + 6x = 1$$

$$x^2 + 6x + 9 = 1 + 9$$

$$(x+3)^2 = 10$$

$$\sqrt{(x+3)^2} = \pm\sqrt{10}$$

$$x + 3 = \pm\sqrt{10}$$

$$x = \boxed{-3 \pm \sqrt{10}}$$

**Assignment:** What needs to be added to the following two polynomials in order to make them a perfect square?

| | |
|---|---|
| 1. $x^2 + 22x$<br>$\left(\frac{22}{2}\right)^2 = (11)^2 = \boxed{121}$ | 2. $x^2 + 100x$<br>$\left(\frac{100}{2}\right)^2 = (50)^2 = \boxed{2500}$ |

Solve the following equations by completing the square.

| | |
|---|---|
| 3. $x^2 + 2x = 8$<br>$\left(\frac{2}{2}\right)^2 = 1$<br>$x^2+2x+1=8+1$<br>$(x+1)^2 = 9$<br>$\sqrt{(x+1)^2} = \pm\sqrt{9}$<br>$x+1 = \pm 3$<br>$x = -1 \pm 3$<br>$x = -1+3 = \boxed{2}$<br>$x = -1-3 = \boxed{-4}$ | 4. $x^2 - 12x = -4$<br>$\left(-\frac{12}{2}\right)^2 = (-6)^2 = 36$<br>$x^2-12x+36 = -4+36$<br>$(x-6)^2 = 32$<br>$\sqrt{(x-6)^2} = \pm\sqrt{32} = \pm\sqrt{16 \cdot 2}$<br>$x-6 = \pm 4\sqrt{2}$<br>$x = \boxed{6 \pm 4\sqrt{2}}$ |
| 5. $x^2 + x = 3/4$<br>$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$<br>$x^2+x+\frac{1}{4} = \frac{3}{4}+\frac{1}{4}$<br>$\left(x+\frac{1}{2}\right)^2 = 1$<br>$\sqrt{\left(x+\frac{1}{2}\right)^2} = \pm\sqrt{1}$<br>$x+\frac{1}{2} = \pm 1$<br>$x = -\frac{1}{2}+1 = \frac{1}{2}$<br>$x = -\frac{1}{2}-1 = -\frac{3}{2}$ | 6. $x^2 - 18x + 4 = 0$<br>$x^2 - 18x = -4$<br>$\left(-\frac{18}{2}\right)^2 = 81$<br>$x^2-18x+81 = -4+81$<br>$(x-9)^2 = 77$<br>$\sqrt{(x-9)^2} = \pm\sqrt{77}$<br>$x-9 = \pm\sqrt{77}$<br>$x = \boxed{9 \pm \sqrt{77}}$ |

**7. $x^2 + .5x = .1$

$\left(\frac{.5}{2}\right)^2 = .25/4$

$= .0625$

$x^2 + .5x + .0625 = .1 + .0625$

$(x+.25)^2 = .1625$

$\sqrt{(x+.25)^2} = \pm\sqrt{.1625}$

$x + .25 \approx \pm .403$

$x \approx -.25 \pm .403$

$x \approx -.25 + .403 = \boxed{.153}$

$x \approx -.25 - .403 = \boxed{-.653}$

8. $4x^2 + 16x = 4$

$\frac{4x^2}{4} + \frac{16x}{4} = \frac{4}{4}$ $\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$x^2 + 4x = 1$

$x^2 + 4x + 4 = 1 + 4$

$(x+2)^2 = 5$

$\sqrt{(x+2)^2} = \pm\sqrt{5}$

$x + 2 = \pm\sqrt{5}$

$x = \boxed{-2 \pm \sqrt{5}}$

9. $3 - 2x - x^2 = 0$

$-x^2 - 2x = -3$ $\left(\frac{2}{2}\right)^2 = 1$

$x^2 + 2x = 3$

$x^2 + 2x + 1 = 3 + 1$

$(x+1)^2 = 4$

$\sqrt{(x+1)^2} = \pm\sqrt{4}$

$x + 1 = \pm 2$

$x = -1 \pm 2 = -1 + 2 = \boxed{1}$

$x = -1 - 2 = \boxed{-3}$

10. $6x + 1 = x^2$

$1 = x^2 - 6x$ $\left(\frac{-6}{2}\right)^2 = 9$

$1 + 9 = x^2 - 6x + 9$

$10 = (x-3)^2$

$\pm\sqrt{10} = x - 3$

$\boxed{3 \pm \sqrt{10}} = x$

11. $.3x^2 - 30x = .3$

$$\frac{.3x^2}{.3} - \frac{30x}{.3} = \frac{.3}{.3}$$

$$x^2 - 100x = 1 \quad \left(\frac{100}{2}\right)^2 = (50)^2$$

$$x^2 - 100x + 2500 = 1 + 2500$$

$$(x-50)^2 = 2501$$

$$\sqrt{(x-50)^2} = \pm\sqrt{2501}$$

$$x - 50 = \pm\sqrt{2501}$$

$$x = \boxed{50 \pm \sqrt{2501}}$$

12. $8x - 2x^2 + 14 = 0$

$$\frac{8x}{-2} - \frac{2x^2}{-2} + \frac{14}{-2} = 0$$

$$-4x + x^2 - 7 = 0 \quad \left(\frac{-4}{2}\right)^2 = 4$$

$$x^2 - 4x = 7$$

$$x^2 - 4x + 4 = 7 + 4$$

$$(x-2)^2 = 11$$

$$\sqrt{(x-2)^2} = \pm\sqrt{11}$$

$$x - 2 = \pm\sqrt{11}$$

$$x = \boxed{2 \pm \sqrt{11}}$$

13. $106x^2 + 106x = 106$

Divide by 106 and get:

$$x^2 + x = 1 \quad \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$x^2 + x + \frac{1}{4} = 1 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \pm\sqrt{\frac{5}{4}}$$

$$x + \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

$$x = \boxed{\frac{-1 \pm \sqrt{5}}{2}}$$

**14. $ax^2 + bx + c$ (Your answer will be in terms of a, b, and c.)

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0 \quad \left(\frac{b}{a}\cdot\frac{1}{2}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\cdot\frac{4a}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

# Enrichment Topic I

**(Statistics)**

Consider the following set of test grades:

{ 92, 99, 85, 95, 79, 64, 79, 91, 56, 82, 81 }

Now arrange them in ascending order:

( 56, 64, 79, 79, 81, 82, 85, 91, 92, 95, 99 }

The **range** is the difference between the highest and lowest numbers:

$99 - 56 = \boxed{43}$

The **mean** is the average:

$$\frac{56 + 64 + \ldots + 95 + 99}{11} = \boxed{82.\overline{09}}$$

The **mode** is the value that occurs most frequently:

(It is possible to have more than one mode or to have none.)

79 occurs twice

The **median** is the number in the middle of the **ordered** list:

(If there are two numbers in the middle, average them.)

82

The **lower quartile** is the median of the lower half of the numbers:

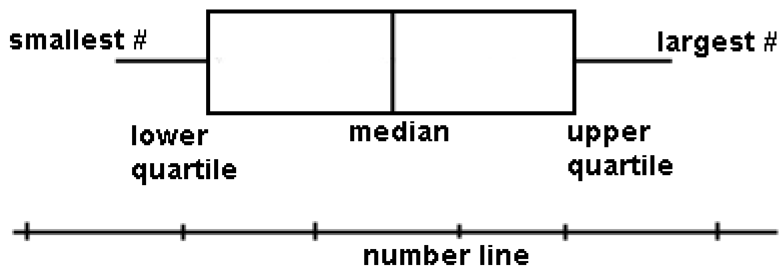
79

The **upper quartile** is the median of the upper half of the numbers:

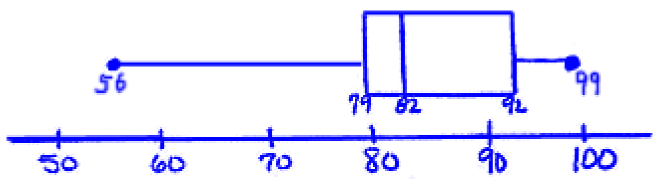
(When there are an odd number of data points, the middle value is not used when finding either quartile.)

92

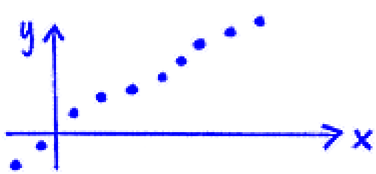
A **box and whisker plot** is often used to display some of the statistics for a set of data:



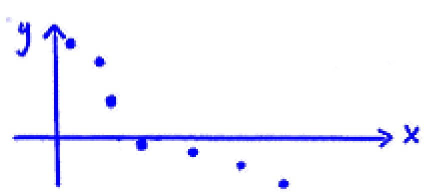

**Example:** Make a box and whisker plot for the data on the preceding page:




**Positive correlation** between variables is when a line of best-fit has a **positive slope**.




**Negative correlation** between variables is when a line of best-fit has a **negative slope**.




It is **possible to have neither** negative nor positive correlation.

See **Calculator Appendix P** and an associated video for how to produce statistics on a graphing calculator.

**Assignment:**

Problems 1-7 refer to the data from the weigh-in at a weight watchers club:
{ 95, 306, 298, 250, 200, 202, 502, 601, 332, 212 }

95, 200, 202, 212, 250, 298, 306, 332, 502, 601
LQ UQ

1. What is the mean?

$\frac{95+200+\ldots+502+601}{10} = \boxed{299.8}$

2. What is the mode?

None, nothing repeated

3. What is the median?

Average 250+298

$\frac{250+298}{2} = \boxed{274}$

4. What is the range?

$601 - 95 = \boxed{506}$

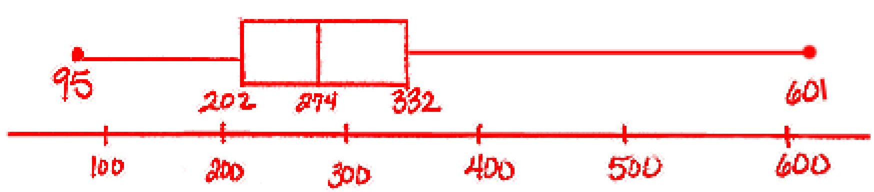
5. What is the upper quartile?

332

6. What is the lower quartile?

202

7. Make a box and whisker plot for this data.



Problems 8-14 refer to weights of the San Pedro Armadillo's JV football team:
{ 130, 195, 218, 180, 160, 180, 401, 145, 121, 192, 146}

121, 130, 145, 146, 160, 180, 180, 192, 195, 218, 401

LQ UQ

8. What is the mean?

$$\frac{121 + 130 + \ldots + 218 + 401}{11} = \boxed{188}$$

9. What is the mode?

180 repeated twice

10. What is the median?

180

11. What is the range?

$401 - 121 = \boxed{280}$

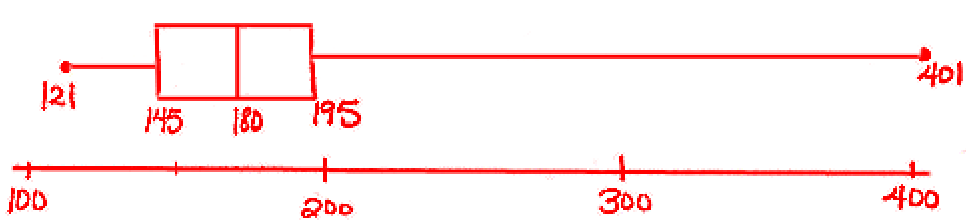
12. What is the upper quartile?

195

13. What is the lower quartile?

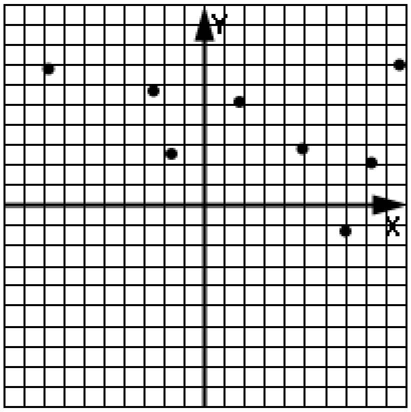
145

14. Make a box and whisker plot for this data.



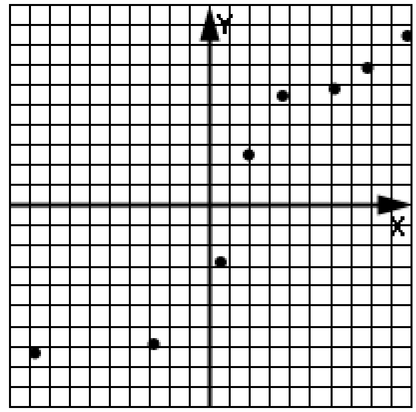
In problems 15-18 decide if there is a negative or positive correlation (or none) between the variables.

15.



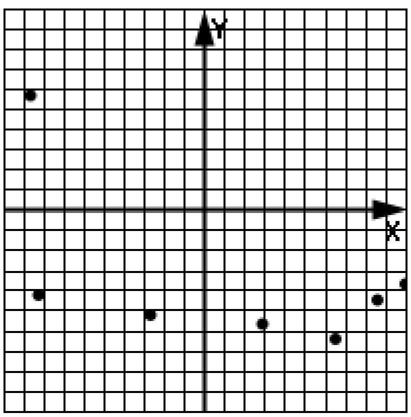

None

16.



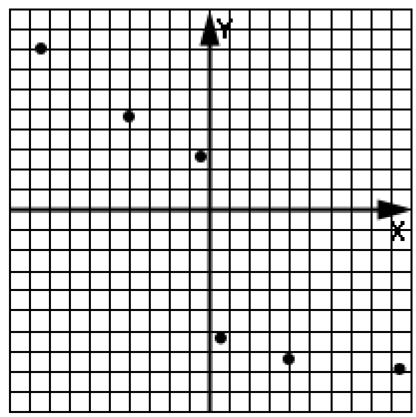

Positive Correlation

17.




None

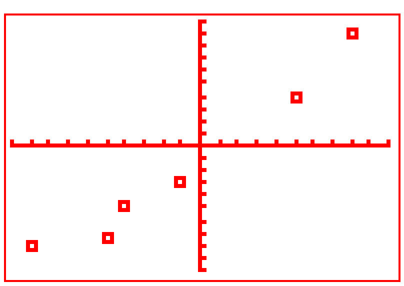
18.




Negative Correlation

*19. Make a scatter-plot of the data presented in the table and decide what type of correlation is represented:

| X | Y |
|---|---|
| 8 | 9 |
| 5 | 4 |
| -1 | -3 |
| -4 | -5 |
| -5 | -7.5 |
| -9 | -8 |



Positive Correlation

# Enrichment Topic J

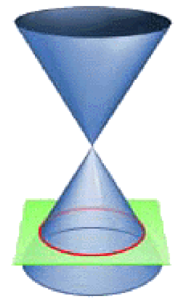
**(Conic Section Applications)**

Applications of the four conic sections are beautifully illustrated at:

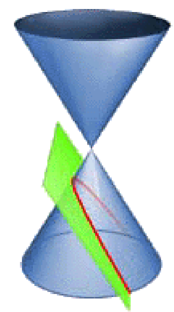
http://britton.disted.camosun.bc.ca/jbconics.htm

Applications of the **circle**:

No applications are given for this obvious category since they abound in everyday life.

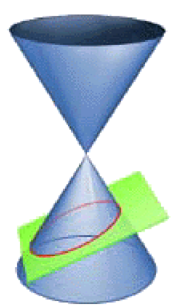


Applications of the **parabola**:



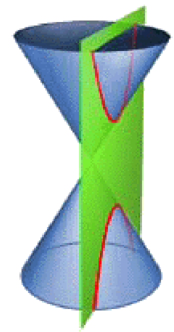
- Some comets pass around the sun in a parabolic path.
- Projectiles follow a parabolic path (a tossed ball).
- The mirrors in flashlights and reflecting telescopes are parabolic in shape.
- The cables of a suspension bridge hang in the shape of a parabola

Applications of the **ellipse**:



- Planets orbit about the sun in an elliptical path with the sun at one of the foci.
- Light passing through one focal point will reflect off the parabola and pass through the other focal point. The same thing happens with sound as evidenced in "whispering galleries".
- A right circular cylinder sliced at an angle produces an ellipse
- Elliptical gears provide variable rotational speed about an axis passing through a focal point.

Applications of the **hyperbola**:



- Some comets pass around the sun in a hyperbolic path.
- Charged particles fired at heavy nuclei of atoms are deflected into a hyperbolic path.
- Hyperbolic tracking (a navigational technique called Loran) uses intersecting hyperbolas to locate the positions of airplanes and ships.
- The shock wave of a sonic boom intersects the ground in the shape of a hyperbola.
- Hyperbolic as well as parabolic mirrors and lenses are used in systems of telescopes.

See www.youtube.com/watch?v=XDLyiEWcj_Y for a humorous video about the applications of conic sections.

# Enrichment Topic K

**Forms of quadratic Equations**

Quadratic functions form parabolas when graphed. A quadratic function is always recognizable because there will be only one *y* term and its exponent will be 1. The other “side” of the equation will be a polynomial with variable *x* of degree 2 (highest power is 2).

**Example 1:** Which of the following are quadratic functions that form parabolas?

(a) $y = 3x^2 – 2x + 6$ (b) $3y - 6x + 2x^2 = 5$ (c) $y^2 = 5x – x^2 + 1$
(d) $y = \sqrt{x^2 - 6x - 79}$ (e) $y = 1/x^2 + 2$ (f) $y = x^3 - 2x + 19$

a, b

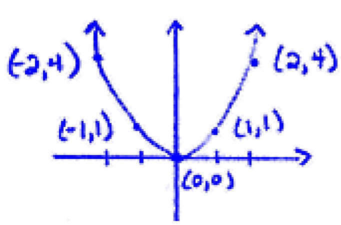
---

The simplest parabola is the parent function:

$y = x^2$

Make an x-y chart using *x* as the independent variable. Then graph the points to form the points of this parent function parabola:

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| −1 | 1 |
| −2 | 4 |




**General form: $y = f(x) = ax^2 + bx + c$**

Vertex → $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

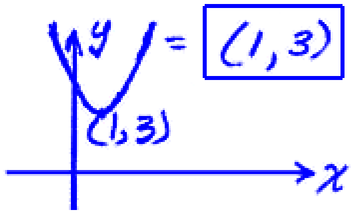
If *a* is positive the parabola goes up:

If *a* is negative the parabola goes down:

**Example 2:** Find the vertex of $f(x) = 2x^2 - 4x + 5$ and then make a rough sketch of the graph of the parabola.

$$\text{Vertex} \rightarrow \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \boxed{(1,3)} \qquad \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$$

$$f(1) = 2(1)^2 - 4(1) + 5$$
$$= 2 - 4 + 5 = 3$$



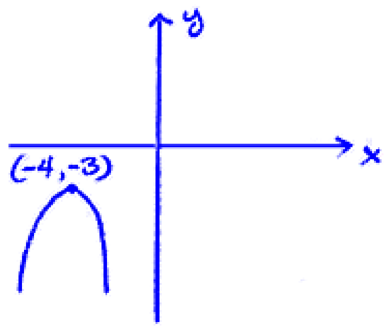

**Vertex form: $y = f(x) = a(x - h)^2 + k$**

Vertex → (h, k)
If *a* is positive the parabola goes up: $\cup$

If *a* is negative the parabola goes down: $\cap$

**Example 3:** Find the vertex of $y = -3(x + 4)^2 - 3$ and then make a rough sketch of the graph of the parabola.

$$y = -3(x+4)^2 - 3$$
$$= -3(x-(-4))^2 - 3$$
$$= a(x - h)^2 + k$$
$$h = -4$$
$$k = -3$$
$$\text{Vertex} \rightarrow \boxed{(-4,-3)}$$




**Root form:** $y = f(x) = a(x - r_1)(x - r_2)$

$r_1$ and $r_2$ are the roots.
If *a* is positive the parabola goes up: $\cup$

If *a* is negative the parabola goes down: $\cap$

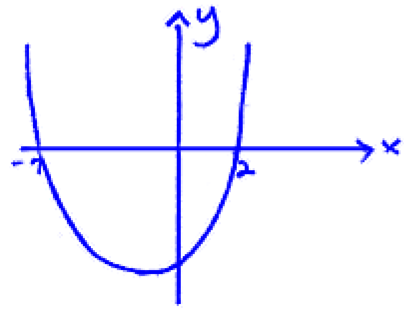
**Example 4:** Identify the roots for y = f(x) = 4(x –2)(x + 7) and then make a rough sketch of the graph of the parabola.

$y = 4(x-2)(x+7)$

$y = 4(x-2)(x-(-7))$

$y = a(x-r_1)(x-r_2)$

$r_1 = \boxed{2}$ $r_2 = \boxed{-7}$



**Assignment:**

Identify the vertex and make a rough sketch of the graphs of the parabolas.

1. $y = -3x^2 - 2x + 1$

$a = -3 \quad b = -2$

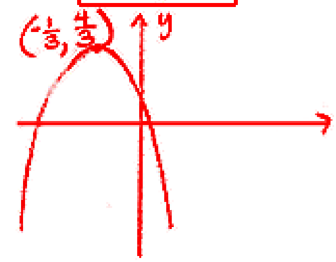
$\frac{-b}{2a} = -\frac{-2}{2(-3)} = \frac{-1}{3}$

$f(-\frac{1}{3}) = -3(-\frac{1}{3})^2 - 2(-\frac{1}{3}) + 1$

$= -\frac{1}{3} + \frac{2}{3} + 1$

$= \frac{1}{3} + \frac{3}{3} = \frac{4}{3}$

Vertex: $\boxed{(-\frac{1}{3}, \frac{4}{3})}$




2. $f(x) = 5x^2 - 6x + 2$

$a = 5 \quad b = -6$

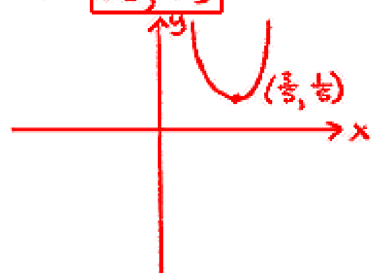
$\frac{-b}{2a} = -\frac{-6}{2(5)} = \frac{6}{10} = \frac{3}{5}$

$f(\frac{3}{5}) = 5(\frac{3}{5})^2 - 6\,\frac{3}{5} + 2$

$= 5\,\frac{9}{25} - \frac{18}{5} + 2$

$= \frac{9}{5} - \frac{18}{5} + \frac{10}{5} = \frac{1}{5}$

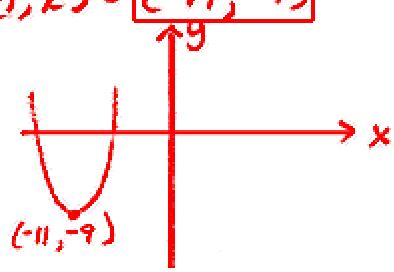
Vertex: $\boxed{(\frac{3}{5}, \frac{1}{5})}$




3. $f(x) = 4(x + 11)^2 - 9$

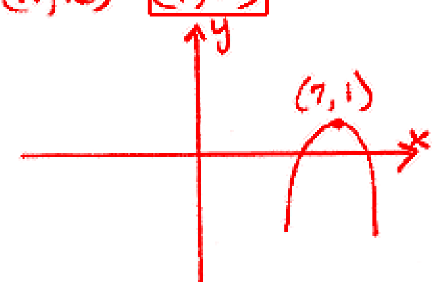
$f(x) = 4(x-(-11))^2 - 9$

$(h, k) = \boxed{(-11, -9)}$




4. $y = -2(x - 7)^2 + 1$

$(h, k) = \boxed{(7, 1)}$

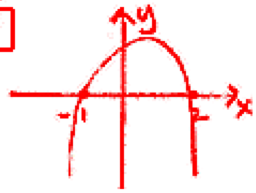


Identify the roots and make a rough sketch of the graphs of the parabolas.

5. $f(x) = -4(x-2)(x+1)$

$y = -4(x-2)(x-(-1))$

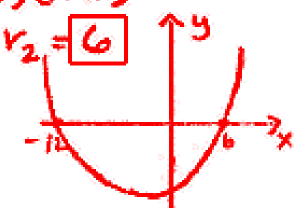
$r_1 = \boxed{2}$ $r_2 = \boxed{-1}$




6. $y = (x+12)(x-6)$

$y = (x-(-12))(x-6)$

$r_1 = \boxed{-12}$ $r_2 = \boxed{6}$




In the following problems, identify the functions that are parabolas. It is possible to have more than one answer on each problem.

7.

(a) $f(x) = 32x - 16x^2 + 1$

(b) $y = 6x$

(c) $y = x^2 + x + x^3$

(d) $g(x) = 1/(x^2 + 1)$

(e) $h(x) = 1/(x^{-2})$

a, e

8.

(a) $f(x) = \sqrt[3]{x^2} + 22$

(b) $y = 1/(x^2 + 2x + 3)$

(c) $k(x) = x^2 + 1/x^{-1} + 10$

(d) $y = x^2 + 1/x + 10$

(e) $y = 2^x + x + 1$

c

9. Write the general form of a quadratic function.

$y = f(x) = ax^2 + bx + c$

10. Write the vertex form of a quadratic function having vertex (h, k).

$y = f(x) = a(x-h)^2 + k$

11. Write the root form of quadratic function having roots $r_1$ and $r_2$.

$y = f(x) = a(x-r_1)(x-r_2)$

12. Write the simplest equation for a parabola (it's called the quadratic parent function).

$y = x^2$

# Enrichment Topic L

## Writing Quadratic Functions

Decide which of the following two forms of quadratic function can be used based on the given information in a problem:

Vertex form: $f(x) = a(x - h)^2 + k$
Root form: $f(x) = a(x - r_1)(x - r_2)$

**Example 1:**
Write the quadratic function having vertex (–7, 2) and passing through the point (1, 5).

$(h, k) = (-7, 2)$
$y = a(x-h)^2 + k$
$y = a(x+7)^2 + 2$
now sub in (1,5)
$5 = a(1+7)^2 + 2$
$5 = a\,64 + 2$
$3 = 64a$
$\frac{3}{64} = a$

$y = a(x-h)^2 + k$
$\boxed{y = \frac{3}{64}(x+7)^2 + 2}$

**Example 2:**
Write the equation for the parabola having roots 1 & –6 and passing through the point (0, 4).

$r_1 = 1 \quad r_2 = -6$
$y = a(x-r_1)(x-r_2)$
$y = a(x-1)(x+6)$
now sub in (0,4)
$4 = a(0-1)(0+6)$
$4 = a(-6)$
$-\frac{2}{3} = a$

$y = a(x-r_1)(x-r_2)$
$\boxed{y = -\frac{2}{3}(x-1)(x+6)}$

**Assignment:** Use the given information to find the equation of the parabolas.

1. Vertex (–8, 2) and passing through (5, –22).

$(h,k) = (-8,2)$
$y = a(x-h)^2 + k$
$y = a(x+8)^2 + 2$
Now sub in $(5,-22)$
$-22 = a(5+8)^2 + 2$
$-22 = a(13)^2 + 2$
$-24 = 169a$
$a = -\frac{24}{169}$
$y = a(x-h)^2 + k$
$\boxed{y = -\frac{24}{169}(x+8)^2 + 2}$

2. Roots 4 & –7 and passing through (–1, 6).

$r_1 = 4, \; r_2 = -7$
$y = a(x-r_1)(x-r_2)$
$y = a(x-4)(x+7)$
sub in $(-1,6)$
$6 = a(-1-4)(-1+7)$
$6 = a(-5)(6)$
$6 = -30a$
$-\frac{6}{30} = a$
$a = -\frac{1}{5}$
$y = a(x-r_1)(x-r_2)$
$\boxed{y = -\frac{1}{5}(x-4)(x+7)}$

3. Vertex down three units from the origin and passing through (–11, 3).

$(h,k) = (0,-3)$
$y = a(x-h)^2 + k$
$y = a(x-0)^2 - 3$, sub in $(-11,3)$
$3 = a(-11)^2 - 3$
$6 = 121a$
$a = \frac{6}{121}$
$y = a(x-h)^2 + k$
$y = \boxed{\frac{6}{121}(x)^2 - 3}$

4. Roots 0 & 4 and passing through (1, –1).

$r_1 = 0, \; r_2 = 4$
$y = a(x-r_1)(x-r_2)$
$y = a(x)(x-4)$ sub in $(1,-1)$
$-1 = a1(1-4)$
$-3a = -1$
$a = \frac{1}{3}$
$y = a(x-r_1)(x-r_2)$
$y = \boxed{\frac{1}{3}(x)(x-4)}$

5. Vertex (–6, 4) and having a root at x = 8.

$(h, k) = (-6, 4)$ point $(8, 0)$
$y = a(x-h)^2 + k$
$y = a(x+6)^2 + 4$
$0 = a(8+6)^2 + 4$
$-4 = 196a$
$a = -\frac{1}{49}$
$y = a(x-h)^2 + k$
$y = \boxed{-\frac{1}{49}(x+6)^2 + 4}$

6. Having two roots, y-intercept –1, and with vertex (–1, 22).

$(h, k) = (-1, 22)$ point $(0, -1)$
$y = a(x-h)^2 + k$
$y = a(x+1)^2 + 22$
$-1 = a(0+1)^2 + 22$
$-23 = a$
$y = a(x-h)^2 + k$
$y = \boxed{-23(x+1)^2 + 22}$

7. Having roots 16 & 1, f(2) = –8.

$r_1 = 16,\ r_2 = 1$ point $(2, -8)$
$y = a(x-r_1)(x-r_2)$
$y = a(x-16)(x-1)$ sub $(2, -8)$
$-8 = a(2-16)(2-1)$
$-8 = a(-14)(1)$
$a = \frac{8}{14} = \frac{4}{7}$
$y = a(x-r_1)(x-r_2)$
$y = \boxed{\frac{4}{7}(x-16)(x-1)}$

*8. f(–2) = 1, f(–6) = 0, and having a root at x = 1.

point $(-2, 1)$ $r_1 = -6,\ r_2 = 1$
$y = a(x-r_1)(x-r_2)$
$y = a(x+6)(x-1)$ sub $(-2, 1)$
$1 = a(-2+6)(-2-1)$
$1 = a\,4(-3)$
$a = -\frac{1}{12}$
$y = a(x-r_1)(x-r_2)$
$y = \boxed{-\frac{1}{12}(x+6)(x-1)}$

9. Double root at x = 7 and passing through (3, 2).

$r_1 = 7$ , $r_2 = 7$

$y = a(x - r_1)(x - r_2)$

$y = a(x-7)(x-7)$ sub (3,2)

$2 = a(3-7)(3-7)$

$2 = 16a$

$a = \frac{1}{8}$

$y = a(x-r_1)(x-r_2)$

$\boxed{y = \frac{1}{8}(x-7)^2}$

10. Vertex at (2, 5) and passing through the y-intercept of 5x – 5y = 15.

$(h,k) = (2,5)$ $-5y = -5x + 15$

$y = a(x-h)^2 + k$ $y = x - 3$

$y = a(x-2)^2 + 5$ $(0,-3)$

sub in $(0,-3)$

$-3 = a(0-2)^2 + 5$

$-3 = 4a + 5$

$-8 = 4a$

$a = -2$

$y = a(x-h)^2 + k$

$\boxed{y = -2(x-2)^2 + 5}$

*11. The line y = –2x + 1 intersects the parabola at x = –2. The vertex of the parabola is at the origin.

$y = -2(-2) + 1$

$y = 5$, point $(-2, 5)$

$(h,k) = (0,0)$

$y = a(x-h)^2 + k$

$y = a(x-0)^2 + 0$

sub in $(-2,-5)$

$5 = a(-2)^2$

$a = \frac{5}{4}$ $y = \frac{5}{4}(x-0)^2 + 0$

$\boxed{y = \frac{5}{4}x^2}$

*12. A horizontal line at y = 6 intersects the parabola at (5, 6) & (9, 6). The vertex is 4 units above this line.

$h = \frac{5+9}{2} = 7$ $k = 6+4 = 10$

$y = a(x-h)^2 + k$

$y = a(x-7)^2 + 10$ sub in $(5,6)$

$6 = a(5-7)^2 + 10$

$6 = 4a + 10$

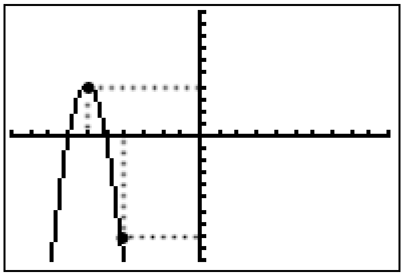
$a = -1$

$y = a(x-h)^2 + k$

$\boxed{y = -1(x-7)^2 + 10}$

In the following two problems, use the information from the graphs to write the corresponding quadratic functions. The tic marks represent 1 unit.

13.



$(h,k)=(-6,4)$ point $(-4,-8)$

$y=a(x-h)^2+k$

$y=a(x+6)^2+4$ sub in $(-4,-8)$

$-8=a(-4+6)^2+4$

$-8=a(2)^2+4$

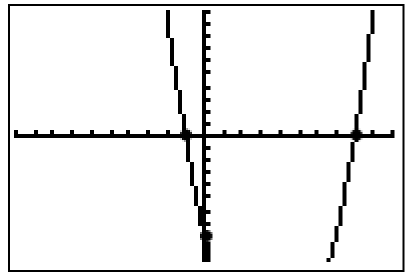
$-12=4a$

$a=-3$

$y=a(x-h)^2+k$

$\boxed{y=-3(x+6)^2+4}$

14.



$r_1=-1$ $r_2=8$ point $(0,-8)$

$y=a(x-r_1)(x-r_2)$

$y=a(x+1)(x-8)$

sub in $(0,-8)$

$-8=a(0+1)(0-8)$

$-8=-8a$

$a=1$

$y=a(x-r_1)(x-r_2)$

$\boxed{y=(x+1)(x-8)}$