Blue Pelican Alg II

First Semester

Absent-student Version 1.01

Alg II Syllabus (First Semester)

Unit 1: Solving linear equations and inequalities

Lesson 01: Solving linear equations

Lesson 02: Solving linear inequalities (See Calculator Appendix A and associated video.)

Lesson 03: *Solving combined (compound) inequalities

Lesson 04: Converting words to algebraic expressions

Lesson 05: Solving word problems with linear equations

Lesson 06: *Graphing calculator solutions of absolute value problems (See Calculator Appendix D & associated video, and Enrichment Topic A.)

Unit 1 review
Test: Unit 1 test

Unit 2: Slope; Solving a linear system of two equations

Lesson 01: Slopes of lines: four different points of view

Lesson 02: Two forms for the equation of a line

Lesson 03: Graphical meaning of the solution to two linear equations

Lesson 04: Algebraic solutions (elimination & substitution) for two linear equations

Lesson 05: Word problems involving two linear equations

Lesson 06: Graphing calculator solutions of linear systems (See Calculator Appendix C and associated video.)

Unit 2 review
Test: Unit 2 test

Unit 3: Graphing linear inequalities in two variables

Lesson 01: Graphing single linear inequalities in two variables

Lesson 02: Graphing systems of linear inequalities in two variables

Lesson 03: *Graphing calculator- graphing systems of linear inequalities in two variables (See Calculator Appendices B & E and associated videos. Also see Enrichment Topic B.)

Cumulative review, unit 3
Unit 3 review
Test: Unit 3 test

Unit 4: Multiplying and Factoring Polynomials

Lesson 01: Simple polynomial multiplication and factoring

Lesson 02: $(a + b)^2$, $(a – b)^2$, $(a – b)(a + b)$--- multiplying and factoring

Lesson 03: More trinomial factoring (Leading coefficient not one)

Lesson 04: Solving equations by factoring

Lesson 05: *Solving word problems with factoring

Lesson 06: *Binomial expansion theorem

Cumulative review, unit 4
Unit 4 review
Unit 4 test

Unit 5: Exponents and radicals

Lesson 01: Exponent rules (This lesson will likely span two days)

Lesson 02: Negative exponents

Lesson 03: More exponent problems

Lesson 04: Simplifying radical expressions

Lesson 05: Fractional exponents

Lesson 06: *Solving equations having rational & variable exponents

Lesson 07: *Solving radical equations

Lesson 8: Rationalizing denominators

Cumulative review, unit 5
Unit 5 review
Unit 5 test

Unit 6: Completing the square, the quadratic formula

Lesson 1: Solving equations by taking the square root

Lesson 2: Completing the square

Lesson 3: *Deriving the quadratic formula

Lesson 4: Using the quadratic formula

Lesson 5: Determining the nature of the roots; The discriminant

Cumulative review, unit 6
Unit 6 review
Unit 6 test

Unit 7: Relations and functions

Lesson 1: Representations of relations and functions

Lesson 2: Independent & dependent variables; Domain & range (See Calculator Appendix F and associated video.)

Lesson 3: Function notation; Evaluating functions

Lesson 4: *Even and odd functions (See Calculator Appendix G and associated video.)

Lesson 5: Putting it all together: x-axis & y-axis associations

Cumulative review, unit 7
Unit 7 review
Unit 7 test

Unit 8: Analyzing and graphing quadratic functions

Lesson 1: Forms of quadratic functions

Lesson 2: Finding intercepts and graphing quadratic equations

Lesson 3: *Analysis of quadratic functions

Lesson 4: Using graphs to analyze quadratic transformations

Lesson 5: *Writing quadratic functions

Lesson 6: Analyzing quadratic functions with a graphing calculator

Lesson 7: *Quadratic inequalities

Cumulative review, unit 8
Unit 8 review
Unit 8 test

Unit 9: Reflections, translations, and inverse functions

Lesson 1: Reflection fundamentals

Lesson 2: Translations and reflection of relations

Lesson 3: *Inverse function fundamentals

Lesson 4: *Determining if two relations are inverses of each other

Cumulative review, unit 9
Unit 9 review
Unit 9 test

Semester summary

Semester review
Semester test

Enrichment Topics

Topic A: Analysis of absolute value inequalities

Topic B: Linear Programming

Topic C: Point-slope and intercept forms of a line

Topic D: The summation operator, Σ

Topic E: An unusual look at probability

Topic F: Rotations

Topic G: Absolute value parent functions

Topic H: Dimension changes affecting perimeter, area, and volume

Topic I: Algebraic solution to three equations in three variables

Topic J: Algebraic solution to quadratic systems of equations.

Topic K: Derivation of the sine law

Topic L: Derivation of the cosine law

Topic M: Tangent composite function derivations

Topic N: Locating the vertex of a standard-form parabola

Topic O: Algebraic manipulation of inverse trig functions

Topic P: Logarithm theorem derivations

Topic Q: Arithmetic and geometric sum formulas

Topic R: Converting general form conics to standard form

Topic S: Conic section applications

Topic T: A close look at composite functions
Restrictions on the domain

Topic U: "Box" method of trinomial factoring

Alg II, Unit 1

Solving linear equations and inequalities

Unit 1: Lesson 01 Solving linear equations

Solve this linear equation: $3x - 4 = 5x + 1$

The goal is to first get all the x's on the left side of the equation and all the "other stuff" on the right side. In earlier courses we did this by first adding -5x to both sides as follows:

$$\begin{array}{rcl} 3x - 4 &=& \cancel{5x} + 1 \\ -5x & & \cancel{-5x} \\ \hline -2x - 4 &=& 1 \end{array}$$

In this course we will begin taking short-cuts to speed things along. Beginning again with $3x - 4 = 5x + 1$, we say that we will "transpose" (move) the 5x to the left side. When we do, we change its sign. The result that is equivalent to the above is:

$$3x - 4 = 5x + 1$$
$$-5x + 3x - 4 = 1$$

Combine the $-5x$ and $3x$ and then transpose the -4 to the right side and change its sign to get:

$$-2x - 4 = 1$$
$$-2x = 1 + 4$$
$$x = \boxed{\frac{5}{-2}}$$

Notice the last step where we divided both sides by -2.

In the following examples, solve for x:

Example 1: 2(4x – 9) = x +2

$$2(4x-9) = x+2$$
$$8x-18 = x+2$$
$$8x-x = 2+18$$
$$7x = 20$$
$$x = \boxed{\frac{20}{7}}$$

Example 2: x – 5x = 2(–3x + 4)

$$x-5x = 2(-3x+4)$$
$$-4x = -6x+8$$
$$-4x+6x = 8$$
$$2x = 8$$
$$x = \boxed{4}$$

Example 3: $\left(\frac{1}{2}\right)x + 6 = 14$

$$\left(\frac{1}{2}\right)x + 6 = 14$$
$$\frac{1}{2}x = 14-6$$
$$\cancel{2}\,\frac{1}{\cancel{2}}x = 8\cdot 2$$
$$x = \boxed{16}$$

Example 4: $\frac{4(x-5)}{2} = 7x$

$$\cancel{2}\,\frac{4(x-5)}{\cancel{2}} = 7x$$
$$2x-10 = 7x$$
$$2x-7x = 10$$
$$-5x = 10$$
$$x = \frac{10}{-5} = \boxed{-2}$$

Assignment:

In problems 1 – 16, solve for the variable:

1. $12x = 24$

2. $x + 3 = 8$

3. $\frac{x}{2} = 11$

4. $-11x = -33$

5. $-4 + \left(\frac{1}{3}\right)x = 4$

6. $3x = -2x + 10$

7. $-x - 2 = 22x + 1$

8. $7(x - 2) = 4(x + 15)$

9. $5(x-6)+\ 1=4(-1+x)$

10. $12x-(x-1)=11+4x$

11. $x(1+5)=x(2-7)+2$

12. $-(x+1)=3(x-6)$

13. $-8(-x-1)+7=2$

14. $6x + 2 = \frac{3x+2}{2}$

15. $\frac{6}{7}x + 1 = -x + 2$

16. (3x + 8)/4 = 1 – x/2

17. Simplify $3m + 2m - 6$

18. Simplify $-2(y - 6) + 8$

19. Simplify (11 – 2)y

20. Simplify $x(4 + 7)$

Unit 1: Lesson 02 Solving linear inequalities

To solve an inequality, treat it just like an equation with this one exception: when multiplying or dividing both sides by a negative number, **reverse the inequality**.

It is suggested that when solving, the variable be isolated on the left side.

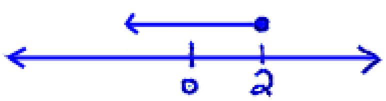
In the following examples, algebraically solve for x and then graph on a number line. (Be sure to graph $\leq$ & $\geq$ with closed circles and $<$ & $>$ with open circles):

Example 1: $24x \leq 48$

$24x \leq 48$

$x \leq \frac{48}{24}$

$x \leq 2$



Example 2: $x - 8 < 5x$

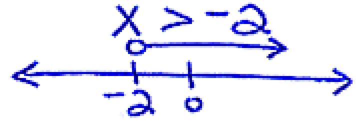
$x - 8 < 5x$

$-5x + x < 8$

$-4x < 8$

$x > \frac{8}{-4}$ * Note sign reversal

$x > -2$



Example 3: The teacher's age (*t*) is at least 3 times your age (*y*). Write this as an inequality in terms of *t* and *y*.

$t \geq 3y$

If instead of "at least," it said "at most", the answer would have been $t \leq 3y$

Example 4: $6(x-4) \geq 22x - 5$

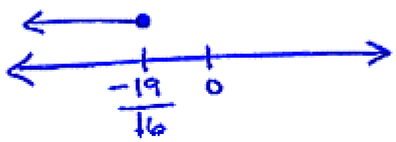
$6(x-4) \geq 22x-5$

$6x-24 \geq 22x-5$

$6x-22x \geq -5+24$

$-16x \geq 19$

(reversal) $x \leq \frac{19}{-16}$



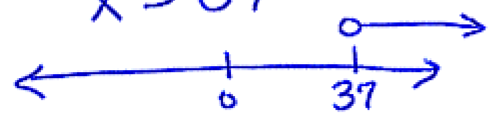
Example 5: $\frac{1}{2}(x-5) > 16$

$\cancel{2}\cdot\frac{1}{\cancel{2}}(x-5) > 16\cdot 2$

$x-5 > 32$

$x > 32+5$

$x > 37$



Assignment:

In problems 1 – 14, solve for the variable:

1. $x + 2 > 6$

2. $\left(\frac{1}{3}\right)x > -6$

3. $2t - 3 \leq -4$

4. $-2z \geq -10$

5. $7x + 4 > 5x - 4$

6. $3x + 2 > 4(x + 5)$

7. $2(p+2)-3>5(p-1)$

8. $4(1+m)\geq 2m-(1-m)$

9. $3[2(\mathrm{x}-1)-\mathrm{x}]<5(x+2)$

10. $12x-(x-1)>11+4x$

11. $x(1+5)>x(2-7)+2$

12. $-(x+1)\leq 3(x-6)$

13. My weight (m) is at least as much as your weight (y). Express this inequality in terms of m and y.

14. Four times the number of hamburgers Bill ate (B) is still no more than what Lucas ate (L). Express this statement as an inequality in terms of B and L.

Unit 1: Lesson 03

*Solving combined (compound) inequalities

In the following examples, algebraically solve for *x* and then graph on a number line.

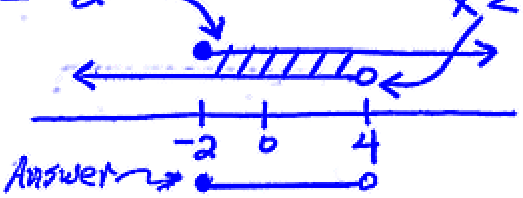
Example 1: $-4 \le 2x < 8$ First, we must recognize that this is really **two** simultaneous inequalities that could have been written as:

$-4 \le 2x$ *and* $2x < 8$ (also written as $-4 \le 2x \cap 2x < 8$)

Solve each half and combine:

$-2x \le 4$ $\qquad$ $x < \frac{8}{2}$

$x \ge -2$ $\qquad$ $x < 4$



Notice above that we wrote our problem as two separate inequalities with the word **and** (or the symbol ∩) between them. This is called a **conjunction** and makes it necessary to find the **intersection** of the two answers (where they overlap) to produce the final answer.

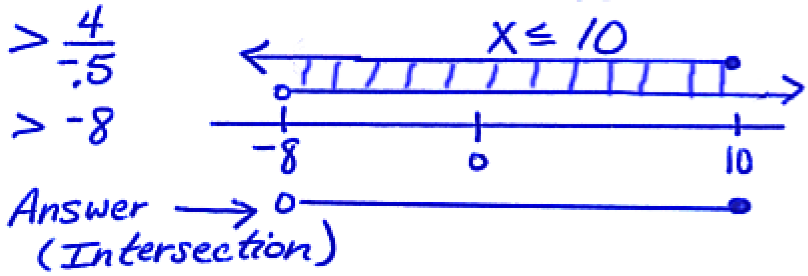
Example 2: $-4 < .5x \le 5$

$-4 < .5x$ $\qquad$ *and* $\qquad$ $.5x \le 5$

$-.5x < 4$ $\qquad$ $x \le \frac{5}{.5}$

$x > \frac{4}{-.5}$ $\qquad$ $x \le 10$

$x > -8$



In the next two examples the word "or" indicates a **disjunction** so we take the **union** of the two answers.

Example 3: $-4 > 2(\mathrm{x}-1)$ or $\mathrm{x} \geq 7$ (also written as $-4 > 2(\mathrm{x}-1) \cup \mathrm{x} \geq 7$)

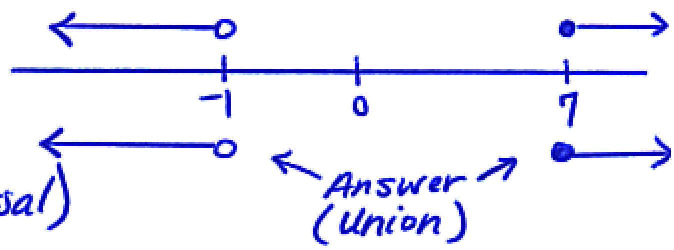
$-4 > 2x - 2$ or $x \geq 7$

$-2x > 4 - 2$

$-2x > 2$

$x < -1$

(reversal)



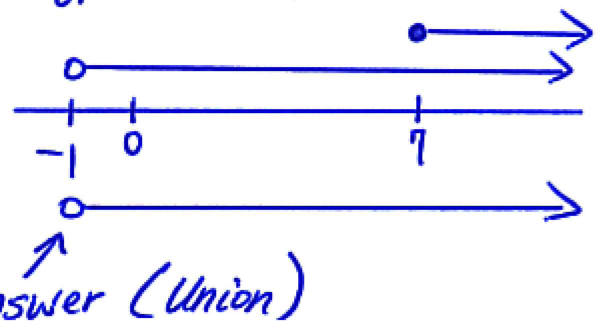
Example 4: $-4 < 2(\mathrm{x}-1)$ or $\mathrm{x} \geq 7$ $(-4 < 2(\mathrm{x}-1) \cup \mathrm{x} \geq 7)$

$-4 < 2x - 2$ or $x \geq 7$

$-2x < 4 - 2$

$-2x < 2$

$x > -1$



Assignment:

In problems 1 – 9, solve for the variable and graph on a number line:

1. $-2 \leq x + 3 < 4$

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2. $z + 3 \leq 1 \ \ or \ \ 1 < z - 2$

---

3. $1 \leq m + 5 < 6$

---

4. $k - 3 \geq 0 \quad \cup \quad k - 1 \leq 3$

5. $1 - 2x > 5 \ \ or \ \ 1 - 2x < -2$

6. $3 - x \leq 2 \ \ or \ x - 3 < 2$

7. $2p - 2 < p + 1 \leq 2p + 3$

8. $6 > x - 1 > 3$

9. $-3x > 9 \quad \cap \quad 8x > -16$

---

10. Answer these questions with either "and" or "or."

Conjunction is associated with what?
Disjunction is associated with what?

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11. What symbol is used for intersection (and)?
What symbol is used for union (or)?

Unit 1: Lesson 04 **Converting words to an algebraic expression**

Before actually doing "word problems" where a variable is ultimately solved, it is necessary to become proficient at converting those words into mathematical language. **Define all variables.**

Example 1. What is the sum of three consecutive integers?

n = 1st int.
$n+1$ = 2nd int.
$n+2$ = 3rd int.

$n + n+1 + n+2$
$= \boxed{3n+3}$

Example 2. What is the product of three consecutive odd integers?

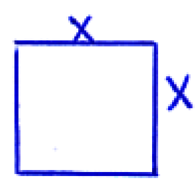
n = 1st odd int.
$n+2$ = 2nd " "
$n+4$ = 3rd " "

$\boxed{n(n+2)(n+4)}$

Example 3. Consider an isosceles triangle. What is the measure of the third angle if the two congruent angles measure *x* degrees?

$\boxed{180-2x}$

Example 4. What is the perimeter of a square with sides of length *x*?



$\boxed{4x}$

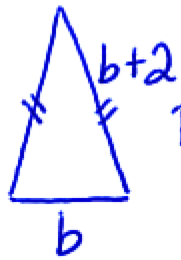
Example 5. A car travels for *h* hours at 50 mph. What is the total distance traveled?

dist = rate(time)
$= \boxed{50h}$

Example 6. What is the number that is 5 more than twice *n*?

$\boxed{2n+5}$

Example 7. What is the perimeter of an isosceles triangle whose base is *b* and the other two sides each being 2 cm longer than the base?



$$P = 2(b+2)+b$$
$$= 2b+4+b$$
$$= \boxed{3b+4}$$

Example 8. What is the cost of *t* movie tickets if the cost of each is $8.00?

$$\boxed{8t}$$

Assignment:

In the following problems, express the answer in terms of the variable given. Simplify when possible:

1. Let *n* be the first of 3 consecutive even integers. What is the sum of those integers?

2. Let *i* be the first of four consecutive integers. What is 3 less than the sum of those integers?

3. Consider four consecutive odd integers. What is the sum of the 2^{nd} and fourth numbers if the first number is *n*?

4. A playing field is 10 m longer that it is wide, *w*. What is its area?

5. If the length of a side of an equilateral triangle is x, what is half of the perimeter?

6. What is the distance traveled by a train moving at a rate of 45mph in t hours?

7. Jim is twice as old as his sister. If s is the sister's age, what is the sum of their ages?

8. Three siblings are one year apart in age. If the oldest is x years old, what is the average of their ages?

9. A car travels for 2 hours at r mph and then decreases its speed by 5 mph for the next 3 hours. What is the total distance traveled?

10. Two buses leave town at the same time going in opposite directions. Bus A travels at v mph while bus B is 10mph faster. How far apart are they in 3 hours?

11. A group of g students have detention and one girl left early. If 8 girls remain, how many boys remain?

****12.** How long is the diagonal in a rectangle whose length, L, is three more than twice its width?

13. What is the average of y plus 1 and three times x?

14. If a factory can produce p parts per hour, what would be the production in an 8 hour day?

Unit 1: Lesson 05 **Solving word problems with linear equations**

Read each problem, set up an equation with an appropriate variable and then solve.

Be sure to define the variable and make drawings when appropriate.

Example 1: Find three consecutive odd integers whose sum is 81.

n = 1st odd integer = $\boxed{25}$

$n+2$ = 2nd " " = $\boxed{27}$

$n+4$ = 3rd " " = $\boxed{29}$

answers

$n+n+2+n+4=81$

$3n+6=81$

$3n=81-6$

$3n=75$

$n=25$

Example 2: An angle is 8 times its supplement. What is the angle?

A = the angle = $\boxed{160}$

$180-A$ = its supplement = $180-160=\boxed{20}$

$A=8(180-A)$

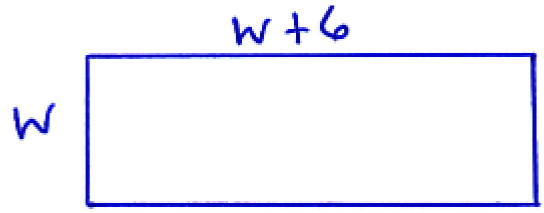
$A=1440-8A$

$A+8A=1440$

$9A=1440$

$A=\frac{1440}{9}=160$

Example 3: A board game is 6 cm longer than it is wide. If the perimeter is 152 find both dimensions.



$P = 2(w+6)+2w = 152$

$2w+12+2w = 152$

$4w = 152-12$

$4w = 140$

$w = 140/4 = 35$

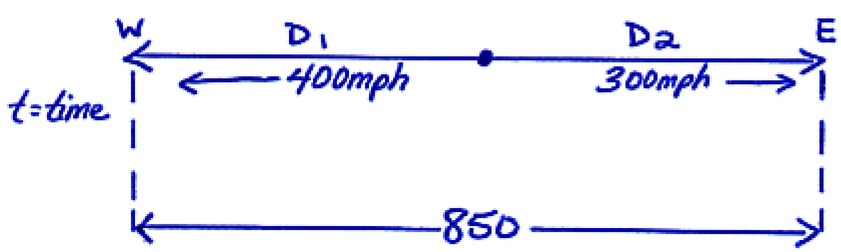
$w = \boxed{35}$

$L = w+6$

$= 35+6$

$= \boxed{41}$

Example 4: Two planes leave an airport at 1:00 PM and fly in opposite directions. The plane going East flies at 300 mph and the one going West has a speed of 400 mph. How long does it take for them to be 850 miles apart? *According to the clock, what time will that be?



$D_1 + D_2 = 850$

Dist = rate(time)

$400t + 300t = 850$

$700t = 850$

$t = 1.21428$

$t = \boxed{1.21 \text{ hrs.}}$

$.21(60) = 12.6$ min.

Clock time $= 1{:}00 + 12.6 \approx \boxed{2{:}13 \text{ PM}}$

Assignment:

Read each problem, set up an equation with an appropriate variable and then solve. Be sure to define the variable and make drawings when appropriate.

1. Two planes that are initially 800 mi apart are flying directly toward each other. One flies at 250 mph and the other at 150 mph. How long will it be before they meet?

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2. Bill's number is 4 more than Mary's. The average of their numbers is 36. What is each person's number?

---

3. Initially the first bank account is $5 less than the second account. After a month the first account had tripled and the second account had doubled. At that time they were equal. What was the amount originally in each account?

4. In an isosceles triangle the base angles are 30 degrees less than the other angle. What is the measure of the third angle?

---

5. The sum of 3 consecutive integers is 153. What are the three integers?

---

6. The sum of 4 consecutive even integers is 44. What is the largest of those four integers?

---

7. An angle is exactly half its complement. What is the angle?

8. A number is multiplied by 2 and then this product is increased by 5. The result is 14 less than twice the opposite of the number. What is the original number?

9. The average of three consecutive integers is 17. what are the integers.

10. Long distance runner A averages 6 mph, while runner B averages 7 mph. They both start from the same starting position; however, runner B begins an hour after A leaves. How long will it take B to catch A?

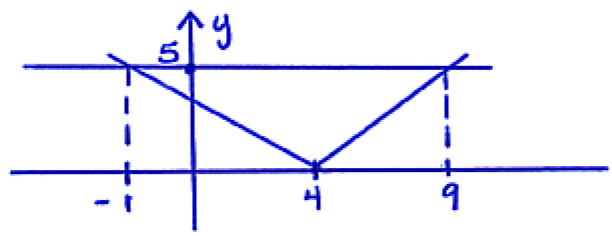
Unit 1: Lesson 06 **Graphing calculator solutions: absolute value problems

Consider the solutions to the three problems: $|x-4|=5$, $|x-4|\leq 5$, and $|x-4|\geq 5$. To solve these on the calculator, all will require us to graph two functions:

Y1 = abs(X – 4) (absolute value bars are replaced with "abs" from the **Math | Num** menu)

Y2 = 5

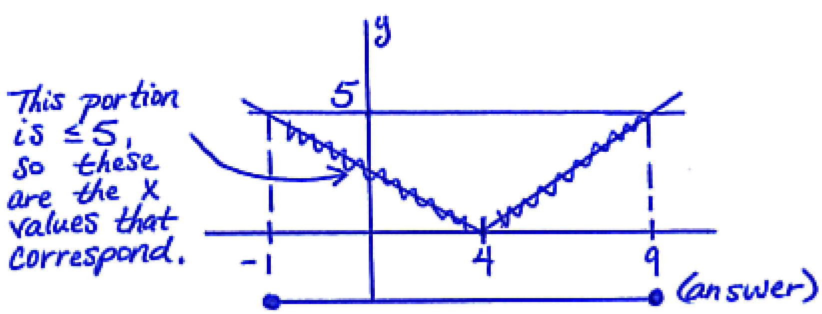
Using a "6. ZStandard" zoom, the simultaneous graph of these two is the following where the intersection points have been determined on the calculator with **2nd Calc | intersect** (See **Calculator Appendix D** and a related video for instructions.):



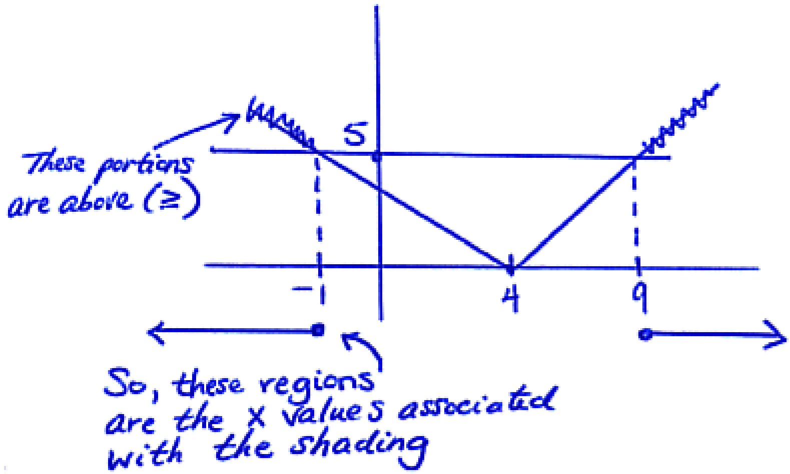
Example1: $|x-4|=5$

For this problem, the answers are –1 and 9 as determined in the drawing above.

Example 2: $|x-4|\leq 5$



Example 3: $|x-4| \geq 5$



Now consider how to **manually** solve a linear equation involving the absolute value of a variable expression.

Example 4: $|x-2| = 3$ When an absolute value is **equal** to a number, create two separate equations as follows and take the union (**or**) of the answers.

$x - 2 = 3$ or $x - 2 = -3$

$x = 3 + 2$ $\qquad$ $x = -3 + 2$

$x = \boxed{5}$ $\qquad$ $x = \boxed{-1}$

two answers

See **Enrichment Topic A** for how to **manually solve inequalities** involving the absolute value of variable expressions.

Assignment:

Solve problems 1-3 with a graphing calculator. Make a sketch of the x-y plot and finally draw the solution to the problem on a number line.

1. $|-2x-1|+2=4$

---

2. $|-2x-1|+2<4$

---

3. $|-2x-1|+2\geq 4$

Solve the following three problems **manually** (no calculator).

4. $\|x+6\|=17$	**5.** $\|2t-3\|=4$

6. $|-2z+8|=10$

Review for test on Unit 1

Unit 1: Review

In problems 1- 4, solve for the variable and show all work:

1. $4(x - 7) = x + 2$	2. $-6(x - 5) = x + 5$
3. $2x(6 - 3) + 4 = x - 2$	4. $\frac{3(2-x)}{2} = -8$

In problems 5 and 6, algebraically solve for the variable and graph the answer on a number line. Show your work.

5. $5(x + 7) > 20$	6. $-3(p + 1) \le -2p$

7. Write an expression for Bob's age. He is 5 years older than 3 times Amy's age (A).

8. Make two associations groups from the words intersection, union, disjunction, conjunction, and, & or.

9. If x is the first even number, write an expression for the average of three consecutive even numbers.

10. An angle is one more than twice its complement. What is the angle?

11. The new court house is 11 meters longer than it is wide. The perimeter of the court house is 175 meters. What are its dimensions?

12. If twice the amount of flour in the pantry exceeds the amount of corn meal by two pounds, what is the amount of flour? There are 15 pounds of corn meal in the pantry.

**13. Solve $|3x - 1| = 11$ for x.

*14. Solve $-3 \leq x - 1 < 6$ for x. Solve algebraically and then present the answer both algebraically and on a number line.

*15. Solve $-3 \geq x - 1 \quad or \quad x + 3 > 7$ for x. Solve algebraically and then present the answer both algebraically and on a number line.

16. **This problem is from Enrichment Topic A: Solve $|7 – 6x| < 12$ for x. Solve algebraically and then present the answer both algebraically and on a number line.

Alg II, Unit 2

Slope, solving a linear system of two equations

Unit 2: Lesson 01

Slopes of lines: four different points of view

The slope (m) of a line is simply a measure of its **steepness**; therefore, a **horizontal line has a slope of m = 0** since it has no steepness. A **vertical line** is the ultimate in steepness so its slope is **infinity** (undefined).

What about lines that are neither horizontal nor vertical? We are going to look at how to determine the slope four different ways.

First method: The two-point slope formula

Given two points (x_1, y_1) and (x_2, y_2) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1:

Find the slope of the line connecting the points (2, 3) and (5, –6).

$x_1 y_1 \quad x_2 y_2$

$m = \frac{-6-3}{5-2} = \frac{-9}{3} = \boxed{-3}$

Example 2:

Find the slope of the line connecting the points (–2, 4) and (–2, –17).

$x_1 \; y_1 \quad x_2 \; y_2$

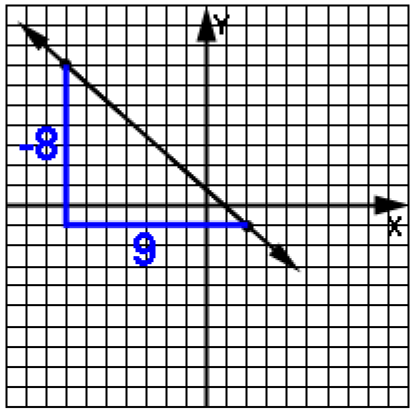
$m = \frac{-17-4}{-2+2} = \frac{-21}{0} =$ Forbidden!

Vertical line, $\boxed{\text{no slope}}$

Second method: $m = \frac{\text{rise}}{\text{run}}$

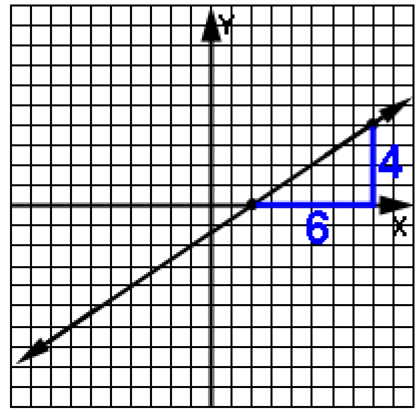
Simply find two places on the line and create a triangle. The horizontal line segment is the run (always considered positive) while the vertical segment is the rise. It can be either positive or negative depending on if it is rising or falling when moving from left to right.

Example 3:



$m = \frac{rise}{run} = \boxed{\frac{-8}{9}}$

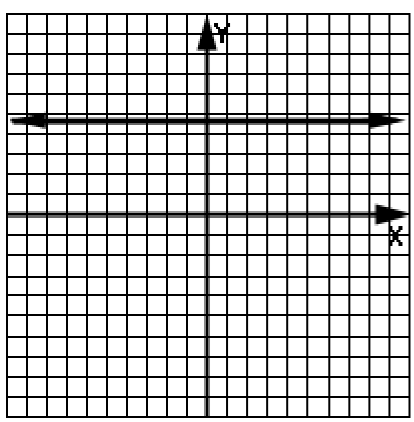
Example 4:



$m = \frac{4}{6} = \boxed{\frac{2}{3}}$

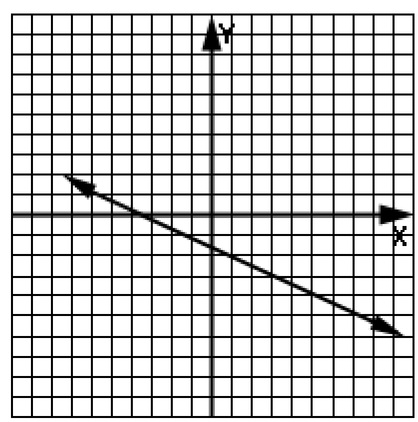
Third method: Just by looking at a line, moving from left to right, determine if the line is going up (positive slope), is going down (negative slope), is horizontal (zero slope), or is vertical (no slope, undefined).

Example 5:



0 slope

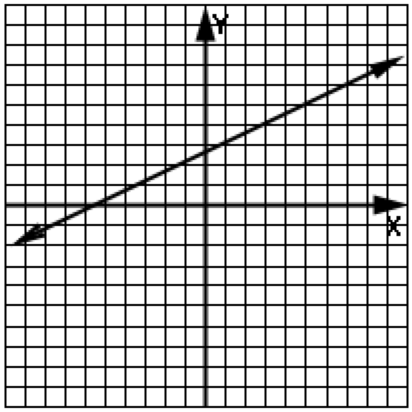
Example 6:



neg. slope

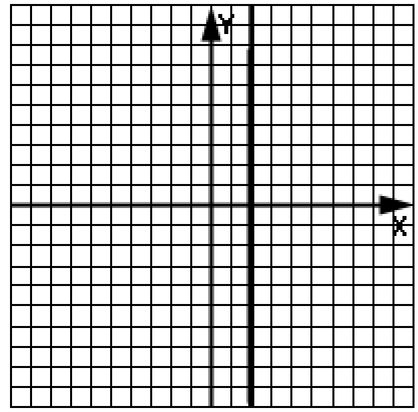
Example 7:

Positive Slope



Example 8:

Undefined, no slope



Fourth method: Convert the equation of a line to slope-intercept form (y = mx + b). The coefficient of *x* is the slope, *m*.

Example 9: What is the slope of the line $y = -13x + 5$?

$m = \boxed{-13}$

Example 10: What is the slope of the line $4x - 5y = 3$?

$-5y = -4x + 3$

$y = \frac{4}{5}x - \frac{3}{5}$

$m = \boxed{\frac{4}{5}}$

Slope relationships: If the slopes of two different lines are equal, the lines are parallel. If the slopes are negative reciprocals of each other (they multiply to give –1), the lines are perpendicular.

Example 11: Determine if the following two lines are parallel, perpendicular or neither:

y = 3x —1 and y — 3x = 5

$m_1 = 3$ $\quad y = 3x + 5$

$m_2 = 3$

slopes are the same, so they are parallel (//)

Example 12: Determine if the following two lines are parallel, perpendicular or neither:

y = 3x —1 and 3y = –x + 2

$m_1 = 3$ $\quad 3y = -x + 2$

$y = -\frac{1}{3}x + \frac{2}{3}$

$m_2 = -\frac{1}{3}$

$m_1 \cdot m_2 = 3(-\frac{1}{3}) = -1$, so they are perpendicular ($\perp$)

Assignment:

1. Find the slope of the line connecting the points (44, –1) and (–11, –6).

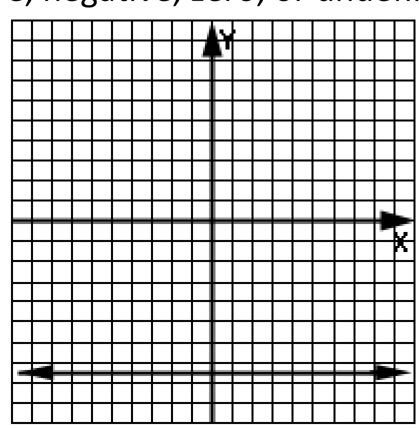
2. Find the slope of the line connecting the points (–1, –1) and (–1, –4).

3. A cone is 486 ft tall. From the bottom of the outside of the cone, the horizontal distance to the center of the cone is 230 ft. What is the slope of the cone?

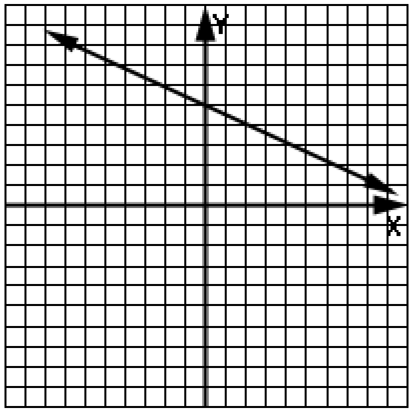
4. What is the slope of a vertical flagpole?

5. Find the slope of the line connecting the points (–4, 12) and (10, –6).

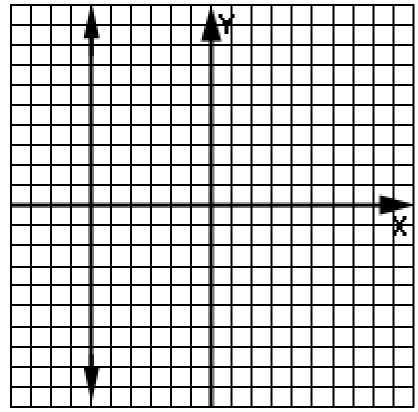
6. Identify the slope of this line as positive, negative, zero, or undefined.



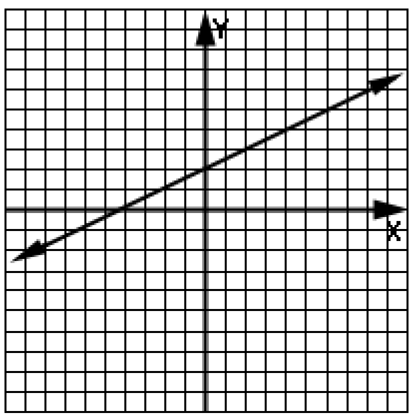
7. Identify the slope of this line as positive, negative, zero, or undefined.



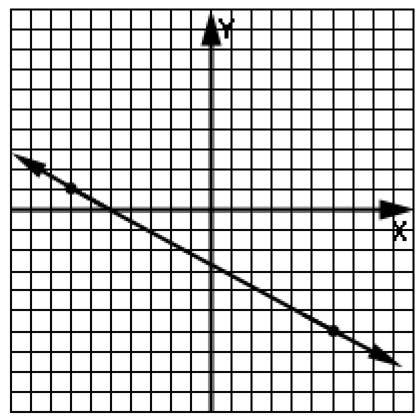
8. Identify the slope of this line as positive, negative, zero, or undefined.



9. Identify the slope of this line as positive, negative, zero, or undefined.



10. Use rise/run to determine the slope.



11. What is the slope of the line given by $3x - 15y = -1$?

12. What is the slope of the line given by $-11y = -4.5x - 2.2$?

13. What is the slope of the line given by $y = -4$?

14. What is the slope of the line given by $y = 13x - 2$?

15. What is the slope of the line given by $y = -21 - 5x$?

16. Find the slope of the line connecting the points (1,-2) and (-3, 4).

17. Determine if the following two lines are parallel, perpendicular or neither:
y = 4x —2 and y — 4x = 5

18. Determine if the following two lines are parallel, perpendicular or neither:
y = 3x —1 and the line connecting (1, 7) and (4, 6).

Unit 2: Lesson 02 Two forms for the equation of a line

Slope-intercept form: y= mx + b

m is the slope and **b** is the y-intercept (where line crosses y-axis)

Standard form: Ax + By = C

In both forms notice that x and y both are raised to the one power. Formally, we say they both are of degree one.

Special cases:

For a **horizontal line the equation is y = a number** that is the y-intercept.

For a **vertical line the equation is x = a number** that is the x-intercept.

(See **Enrichment Topic C** for two more forms of a line (point-slope & intercept).

In the following problems, use the provided information to write the slope-intercept form of the equation of the line.

Example 1: Slope is –2 and the y intercept is –5.

$m = -2 \quad b = -5$

$y = mx + b$

$\boxed{y = -2x - 5}$

Example 2: Slope is 4 and the line passes through the point (11, –2).

$m = 4$

$y = 4x + b$ (Now sub in (11, -2))

$-2 = 4(11) + b$

$-2 = 44 + b$

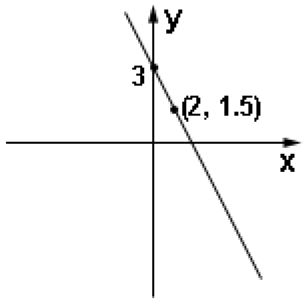
$-2 - 44 = b$

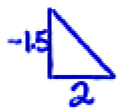
$-46 = b$

$y = mx + b$

$\boxed{y = 4x - 46}$

Example 3:





$m = \frac{-1.5}{2}$

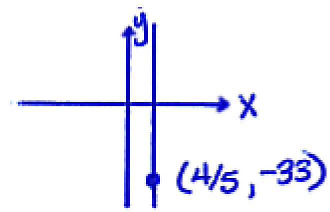
$m = \frac{-1.5}{2} = -.75$

$b = 3$

$y = mx + b$

$\boxed{y = -.75x + 3}$

Example 4: Slope is undefined and the line passes through the point (4/5, –33).



Vert. line, so equation is of the form $x = \#$

$\boxed{x = 4/5}$

Use a graphing calculator to graph two lines at the same time so as to compare the effect of different y-intercepts and slopes. (See **Calculator Appendix A** for instructions and a related video.)

Example 5: Slope is 2 and the x intercept is –4.

x int. is the point $(-4,0)$

$y = 2x + b$, now sub in $(-4,0)$

$0 = 2(-4) + b$

$0 = -8 + b$

$8 = b$

$y = mx + b$

$\boxed{y = 2x + 8}$

Example 6: The line passes through the points (1, –1) and (2, –7).

(x_1, y_1) (x_2, y_2)

$m = \frac{-7-(-1)}{2-1} = \frac{-6}{1} = -6$

$y = -6x + b$, now sub in $(1,-1)$

$-1 = -6(1) + b$

$-1 = -6 + b$

$-1 + 6 = b$

$5 = b$

$y = mx + b$

$\boxed{y = -6x + 5}$

Example 7: Parallel to the line given by 3x –2y = 17 and passing (1, 2).

$-2y = -3x + 17$

$y = \frac{3}{2}x - \frac{17}{2}$, So our line has a slope of 3/2.

$y = \frac{3}{2}x + b$, Now sub in (1,2)

$2 = \frac{3}{2}(1) + b$

$2 - \frac{3}{2} = b$

$\frac{1}{2} = b$

$y = mx + b$

$\boxed{y = \frac{3}{2}x + \frac{1}{2}}$

Example 8: Perpendicular to the line given by y = .5x + 3 and having a y intercept of 2.

$m = \frac{1}{2}$, neg. recip is

$m = -2$

$y = mx + b$

$\boxed{y = -2x + 2}$

Example 9: x-intercept 5 and y-intercept 6

$b = 6$ x int. (5,0)

$y = mx +$, sub in (5,0)

$0 = m(5) + 6$

$-6 = 5m$

$-\frac{6}{5} = m$

$y = mx + b$

$\boxed{y = -\frac{6}{5}x + 6}$

Example 10: Parallel to x + y –1= 0 and having a y intercept of 100.

$b = 100$

$x + y - 1 = 0$

$y = -x + 1$

$m = -1$

$y = mx + b$

$\boxed{y = -1x + 100}$

Example 11: Convert y = –8x + 2 to standard form.

$y = -8x + 2$

$\boxed{8x + y = 2}$

Example 12: Convert 44x –2y = 9 to slope-intercept form.

$44x - 2y = 9$

$-2y = -44x + 9$

$\boxed{y = 22x - \frac{9}{2}}$

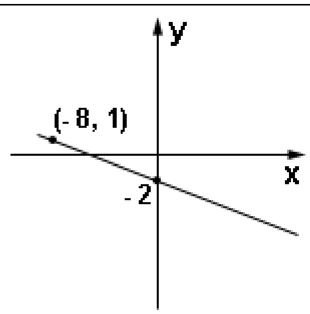
Assignment:

In each problems 1-10 find the equation of the line.

1. Slope is –3 and the y intercept is – 4.

2. Slope is 2 and the line passes through the point (5, 6).

3.



4. Slope is undefined and the line passes through the point (2, 10).

5. Slope is 1 and the x intercept is –5.

6. The line passes through the points (1, 0) and (–1, 7).

7. Parallel to the line given by $3x - 2y = 17$ and passing (3, 9).

8. Perpendicular to the line given by $y = 2x + 3$ and having a y intercept of -1.

9: x-intercept 3 and y-intercept 8

10. Parallel to $x + 7y - 1 = 0$ and having a y intercept of -20.

11. Convert $y = -6x + 5$ to standard form.

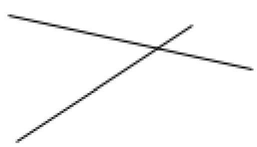
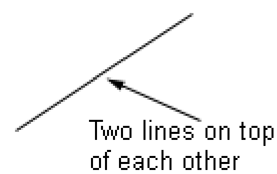
12. Convert $28x - 2y = 8$ to slope-intercept form.

Unit 2: Lesson 03 Graphical meaning of the solution to two linear equations

Consider the following two equations:

$$2x - 4y = 9$$
$$11x - 5y = -8$$

What does it mean to **solve this system of equations**? Very simply, it means to **find all the points of intersection** of these two lines. There are three distinct possibilities as shown below:

The two lines intersect in a single point. The *x* and *y* values of that point are the solutions to the system.	The two lines never intersect because the lines are parallel but separate.	The two lines are directly on top of each resulting in an infinite number of intersection points.
		

So, how can we tell by just looking at the equations of two lines which of the three pictures above represents their orientation?

If the **slopes of the two lines are different**, then it's the left picture above and we have only **one point** of intersection.

If the **slopes are the same** and the **y-intercepts are different**, then it's the middle picture above and we have **no points** of intersection.

If the **slopes are the same** and the **y-intercepts are the same** (they are actually the same line), then it's the right-hand picture above and there are **infinitely many points** of intersection.

In each example below, examine the slope and y-intercept. Then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

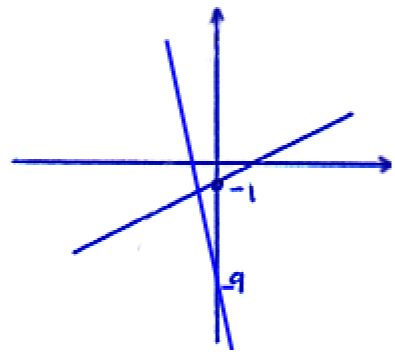
Example 1: $-4x = y + 9$ and $y = x - 1$

$-4x - 9 = y$

$m_1 = -4$ $\quad$ $m_2 = 1$

Slopes are different

(one intersection point)



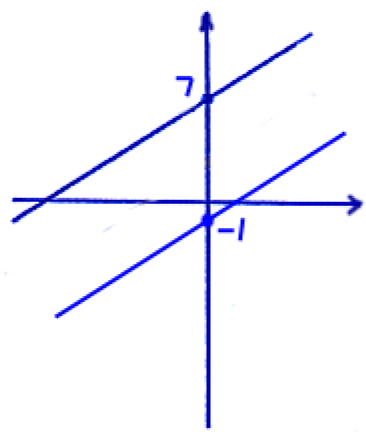
Example 2: $-x + y = 7$ and $y = x - 1$

$y = x + 7$ $\quad$ $m_2 = 1$

$m_1 = 1$

$m_1 = m_2$ so the lines are parallel. They have dif. b values so separate.

(No Solution)

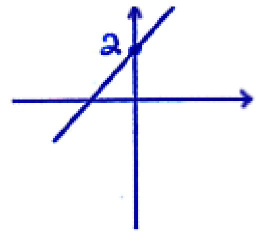


Example 3: $-x + y = 2$ and $6 + 3x - 3y = 0$

$y = x + 2$ $\quad\quad$ $-3y = -3x - 6$

$m_1 = 1$ $\quad\quad$ $y = x + 2$

$m_2 = 1$



Slope are the same; infact, the equations are the same. They sit on top of each other. So there are an infinite number of points as given by $y = x + 2$

$\boxed{\text{Infinitely many solutions}}$

Assignment:

In the following problems, examine the slopes and y-intercepts of the two lines and then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

1. $x + 4 = y \ \ and \ \ y = 3x - 8$

2. $2x - 3y = -1$ and $8x - 12y = -4$

3. The line given by (1, –5) & (6, –10) and $3y = -3x + 11$

4. Line A has a slope of 3/5 and crosses the y-axis at –8. Line B has a slope of 3/5 and crosses the y-axis at 22.

5. Line A has a slope of 1 and crosses the y-axis at 3. Line B has a slope of 3 and crosses the y-axis at –1.

6. The product of the slope of the two lines is –1. Hint: This has something to do with the orientation of the line (parallel, perpendicular, etc).

7. $y = 4.1x + 2 \quad and \quad 2y = 4.1x + 2$

8. The line connecting (0, 0) & (1,1) and the line connecting (–2, –2) & (5, 5)

9. y = 3 and x = 3

10. y = 4 and y = –5

Unit 2: Lesson 04 — Algebraic solutions for two linear equations (elimination & substitution)

There are two primary algebraic ways to **find the intersection point** (the solution) of a system of two linear equations: **Substitution and elimination**

Substitution method:

Example 1: y = 3x – 2 and x + y = 6

$$x+(3x-2)=6$$
$$x+3x=6+2$$
$$4x=8$$
$$x=2$$

$$y=3x-2$$
$$y=3(2)-2$$
$$=6-2$$
$$y=4$$

$$\boxed{x=2 \quad y=4}$$

Example 2: x + 2y = 9 and x + y = 4

$$x=9-2y$$
$$9-2y+y=4$$
$$9-y=4$$
$$-y=4-9$$
$$-y=-5$$
$$y=5$$

$$x=9-2\cdot 5$$
$$x=9-10$$
$$x=-1$$

$$\boxed{x=-1} \quad \boxed{y=5}$$

Elimination method:

Example 3: 5x –2y = 8 and –3x + 2y = 0

$$5x-\cancel{2y}=8$$
$$-3x+\cancel{2y}=0$$
$$2x=8$$
$$x=4$$

$$-3\cdot 4+2y=0$$
$$-12+2y=0$$
$$2y=12$$
$$y=6$$

$$\boxed{x=4} \quad \boxed{y=6}$$

Example 4: 2x – 3y = 1 and 5x + 2y = 0

$$2(2x-3y)=1\cdot 2 \rightarrow 4x-\cancel{6y}=2$$
$$3(5x+2y)=0\cdot 3 \rightarrow 15x+\cancel{6y}=0$$
$$19x=2$$
$$\boxed{x=\frac{2}{19}}$$

$$2\left(\frac{2}{19}\right)-3y=1$$
$$\frac{4}{19}-3y=1$$
$$-3y=1-\frac{4}{19}$$
$$-3y=\frac{15}{19}$$
$$y=\frac{15}{19(-3)}=\boxed{\frac{-5}{19}}$$

Special cases:

Case 1:

If during the process of solving two linear equations, both variables cancel out and we are left with a **true** statement:

The interpretation is that we have **two lines right on top of each other**. There are infinitely many solutions and can be specified by giving the equation of either line.

Case 2:

If during the process of solving two linear equations, both variables cancel out and we are left with an **untrue** statement:

The interpretation is that we have **two parallel lines** that are separated by some distance and never meet. Therefore, there are **no solutions**.

Example 5:

$7x + y = 6$ and $2y = -14x + 12$

$-2(7x+y) = 6(-2) \rightarrow -14x - 2y = -12$

$14x + 2y = 12 \rightarrow 14x + 2y = 12$

$0 = 0$

true

Equations are the same…
Infinitely many solutions:

All points on y = -7x + 6

Example 6:

$$y = \left(\frac{3}{2}\right)x - 1 \quad \text{and} \quad 2y = 3x + 8$$

$y = \frac{3}{2}x - 1 \qquad 2y = 3x + 8$

$2\left(\frac{3}{2}x - 1\right) = 3x + 8$

$3x - 2 = 3x + 8$

$3x - 3x = 8 + 2$

$0 \neq 10$

Not true

$\boxed{\text{No solution}}$

Slopes are the same, but y-int are different… Two parallel & separate lines.

See **Enrichment Topic I** for how to solve three equations for three variables.

Assignment:

In the following problems, solve for the intersection point(s) of the systems of linear equations using the **substitution** method:

1. $y = 8x - 11$ and $x - 2y = 1$

2. $x = 4y + 4$ and $y = -3x + 1$

3. $x + y = 6$ and $2y + 2x = 0$

4. $\left(\frac{1}{2}\right)y = \left(\frac{1}{3}\right)x + 2$ and $x - y - 1 = 0$

5. $x = 1$ and $y = x + 2$

6. $18x - .5y = 7$ *and* $y = 8$

In the following problems, solve for the intersection point(s) of the systems of linear equations using the **elimination** method:

7. $\mathrm{x} + \mathrm{y} = 5 \quad \text{and} \quad -\mathrm{x} + 11\mathrm{y} = 0$

8. $4 = \mathrm{x} + 2\mathrm{y} \quad \text{and} \quad -2\mathrm{y} + \mathrm{x} = 1$

9. $.4\mathrm{x} + .2\mathrm{y} = .1 \quad \text{and} \quad 6\mathrm{x} + 18\mathrm{y} = 1$

10. $\mathrm{x} + \mathrm{y} + 1 = 0 \quad \text{and} \quad 2\mathrm{x} - 9\mathrm{y} = 3$

11. $x = 2 \quad and \quad x - 5y = 2$

12. $\mathrm{y} = 5\mathrm{x} - 6 \quad \text{and} - 15\mathrm{x} + 3\mathrm{y} = -18$

In the following problems, use any technique. Hint: Make a sketch of the two lines in which case you might be able to “see” the answer. This is called solving by “inspection.”

13. $x = -5 \quad and \quad y = 11$	**14.** $x + 2 = 0 \quad and \quad 2(8 - y) = 0$

Unit 2: Lesson 05 Solving word problems with two linear equations

Solve the following word problems by first defining the **two variables**. Then set up **two linear equations** in the two variables and solve by either substitution or elimination. Note that when we solve for **two variables** we must have **two equations**. Later, when we have three variables, we will need three equations, etc.

Example 1: Larry has $1.15 worth of coins in nickels and quarters. He has one more quarter than nickels. How many of each coin does he have?

N = # of Nickels
Q = # of Quarters

$.05N + .25Q = 1.15$
$Q = N + 1$

Solve by substitution

$.05N + .25(N+1) = 1.15$

$.05N + .25N + .25 = 1.15$

$.30N = 1.15 - .25$

$.30N = .90$

$N = \boxed{3}$

$Q = 3 + 1 = \boxed{4}$

3 Nickels, 4 Quarters

Example 2: Four years ago Bob was twice as old as his little sister. Five years from now he will be six years older than his sister. How old is each now?

B = Bob's age <u>Now</u>

S = Sister's age <u>Now</u>

$$\left.\begin{aligned} B-4&=2(S-4)\\ B+5&=(S+5)+6 \end{aligned}\right\}\text{ 2 equations}$$

$$B-4=2S-8$$
$$B-2S=-4$$

$$B+5=S+11$$
$$B-S=6$$
$$-2(B-S)=-2\cdot 6$$
$$-2B+\cancel{2S}=-12$$
$$B-\cancel{2S}=-4$$
$$-B=-16$$
$$\boxed{B=16}$$

$$16-S=6$$
$$-S=6-16$$
$$-S=-10$$
$$\boxed{S=10}$$

Assignment:

Solve the following word problems by first defining the two variables. Then set up two linear equations in the two variables and solve by either substitution or elimination.

1. The sum of two integers is 16 and their difference is 12. What are the numbers?

---

2. An airplane flying into the wind can go 2000 miles in 6 hrs. On the return trip with the wind the flight time is 5 hr. What is the speed of the plane in still air and the speed of the wind?

3. Juan is 8 years older than his brother Two. In another 5 years, Juan will be twice as old as Two. How old are they now?

4. A pile of 34 coins is worth $5.10. There are two nickels and the remainder are quarters and dimes. How many quarters and dimes are there?

5. The 10's digit of a number is four more than the units digit of the number. The sum of the digits is 10. What is the number?

6. The difference of two supplementary angles is 32 degrees. What are the angles?

7. Twins Jose and HoseB have a combined weight of 430 pounds. HoseB is 58 pounds heavier than Jose. How much does each weigh?

8. Angus earns $8.80 an hour at his Saturday job and $7.50 per hour at his after-school job. Last week he earned a total of $127.80. The hours he worked after school were 4 hours more than he worked on Saturday. How many hours did he work on Saturday?

Unit 2: Lesson 06

Graphing calculator solutions of linear systems

Finding the intersection point of two lines:

Using the **Y=** button, enter the two linear equations as Y1 and Y2. Press the **Graph** button to display the two lines simultaneously.

If the intersection point of the lines is not visible, use the **Zoom** button and then zoom **In** or **Out**. The **ZStandard** zoom will often be the most useful. If none of these show the intersection point, make an estimate of where it is and use the **Window** button to adjust the max and min values accordingly.

With the intersection point displayed, access **2^{nd} Calc | 5.intersect**. You will then be asked to identify the "first curve." Move the blinker with the left and right arrows until it is clearly on one of the lines. Press Enter. You will then be asked to similarly identify the "2^{nd} curve." Finally, you are asked to "guess" the intersection. Move the blinker until it is very close to the intersection point and press Enter. The *x* and *y* values of the intersection point will be given at the bottom of the display.

See **Calculator Appendix C** and a related video for more details on the finding the intersection point of two lines.

Using the techniques described above, find the intersection point of the following systems of linear equations. Make a rough sketch of the calculator display.

Example 1:

$y = 3x + 4$ and $y = -2x - 9$

Example 2:

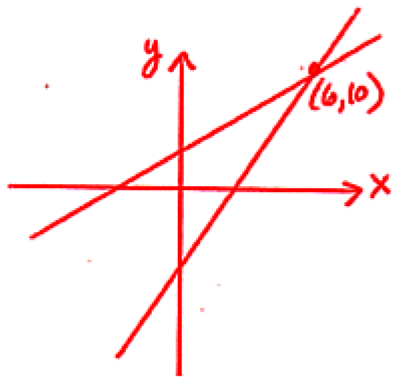
$y = -3$ and $2y - 4 + x = 0$

Assignment:

Using a graphing calculator, find the intersection point of the following systems of linear equations. Make a rough sketch of the calculator display.

1. $x + 4 = y$ *and* $y = 3x - 8$

$y_1 = x+4 \quad y_2 = 3x-8$

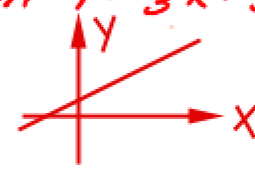


2. $2x - 3y = -1$ and $8x - 12y = -4$

$-3y = -2x - 1 \quad -12y = -8x - 4$

$y_1 = (-2x-1)/-3 \quad y_2 = (-8x-4)/-12$

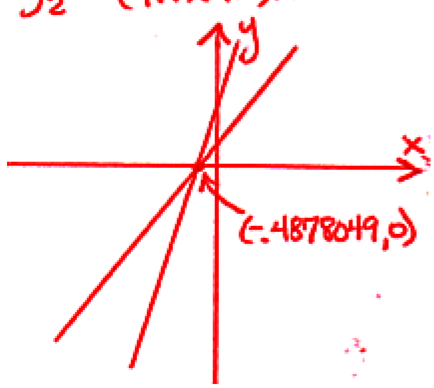
The two lines are right on top of each other.
Infinite # of Solutions
on $y = \frac{2}{3}x + \frac{1}{3}$



3. $y = 4.1x + 2$ and $2y = 4.1x + 2$

$y_1 = 4.1x+2$

$y_2 = (4.1x+2)/2$

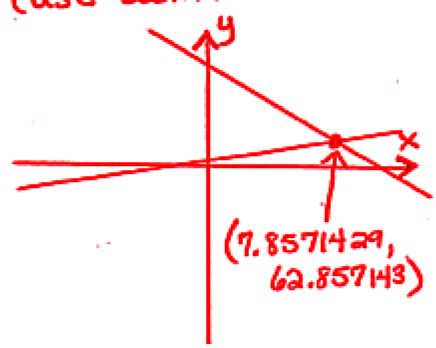


4. y = – 6x + 110 and y = x + 55

$y_1 = -6x+110$

$y_2 = x+55$

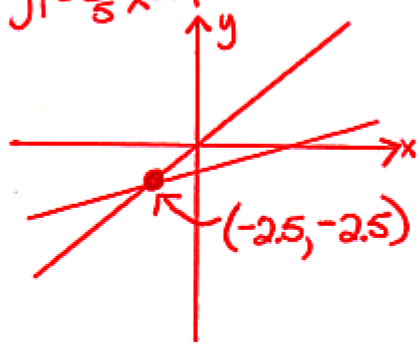
(use Zoom | Zoom Fit)



5. $\left(\frac{3}{5}\right)x - y = 1 \quad and \quad y = x$

$-y = \left(-\frac{3}{5}\right)x + 1 \qquad y_2 = x$

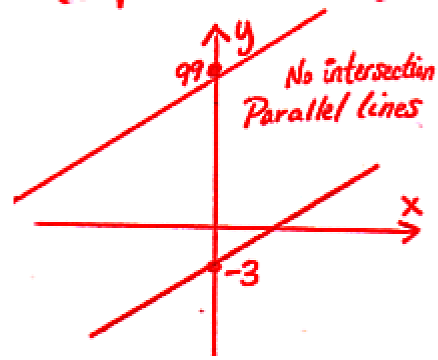
$y_1 = \frac{3}{5}x - 1$



6. y = x + 99 and y – x = -3

$y_1 = x + 99 \qquad y_2 = x - 3$

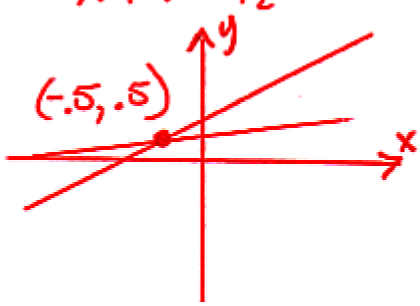
(slopes are the same)



7. .1 x + .2 = .3y and y = x + 1

$(.1x + .2)/.3 = y_1$

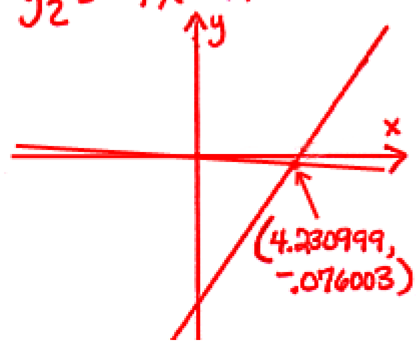
$x + 1 = y_2$



8. 1002y = –5x – 55 and y= 4x – 17

$y_1 = (-5x - 55)/1002$

$y_2 = 4x - 17$



Unit 2: Review

1. What is the slope of the line containing the origin and $(-3, -5)$?	2. What is the slope of a horizontal line after being rotated 90 degrees?
3. A hill falls 30 feet in altitude when moving from left to right. The horizontal distance covered is 150 feet. What is the "mathematical slope" of the hill?	4. Convert $y = -8x + 2$ to general form.
5. What is the slope and y-intercept of the line given by $12x - 5y = 7$?	6. How many points are there in the solution for the system, $y = 5x - 8$ and $3y - 15x = 11$?
7. How many points are there in the solution for the system, $y = 7x + 5$ and $y = -7x - 5$?	8. State whether the following two lines are parallel, perpendicular, or neither: $y = 4x - 1$ and $4y = -x + 2$

9. Solve by elimination:
$5x - 2y = 8$ and $-3x + 2y = 2$

10. Solve by substitution:
$y = 3x - 4$ and $x - y = 6$

11. What is the intersection point of the horizontal line with y-intercept 6 and a vertical line with x-intercept –3?

12. Solve with substitution:
$-x + y = 4$ and $6x + 7y = 2$

13. The nerds in Saturday detention outnumbered the geeks by 6. Altogether there were 22 nerds and geeks. How many nerds and how many geeks were there? Define two variables and then set up two equations. Solve using either substitution or elimination.

14. The smaller of two supplementary angles is exactly one-third the larger one. What are the two angles? Define two variables and then set up two equations. Solve using either substitution or elimination.

15. Using a graphing calculator, solve

$-13x - 5y = 1.8$ and $x + y = 20.2$

16. Using a graphing calculator, solve

$y = 5$ and $y = x + 11.5$

Alg II, Unit 3

Graphing linear inequalities in two variables

Unit 3: Lesson 01

Graphing a linear inequality in two variables

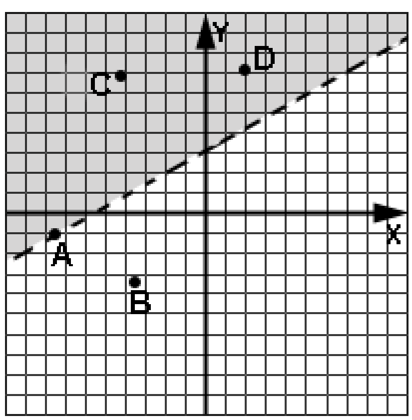
To graph an inequality like y < 3x – 5 , we first **draw the line** y = 3x – 5. Then do the following:

- If the inequality is **≥ or ≤** make the line **solid**. If the inequality is **< or >** make it **dotted**.
- If the inequality is **≤ or <**, shade **below** the line. If it is **≥ or >,** shade **above** the line. (This assumes *y* has been solved-for on the left.)
- If the line is vertical then, **≤ or <**, dictates that we shade to the left. Shade to the right if **≥ or >.**

  All the shaded points and/or a solid line are the solutions to the inequality.

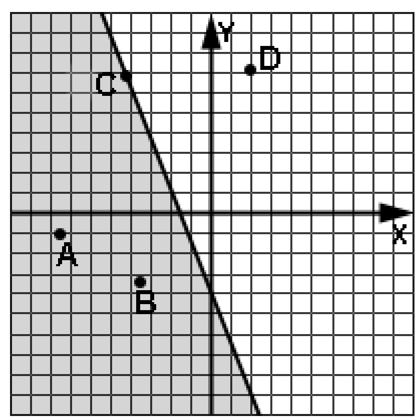
In examples 1 and 2, identify those points that are solutions to the inequality.

Example 1:



C, D

Example 2:



A, B, C

In examples 3 and 4, determine algebraically if the point is part of the solution to the inequality.

Example 3: 3x – 7y ≤ –2 (–4, 10)

$3(-4)-7(10) \le -2$

$-12-70 \le -2$

$-82 \le -2$ ✓

True, so the point (-4,10) is part of the solution. Yes!

Example 4: x < 2y – 17 (–8, 1)

$-8 < 2(1)-17$

$-8 \not< -15$

False!

(-8,1) is not part of the solution. No!

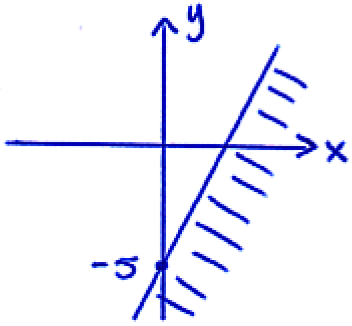
In examples 5 - 8, graph the inequality. Remember when dividing or multiplying by a negative number to reverse the inequality.

Example 5: 2x – y ≥ 5

$-y \geq -2x + 5$

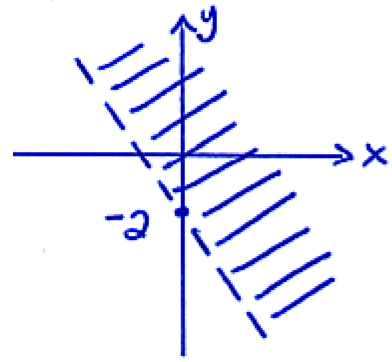
$y \leq 2x - 5$

Note reversal



Example 6: x + y > –2

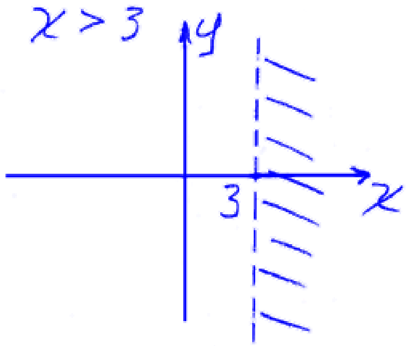
$y > -x - 2$



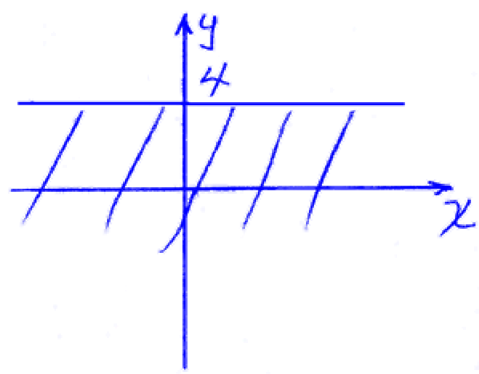
Example 7: –x < –3

$-x < -3$

$x > 3$



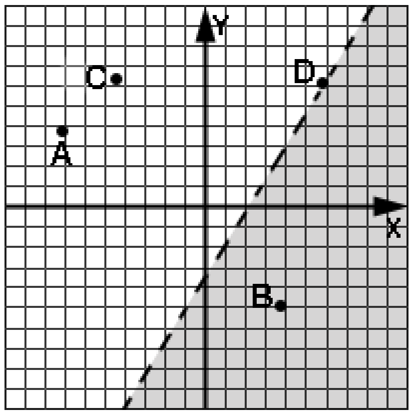
Example 8: y ≤ 4



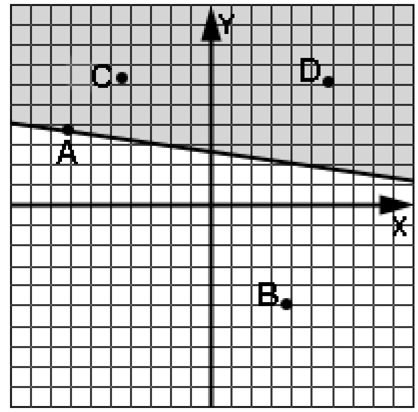
Assignment:

In problems 1 and 2, identify those points that are solutions to the inequality.

1.



2.



In problems 3 and 4, determine algebraically if the point is part of the solution to the inequality.

3. $77x - y < 2x - 1$ (0, 0)

4. $10 \geq 4x - 7y$ (–1, –2)

In problems 5 – 12, graph the inequality.

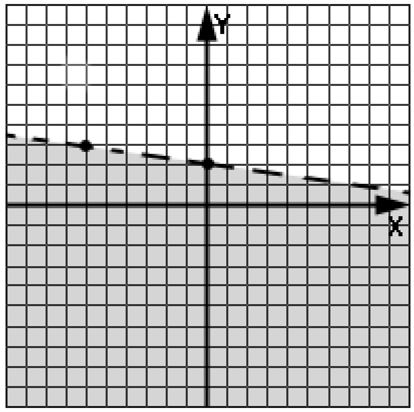
5. $x \geq -2$

6. $y < 7$

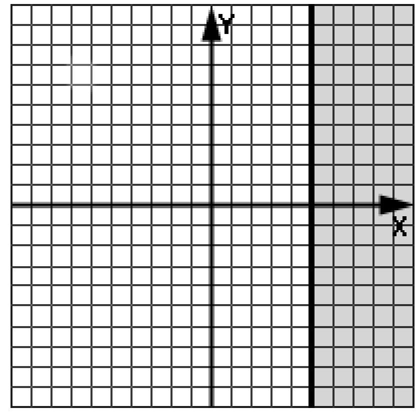
7. $y \geq 3x - 4$	**8.** $x - y > 18$
9. $x \geq \pi$	**10.** $x < -y + 1$
11. $3y < 12x$	**12.** $y < \sqrt{2}$

In problems 13 and 14, state the inequality represented by the graph.

13.



14.



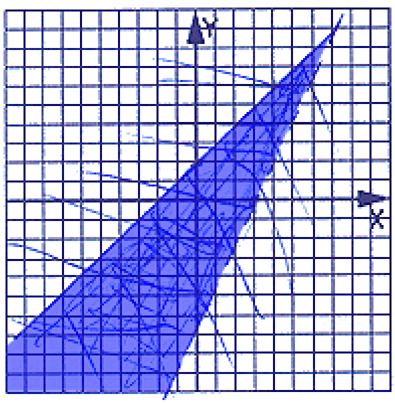
Unit 3:
Lesson 02

Graphing systems of linear inequalities in two variables

In this lesson we will simultaneously graph more than one linear inequality in two variables. The solution to this "system" will be the region in which they all **intersect** (overlap).

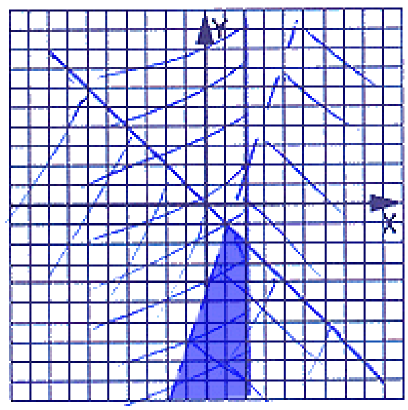
In the following examples, sketch the graph of the inequalities on the same coordinate system and then with heavier shading, show where they all intersect (overlap).

Example 1:



$y > 2x - 6$ and $y \leq x + 2$

Example 2:



$3x - y > 4$, $x \leq 2$, and $y \leq -x$

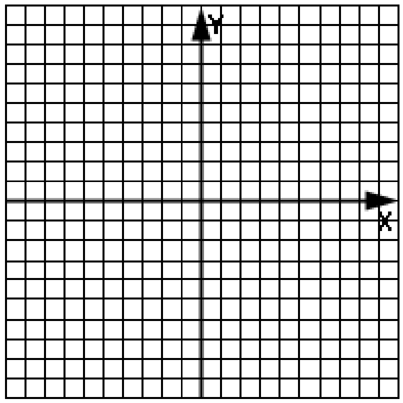
$-y > -3x + 4$
$y < 3x - 4$

See **Enrichment Topic B** for a "real world" application of linear inequality systems in two variables… **Linear Programming**.

Assignment:

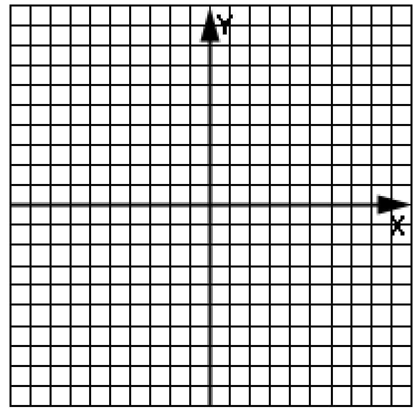
In the following problems, sketch the graph of the inequalities on the same coordinate system and then with heavier shading, show where they all intersect.

1.



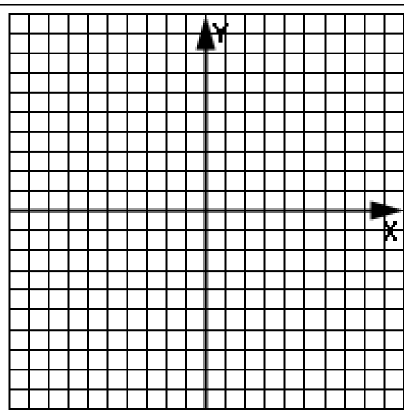
$x > 0, \quad y \le 2$

2.



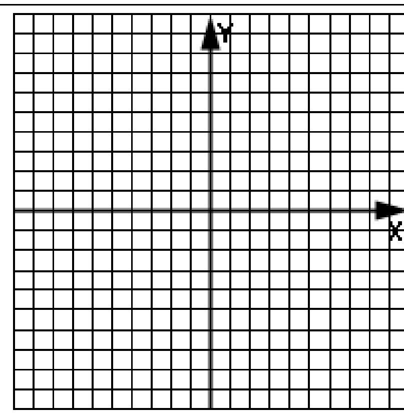
$y < x, \quad y < -x$

3.



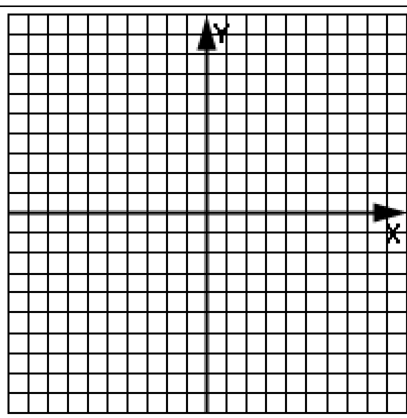
$y \le x + 2, \quad x \ge 2.5$

4.



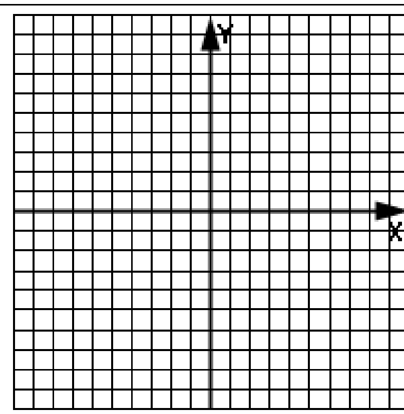
$y > -5, \quad y \le 6$

5.



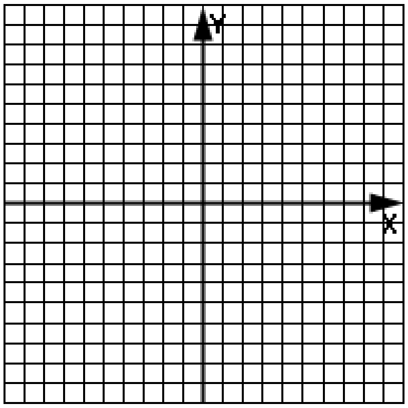
$x - y < 1, \quad 2y + x \le 10$

6.



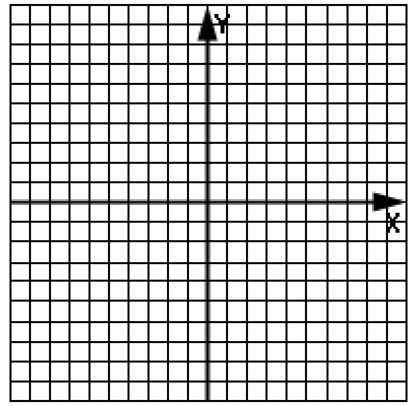
$y < -x + 3, \quad y > -x - 3$

7.



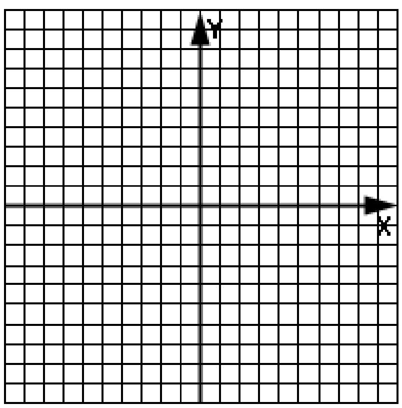
$x + y + 1 \leq 0, \quad -y \leq x - 2$

8.



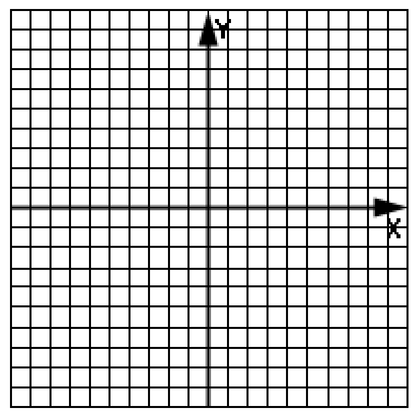
$x - 7 < 0, \quad x + y \geq 0$

***9.**



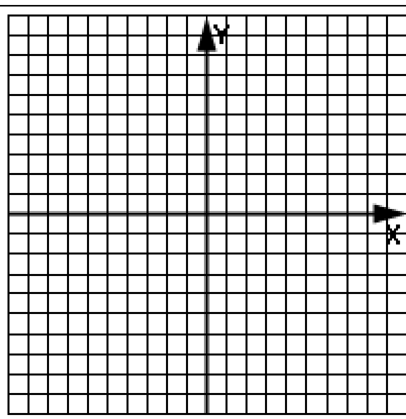
$y > x + 6, \quad y = -x$

***10.**



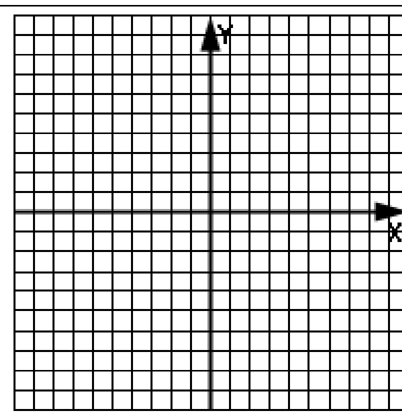
$y = x - 2, \quad y \geq -3$

11.



$y > x, \quad y < x + 2$, and $x < 3$

12.



$x < 4, \ x > -6, \ y > 0$, and $y < 8$

Unit 3: Lesson 03

Graphing calculator- linear inequalities in two variables

The basics of graphing an inequality:
To graph an inequality in two variables on the calculator enter the function with the **Y=** button. After the function has been entered as **Y1**, use the left arrow to move the cursor to the left of **Y1**.

Repeatedly press **Enter** until a short diagonal line segment is displayed to the left of **Y1** with shading above the segment icon( ). This, in effect, converts **Y1 =** into **Y1 ≥**. Similarly, a short diagonal segment icon with shading below ( ) means **≤**.

Select an appropriate zoom level (either with **Zoom** and/or **Window** buttons and then press **Graph**. The line will be drawn with the proper shading. You are on your own with regard to the difference between ≤ & < and ≥ & >. The calculator makes no distinction between them.

Multiple inequalities (Y1, Y2, etc.) can be displayed with this technique. The intersection representing the solution of the system will be evidenced by a dark cross-hatched region.

See **Calculator Appendix E** for more details.

How to graph a vertical line and associated inequality:
Strictly speaking, a vertical line cannot be graphed on a TI 84 or similar graphing calculator because a vertical line is multi-valued; therefore it is not a function (these calculators only graph functions). It is possible, however to closely **approximate a vertical line**.

> Suppose we need to graph the line x = 5 in for Y2. Use a very large negative number (compared to your window-span values) as follows:

Y2 = –9999999999(x – 5)

This will draw what will appear to be a vertical line through x = 5. It is actually a line with a huge negative slope and a huge negative y-intercept.

This "vertical" line can be shaded to the left using the technique described above ( ) to the left of Y2. In effect, this turns the line into x < 5.

Similarly, with ( ), we are graphing x > 5 and the shading will be to the right.

See **Calculator Appendix B** for more details and a video on graphing vertical lines.

Assignment:

Use the graphing calculator to produce the solution set to the following systems of inequalities. Make a rough sketch of the answers. (These are the same problems from Lesson 2.)

1. $x > 0, \quad y \le 2$	2. $y < x, \quad y < -x$
3. $y \le x + 2, \quad x \ge 2.5$	4. $y > -5, \quad y \le 6$
5. $x - y < 1, \quad 2y + x \le 10$	6. $y < -x + 3, \quad y > -x - 3$

7. $x + y + 1 \le 0$, $-y \le x - 2$

8. $x - 7 < 0$, $x + y \ge 0$

9. $y > 2x - 6$ and $y \le x + 2$

10. $3x - y > 4$, $x \le 2$, and $y \le -x$

Unit 3: Cumulative Review

1. Solve for x: $4x - x + 2 = 5 - x$

2. Solve $3 - x \geq 18$ and present the answer on a number line.

3. What is the slope of the line containing the points $(-4, -2)$ and $(6, -17)$?

4. What is the equation of the line parallel to $x + y = 17$ and containing the point $(-3, 1)$?

5. Find the point of intersection of the line $x + y = 5$ and $y = 6x - 1$

6. Determine if the lines $3x - y = 5$ and $-6x = 2y + 27$ are parallel, perpendicular or neither.

7. What is the relationship between the slopes of two lines that are perpendicular to each other?

8. Solve $\frac{4x-1}{4} = -x$ for x.

Unit 3:
Review

1. Consider the inequality $x < -4$. Which of these points are solutions to this inequality?
$(-6, 2), (-4, -7), (-1, 13), (1, 4)$

2. Consider the inequality $y \geq 2$. Which of these points are solutions to this inequality?
$(-8, 2), (4, 7), (0, 0), (-2, -6)$

3. Determine algebraically if the point is included in the solution to the inequality.
$2x - 5y > 2 \quad (3, 1)$

4. Determine algebraically if the point is included in the solution to the inequality.
$5x \leq y + 1 \quad (0, -1)$

5. Graph the solution to $x - y \geq 5$.

6. Graph the solution to $y < -x + 2$.

7. Graph the solution to $x < 22$.

8. Graph the solution to $x - y \geq 2$ and $-3x + 1 < y$.

9. Graph the solution to $y > x$ and $y \leq -x$.

10. Use a graphing calculator to show the solution to $x \leq 0$, $y \leq 0$, and $y > x$.

Alg II, Unit 4

Multiplying and Factoring Polynomials

Unit 4: Lesson 01 Simple polynomial multiplication & factoring

An essential fact to remember here is to add exponents when multiplying like variables: $x^2x^3 = x^5$

GCF stands for "greatest common factor"

Example 1: Multiply $3x(x^2 + 1)$

$3x(x^2+1)$

$= \boxed{3x^3 + 3x}$

Example 2: Multiply $3(5x - 7y + 1)$

$3(5x-7y+1)$

$= \boxed{15x - 21y + 3}$

Example 3: Factor out a GCF.
$6x^2 - 2x - 12$

$(6x^2-2x-12)$

$= \boxed{2(3x^2-x-6)}$

Example 4: Factor out a GCF.
$x^4 - 3x^3 - 17x^2$

$(x^4-3x^3-17x^2)$

$= \boxed{x^2(x^2-3x-17)}$

Example 5: Multiply $(x - 5)(x + 3)$

x^2 -15

$(x-5)(x+3)$

$-5x$

$3x$

$= \boxed{x^2-2x-15}$

Example 6: Multiply $(x + 4)(x - 7)$

x^2 -28

$(x+4)(x-7)$

$4x$

$-7x$

$= \boxed{x^2-3x-28}$

Example 7: Factor $x^2 + 4x + 3$

$=(x+?)(x+?)$

What two numbers add to give 4 & multiply to give 3 ⟿ 3 & 1

$= \boxed{(x+1)(x+3)}$

Example 8: Factor $x^2 + 3x - 10$

$= (x+?)(x+?)$

What two numbers add to give 3 & multiply to give −10 ⟿ 5 & −2

$= \boxed{(x-2)(x+5)}$

Assignment:

1. Multiply $2(x - 11)$

2. Multiply $23(x^2 + 2)$

3. Multiply $2x(x + y + 2)$

4. Multiply $ab(b^2 - 2b + 1)$

5. Multiply $-5x(2x^2 - x + 8)$

6. Multiply $-2y(-y^3 - 4)$

7. Factor out a GCF $-x^4 - 2x^2 + x$

8. Factor out a GCF $x^2 + z + y^2$

9. Factor out a GCF $2x^2 + 2z^2 + 4y^2$

10. Factor out a GCF $45z^2 - 15z - 30yz$

11. Multiply $(x - 11)(x + 2)$

12. Multiply $(x + 2)(x + 3)$

13. Multiply $(x + 7)(x - 20)$

14. Multiply $(x + 9)(x - 3)$

15. Multiply $(x - 3)(x - 3)$

16. Multiply $(x + 7)(x - 7)$

17. Factor $x^2 + 2x - 3$

18. Factor $x^2 + 4x + 4$

19. Factor $x^2 + 18x - 40$

20. Factor $x^2 + 7x + 14$

***21.** Factor $3x^2 - 24x + 36$

***22.** Factor $2x^2 - 14x - 60$

Unit 4: Lesson 02 — $(a + b)^2$, $(a - b)^2$, $(a - b)(a + b)$--- multiplying and factoring

Consider the multiplication of $(a + b)^2$. There are two ways to do this:

$$\begin{array}{r} a+b \\ \underline{a+b} \\ ab+b^2 \\ \underline{a^2+ab} \\ a^2+2ab+b^2 \end{array}$$

a^2 $\quad$ b^2

$$(a+b)^2 = (a+b)(a+b)$$

ab

$ab \rightarrow 2ab$

$$= \boxed{a^2+2ab+b^2}$$

However, in this and all future lessons, we will use the following short-cut formula:

$$\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$$

Similarly, we will also use:

$$\mathbf{(a - b)^2 = a^2 - 2ab + b^2} \qquad \text{and} \qquad \mathbf{(a - b)(a + b) = a^2 - b^2}$$

When applying any of these rules to a factoring problem, always factor out a GCF first.

Example 1: Multiply $(x + 3)^2$

$a = x$; $b = 3$

$(a+b)^2 = x^2 + (2)(x)(3) + 9$

$= \boxed{x^2+6x+9}$

Example 2: Multiply $(3x - 2)^2$

$a = 3x$; $b = 2$

$(a-b)^2 = (3x)^2 - 2(3x)2 + 4$

$= \boxed{9x^2-12x+4}$

Example 3: Multiply $(5 - y)(5 + y)$

$a = 5 \quad b = y$

$(a-b)(a+b) = \boxed{25-y^2}$

Example 4: Multiply $(2x + 5y^2)(2x - 5y^2)$

$a = 2x$; $b = 5y^2$

use $(a-b)(a+b) = a^2 - b^2$

$= (2x)^2 - (5y^2)^2$

$= \boxed{4x^2-25y^4}$

Example 5: Factor $x^2 - 10x + 25$

use $a^2 - 2ab + b^2 = (a-b)^2$

$a = x$; $b = 5$

$= \boxed{(x-5)^2}$

Example 6: Factor $h^2 + 14h + 49$

use $a^2 + 2ab + b^2 = (a+b)^2$

$a = h$; $b = 7$

$= \boxed{(h+7)^2}$

Example 7: Factor $x^2 - 121$

use $a^2 - b^2 = (a-b)(a+b)$

$x^2 - 121 = (x)^2 - (11)^2$

$= (a)^2 - (b)^2$

$= (a-b)(a+b)$

$= \boxed{(x-11)(x+11)}$

Example 8: Factor $3xy^2 - 48x$

$GCF = 3x$

$= 3x(y^2 - 16)$

$= 3x[(y)^2 - (4)^2]$ $a = y$, $b = 4$

$= \boxed{3x(y-4)(y+4)}$

Assignment: When factoring, be sure to factor completely.

1. Multiply $(x-4)^2$

2. Multiply $(y+9)^2$

3. Multiply $(x-3)(x-3)$

4. Multiply $(z+5)(z-5)$

5. Multiply $2(x-6)^2$

6. Multiply $2d(x-12)(x+12)$

7. Factor x^2-16

8. Factor x^2-100

9. Factor $2x^2 - 50$

10. Factor $100 - 9x^2$

11. Factor and simplify $(x + 2)^2 - y^2$

12. Factor and simplify $3x^2y - 27y$

13. Factor $x^2 - 16x + 64$

14. Factor $x^2 - 10x - 25$

15. Factor $x^2 + 16$

16. Factor $z^2 + 2z + 1$

*17. Factor completely $x^4 - 81$

*18. Multiply $((x-3)^2 + 4)^2$

*19. Factor $x^{2p} - 25$

*20. Multiply $((y+2)+z)((y+2)-z)$

21. Factor $x^2 + 4x + 4$

22. Factor completely $10m^2 - 1000$

Unit 4: Lesson 03 — More trinomial factoring. Leading coefficient not one

Consider a trinomial with a **leading coefficient of 1**:

Example, $\mathbf{x^2 + 4x + 3}$

All we need to do is find two numbers that multiply to give 3 and add to give 4. The numbers 1 and 3 satisfy those requirements; therefore, the factors are (x + 1)(x+ 3).

Factoring is not quite as simple if **the leading coefficient is not 1**:

Example, $\mathbf{12x^2 - 17x - 5}$

Begin by thinking of all the ways to multiply to produce $12x^2$:

12x(1x), 3x(4x), 6x(2x)

Now think of all the ways to produce –5 with multiplication:

5(–1), –5(1).

Next, build some trial factors from these possibilities: for example, (6x + 5)(2x – 1) and test to see if the full multiplication produces the original middle term (–17x).

$(6x+5)(2x-1)$

$10x$

$-6x$

$10x - 6x \neq -17x$

This one does <u>not</u> work

$(4x+1)(3x-5)$

$3x$

$-20x$

$3x - 20x = -17x$

Yes! It works.

Factors are

$\boxed{(4x+1)(3x-5)}$

Example 1: $2x^2 - 1x - 10$

$(2x-5)(x+2)$

$-5x$

$4x$

$-5x + 4x = -1x$

$\boxed{(2x-5)(x+2)}$

Example 2: $14x^2 + 4x - 10$

$= 2(7x^2 + 2x - 5)$

$= \boxed{2(7x-5)(x+1)}$

$-5x$

$7x$

See **Enrichment Topic U** for a methodical way ("Box" method) of factoring trinomials in which the leading coefficient is not 1.

Assignment:

Factor completely. Remember to first factor out a GCF if possible and be aware that some problems are from the previous lesson.

1. $4x^2 - 13x + 3$
2. $4x^2 + 2x - 6$
3. $9x^2 - 16$
4. $4x^2 - 6x - 10$
5. $3x^2 - 12$
6. $x^2 - 10x + 21$
7. $13x^2 - x + 4$
8. $3x^2 - 12x - 5$
9. $x^2 - 100$
10. $12x^2 - 30x + 12$

*11. $6x^4 - 7x^2 - 20$

*12. $72x^2 - 54x + 4$

13. $x^2 - 7x - 18$

14. $3x^2 + 3x + 3$

Unit 4 Lesson 04 **Solving equations by factoring**

A quadratic equation is one in which the highest power of *x* (the degree of the equation) is two. That predicts that it will have **two solutions**. Similarly, a degree three equation (example: $4x^3 - 6x^2 + 2x - 1 = 0$) will have three solutions.

If an equation can be factored, it can easily be solved. Even if it can't be factored there are more advanced techniques that we will learn later by which it can be solved.

Step 1: Get all terms on the left side of the equation with 0 remaining on the right side.

Step 2: Factor

Step 3: Set each factor involving *x* equal to 0 and solve for *x*.

The solutions are also called **roots** or **zeros**.

In the following examples, factor and solve for *x*.

Example 1: $x^2 - 9 = 0$

$(x-3)(x+3)=0$

$x-3=0 \quad x+3=0$

$x=\boxed{3} \quad x=\boxed{-3}$

Example 2: $x^2 = 3x + 10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x-5=0 \quad x+2=0$

$x=\boxed{5} \quad x=\boxed{-2}$

Example 3: $3x^2 + x = 0$

$x(3x+1)=0$

$x=\boxed{0}$ $\quad 3x+1=0$

$3x=-1$

$x=\boxed{\frac{-1}{3}}$

Example 4: $2x^2 + 4x - 126 = 0$

$2(x^2+2x-63)=0$

$2(x+9)(x-7)=0$

$x+9=0 \quad x-7=0$

$x=\boxed{-9} \quad x=\boxed{7}$

Do not set this one equal to 0.

Assignment:

Factor and solve for x.

1. $x^2 - 4 = 0$

2. $x^2 = 16$

3. $2x^2 - 18 = 0$

4. $x^3 - 25x = 0$

5. $(x - 5)^2 = 0$

6. $x^2 - 13x + 36 = 0$

7. $(x + 2)(x - 5)(x + 11) = 0$

8. $x^2 = -1$

9. $23x + 100 = -2x$

10. $2x^2 + 1x - 15 = 0$

11. $x^2 + 9x - 22 = 0$

12. $x^2 + 2x - 35 = 0$

13. $3x^2 - 6 = -7x$

14. $x^2 - 49 = 0$

15. How many solutions does this equation have? $x^7 - 2x^6 + 3x^2 = 17$

16. How many roots does this equation have? $x^3 - 22x + 5x^6 - 2x = 0$

17. What is the degree of the polynomial in problem 15?

18. What is the degree of the polynomial in problem 16?

***19.** Solve $72x^2 – 54x + 4 = 0$.

***20.** Solve $12x^2 – 30x = – 12$.

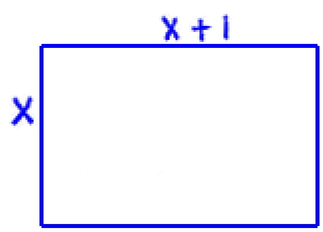
Unit 4: Lesson 05 *Solving word problems with factoring

Word problems can be solved by setting up an equation (often a quadratic) which can then be solved by factoring.

In all such problems be sure to **define the variables used**. If a drawing can be made, then variables can often be defined by labeling the drawing.

When solving be aware of **meaningless answers**. For example, when solving for the length of a rectangle, a negative answer would be meaningless.

Example 1: A rectangle is one foot longer than it is wide. Its area is 20. What are the dimensions?



$x(x+1)=20$

$x^2+1x-20=0$

$(x+5)(x-4)=0$

$x+5=0 \quad x-4=0$

$x=\cancel{-5}$ reject $\quad x=4$

Length $= x+1=4+1=\boxed{5}$

Width $= x=\boxed{4}$

Example 2: When three more than a number is multiplied times four less than the number, the result is 18. What are the possible values for the number?

$x=$ the number $\quad (x+3)(x-4)=18$

$x^2-x-12=18$

$x^2-x-12-18=0$

$x^2-x-30=0$

$(x-6)(x+5)=0$

$x-6=0 \quad x+5=0$

$x=\boxed{6} \quad x=\boxed{-5}$

Assignment:

1. A rectangle of area 50 ft^2 is twice as long as it is wide. What are the dimensions of the rectangle?

*2. If the radius of a circle is decreased by 6, the old area is 9 times the new area. What was the original radius?

3. The volume of a box 2 inches high is 160 in^3. If the length is two more than the width, what is the length and width?

4. Bob claims that if you square his age and then subtract 38 times his age, the result is 80. How old is Bob?

5. A triangle whose height is 11 less than its base has an area of 13. What are the measures of the base and height?

6. Jim's age is 4 times that of his sister. The product of their ages is 36. How old is each?

*7. The first number is two more than twice a second number. If their product is 24, what are the numbers?

Unit 4: Lesson 06 *Binomial expansion

Consider the task of raising the **binomial** (a + b) to integer powers. The only ones we have the knowledge to do immediately are:

$$(a + b)^0 = 1$$

$$(a + b)^1 = a + b$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

What about $(a + b)^3$, $(a + b)^4$, etc.?

One way to obtain these higher powers is with repeated multiplication.
For example, $(a + b)^3 = (a + b)(a + b)(a + b)$

$$= (a^2 + 2ab + b^2)(a + b)$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

This process does work but is **very tedious and time consum**ing, especially for the higher powers. What we need is a **shortcut**.

The binomial expansion (a shortcut**)**:

As an example, consider $(a + b)^5$.

- Obtain the powers of *a* by starting the powers at 5 and stair-stepping steadily **down** to 0.

$$(a + b)^5 = ?\mathbf{a^5}b^? + ?\mathbf{a^4}b^? + ?\mathbf{a^3}b^? + ?\mathbf{a^2}b^? + ?\mathbf{a^1}b^? + ?\mathbf{a^0}b^?$$

- Obtain the powers of *b* by starting the powers at 0 and stair-stepping steadily **up** to 5.

$$(a + b)^5 = ?a^5\mathbf{b^0} + ?a^4\mathbf{b^1} + ?a^3\mathbf{b^2} + ?a^2\mathbf{b^3} + ?a^1\mathbf{b^4} + ?a^0\mathbf{b^5}$$

- Obtain the coefficients from Pascal's triangle (next page)

$$(a + b)^5 = \mathbf{1}a^5b^0 + \mathbf{5}a^4b^1 + \mathbf{10}a^3b^2 + \mathbf{10}a^2b^3 + \mathbf{5}a^1b^4 + \mathbf{1}a^0b^5$$

Pascal's triangle:

- The top and outside numbers are all 1.
- Each number in the interior is the sum of the two nearest numbers in the row directly above it.

1............................ for $(a + b)^0$

1 1......................... for $(a + b)^1$

1 2 1..................... For $(a + b)^2$

1 3 3 1................. for $(a + b)^3$

1 4 6 4 1........... for $(a + b)^4$

1 5 10 10 5 1....... for $(a + b)^5$

1 6 15 20 15 6 1.... for $(a + b)^6$

Example 1: Expand $(a + b)^4$

$$(a + b)^4 = \boxed{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$$

The variables *a* and *b* can be thought of as place holders for more complicated expressions.

Example 2: Expand $(2x – 4y^2)^3$

$(2x - 4y^2)^3 = (\ 2x + (-4y^2)\ )^3 \qquad a = 2x;\ \ b = -4y^2$

$(2x - 4y^2)^3 = 1(2x)^3(-4y^2)^0 + 3(2x)^2(-4y^2)^1 + 3(2x)^1(-4y^2)^2 + 1(2x)^0(-4y^2)^3$

$(2x - 4y^2)^3 = 8x^3 \quad + 3(4x^2)(-4y^2) \quad + 3(2x)(16y^4) \quad + 1(1)(-64y^6)$

$(2x - 4y^2)^3 = 8x^3 \quad - 48x^2y^2 \quad + 96xy^4 \quad - 64y^6$

$(2x - 4y^2)^3 = \boxed{8x^3 - 48x^2y^2 + 96xy^4 - 64y^6}$

Assignment: In the following problems, perform a binomial expansion on the given binomial raised to the indicated power.

1. $(c + d)^3$

2. $(a + 2)^4$

3. $(x + y)^5$

4. $(a + b)^6$

5. $(z^3 + 2y)^4$

6. $(x^2y - 2j)^3$

7. Write the row of Pascal's triangle corresponding to $(a + b)^7$.

Unit 4:
Cumulative Review

1. Solve $x^3 - 25x = 0$

2. What is the equation of a line perpendicular to $3x - y = 4$ and having the same y-intercept as $x + 2y = 15$?

3. Convert $17x - 8y = 2$ to slope-intercept form.

4. Sketch the graph of $8x - 2y \le 11$

5. Find the intersection point of the two lines given by $y = 3x - 7$ and $-x + 4y = -11$. Use the substitution method.

6. What is the equation of a line parallel to the line given by $x - 6 = 0$ and passing through the x-axis at -5?

7. What is the slope of the line containing $(-11, 5)$ and $(-11, -2)$?

8. Graph the solution to $2x > -5x + 14$ on a number line.

9. Graph the solution to the following system of inequalities.

$y > 3x$ $\qquad$ $x < -y + 2$

10. What is the equation of the line in slope-intercept form having an x-intercept at 5 and y-intercept at 2?

11. What is the equation of the line perpendicular to the y-axis and having the same y-intercept as $y + 4 = 18x$?

Unit 4: Review

1. Multiply $(3x + 2)(-x)$

2. Factor $2x^3 + 7x^2 - 9x$

3. Multiply $(3x - 2)(2x + 4)$

4. Factor $x^2 - 3x - 28$

5. Factor $36 - z^2$

6. Factor $4x^2 - y^2$

7. Factor $4xy^2 - 11xy - 3x$

8. Multiply $(x + 11)(x - 2)$

9. Factor $x^2 + 14x + 24$

10. Factor $x^2 - 13x + 42$

11. Multiply $(p - 8)^2$

12. Multiply $(q - 5m)^2$

13. Multiply $(3x + 1)(6x - 4)$

14. Factor and solve $6x^2 - 24 = 0$

15. Factor $12x^2 + 2x - 2$

16. Factor $3x^2 - 19x - 22$

17. Factor and solve $x^2 + 6x + 5 = 0$

18. Factor and solve $3x^2 + x - 2 = 0$

In the following problems, define any variables you use (can sometimes be done by labeling a drawing). Set up an equation and solve by factoring.

19. A rectangle's short side is two feet less than the long side. Its area is 24. What are the dimensions of the rectangle?

20. The new ticket number is one more than the square of the old ticket number. If the new ticket number is 10, what was the old ticket number?

21. Build the first five rows of Pascal's triangle.

22. Use the binomial expansion technique to raise $(w + 3b^2)$ to the 4^{th} power.

Alg II, Unit 5

Exponents and Radicals

Unit 5: Lesson 01 Exponent rules

Rule 1: $x^p x^q = x^{p+q}$

Rule 2: $\frac{x^p}{x^q} = x^{p-q}$

Rule 3: $(x^p)^q = x^{pq}$

Rule 4: $(xy)^p = x^p y^p$

Rule 5: $\left(\frac{x}{y}\right)^p = \frac{x^p}{y^p}$

Rule 6: $x^0 = 1,\ x \neq 0$

Example 1: $[(-3)^3]^2$

$= (-3)^6 = \boxed{729}$

Example 2: $(-2)^4(-2)$

$= (-2)^{4+1} = (-2)^5$

$= \boxed{-32}$

Example 3: $\left(\frac{1}{x^5}\right)x$

$= \frac{x}{x^5} = \boxed{\frac{1}{x^4}}$

Example 4: $(2^2xy^3)^2$

$= \boxed{2^4x^2y^6}$

Example 5: $(3x^5y^7)^0$

Anything to the zero power is 1.

$= \boxed{1}$

Example 6: $zx^2(4x^2+yz)$

$= \boxed{4zx^4 + yz^2x^2}$

Example 7: $\left(\frac{m^5}{n^4}\right)(m^3n)^2$

$= \frac{m^5}{n^4} \cdot \frac{m^6n^2}{1} = \frac{m^{11}\cancel{n^2}}{n^{\cancel{4}\,2}}$

$= \boxed{\frac{m^{11}}{n^2}}$

Example 8: $5^{x+4}5^3$

$= 5^{x+4+3}$

$= \boxed{5^{x+7}}$

Assignment: Simplify

1. $(5x^2y^3)^2$

2. $(-4^2)^3$

3. $4m^3(3m^8)$

4. $3x(x^2 - 8x^2 - 2)$

5. $\dfrac{5z^3x^8}{-3z^2x^3}$

6. $(-2xm^2n^3)^4$

7. $\left(\dfrac{pq^9}{q^5}\right)$

8. $(2xy^4z^3)^4$

9. $\frac{13x^3y^4}{xy}$

10. $(11x^3y)^2$

11. $4p^2(2pq^4)^2$

12. $(-j^4)^3$

13. $(10x^2z)^2/(5xz^9)^0$

14. $(-c^2)^3$

15. $(8/(a^2b^2))^2$

16. $\left(\frac{(3^5x^2)^0}{y^7}\right)(xy^3)^2$

17. $xy^2z^3(x^2 - 4y)$

18. $\left(\frac{a^2b^3}{ab^2}\right)^5$

19. $3x^2y^4 + 3(xy^2)^2$

20. x^5x^{p+3}

Unit 5: Lesson 02 **Negative exponents**

$$x^{-p} = \frac{1}{x^p} \qquad\qquad \frac{1}{x^{-q}} = x^q$$

Rule: When moving an expression having an exponent either up (to the numerator) or down (to the denominator), **change the sign of the exponent**.

Example 1: $(3^{-2})^3$

$= 3^{-6} = \frac{1}{3^6} = \boxed{\frac{1}{729}}$

Example 2: $8^{-3}8^2$

$= 8^{-3+2} = 8^{-1} = \boxed{\frac{1}{8}}$

Example 3: $\left(\frac{2}{3}\right)^{-2}$

$= \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

Example 4: $\left(\frac{2}{3}\right)^{-2}$ On this problem use this shortcut: "If a fraction is raised to a power as in example 3, invert the fraction and change the sign of the exponent."

$= \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \boxed{\frac{9}{4}}$

Example 5. $\dfrac{2^{-3}2^{-4}}{2^{-5}}$

$$= \frac{2^{-3-4}}{2^{-5}} = \frac{2^{-7}}{2^{-5}}$$

$$= \frac{1}{2^{7}2^{-5}} = \frac{1}{2^{2}}$$

$$= \boxed{\frac{1}{4}}$$

Example 6. $\dfrac{3x^{-3}y^{5}}{6y^{-4}}\dfrac{2x^{4}y^{3}}{4x^{3}}$

$$= \frac{6xy^{8}}{24y^{-4}x^{3}} = \frac{1\,y^{4}y^{8}}{4x^{3}x^{-1}}$$

$$= \boxed{\frac{y^{12}}{4x^{2}}}$$

Example 7. $\dfrac{y^{3}x^{-2}m^{1}}{x^{-3}y^{2}}\left(\dfrac{y^{-2}xm}{m^{2}}\right)^{-2}$

$$= y^{3}y^{-2}mx^{-2}x^{3}\left(\frac{m^{2}}{y^{-2}xm}\right)^{2}$$

$$= ymx\ \frac{m^{4}}{y^{-4}x^{2}m^{2}}$$

$$= \frac{m^{5}yy^{4}}{m^{2}x^{2}x^{-1}} = \frac{m^{5}y^{5}}{m^{2}x}$$

$$= \frac{m^{5}m^{-2}y^{5}}{x} = \boxed{\frac{m^{3}y^{5}}{x}}$$

Assignment:

Simplify and leave the answer with all positive exponents.

1. $(2^3)^{-3}$	2. $(3)^2(3)^{-4}$
3. m^5m^{-3}	4. $(z^{-5})^{-2}$
5. $(p^{-4})^6$	6. $7x^{-2}z$
7. $(-5x^2q)^3$	8. $(x^3y^3)^{-2}$

9. $(5xy^2)^{-3}$

10. $\dfrac{\left(\frac{1}{3}\right)^{2}}{\left(\frac{1}{3}\right)^{-2}}$

11. z^4/z^{-1}

12. $\left(\dfrac{5x}{2y}\right)\left(\dfrac{y^3x^{-2}}{x^4}\right)$

13. $\left(\dfrac{2z^{-2}y^5}{8zy^2}\right)^{-2}$

14. $(-2b^2)^4$

15. $4b^0c^4$

16. $\left(\frac{-3x^4}{x^{12}}\right)\frac{2x^9y}{12y^{-2}}$

17. $-4x^{-3}y^0$

18. $(x^{-5}z)/(xz^{-2})$

19. $\left(\frac{ab^{-4}}{3b^{-2}}\right)^2\left(\frac{9b}{3a^5}\right)$

20. $\frac{10y}{2y^{-2}}\left(\frac{b^2}{y^{-2}}\right)^{-2}$

Unit 5: Lesson 03

More exponent problems

Assignment: Simplify and manipulate the answers so that all exponents are positive.

1. $\frac{1}{12x^{-4}}$

2. $x^{-3}x^{84}x-5$

3. $6x^{-4}$

4. $(2z)^{-5}$

5. $\left(\frac{m^3}{n^3}\right)^{-2}$

6. $\left(\frac{3x^{-3}}{-6}\right)^3$

7. $(3a^5)^{-2}$

8. $\frac{1}{(2x)^{-3}}$

9. $\frac{11x^{-4}}{22x^{-9}}$

10. $(-3a^2b^3c^5)^3$

11. $(3x^{-2})^{-3}$

12. $2(pq^{-4})^0$

13. $\frac{x^6y^{-10}}{x^{-2}y^8}$

14. $(5x^3y^{-5})(-2(2xy)^2)$

15. $3x^{-7}4y^{-1}x^{4}$

16. $(-a^{2}b^{-5})^{3}$

17. $\left(\frac{3x^{-2}}{9y^{-2}}\right)^{-2}\left(\frac{3x^{-3}y^{3}}{6m^{-4}}\right)$

18. $(-2a^{-3}b^{5})^{-2}$

19. $\left(\frac{3x^{-2}m}{9y^{-2}}\right)^{-2}\left(\frac{3x^{-3}y^{3}}{6m^{-4}}\right)$

20. $\frac{100x^{-4}}{y^{4}z^{-4}}\frac{2x^{-3}y^{0}}{25z^{2}}$

Unit 5: Lesson 04 Simplifying radical expressions

In simplifying radicals, think of this:

The square root of "something" squared is that "something."

$$\sqrt{x^2} = x$$

Similarly, the cube root of "something" cubed is that "something."

$$\sqrt[3]{x^3} = x$$

Example 1: $\sqrt{72}$

$= \sqrt{36 \cdot 2} = \boxed{6\sqrt{2}}$

Example 2: $\sqrt[3]{24x^4}$

$= \sqrt[3]{8 \cdot 3x^3 \cdot x}$

$= \boxed{2x\sqrt[3]{3x}}$

Example 3: $-\sqrt[3]{48}$

$= -1\sqrt[3]{8 \cdot 6} = -1 \cdot 2\sqrt[3]{6}$

$= \boxed{-2\sqrt[3]{6}}$

Example 4: $\sqrt[5]{-64x^4y^{15}}$

$= \sqrt[5]{(-32)2x^4(y^3)^5}$

$= \boxed{-2y^3\sqrt[5]{2x^4}}$

Multiplication and division rules:

$$\sqrt[p]{\mathbf{x}}\,\sqrt[p]{\mathbf{y}} = \sqrt[p]{\mathbf{xy}} \qquad \frac{\sqrt[p]{\mathbf{x}}}{\sqrt[p]{\mathbf{y}}} = \sqrt[p]{\frac{\mathbf{x}}{\mathbf{y}}}$$

Example 5: $\frac{\sqrt[3]{8}}{\sqrt[3]{48}}$

$= \sqrt[3]{\frac{8}{48}} = \sqrt[3]{\frac{1}{6}}$

$= \frac{\sqrt[3]{1}}{\sqrt[3]{6}} = \boxed{\frac{1}{\sqrt[3]{6}}}$

Example 6: $2\sqrt{8}\sqrt{12}$

$= 2\sqrt{4\cdot 2(4\cdot 3)}$

$= 2\sqrt{16\cdot 6}$

$= 2\cdot 4\sqrt{6} = 8\sqrt{6}$

Example 7: $\sqrt[5]{2x^2y^4}\,\sqrt[5]{16x^4y^7}$

$= \sqrt[5]{2x^2y^4 16x^4y^7}$

$= \sqrt[5]{32x^6y^{11}}$

$= 2\sqrt[5]{x^5x^1(y^5)^2y^1} = \boxed{2xy^2\sqrt[5]{xy}}$

Example 8: $\sqrt[3]{\frac{16}{27}}$

$= \sqrt[3]{\frac{8\cdot 2}{27}} = \frac{\sqrt[3]{8\cdot 2}}{\sqrt[3]{27}}$

$= \boxed{\frac{2\sqrt[3]{2}}{3}}$

Rule: Radicals can only be added or subtracted when **both** the index and radicands are the same.

$\sqrt[p]{r}$ (p is the index and r is the radicand)

Example 9: $3\sqrt{11} - 5\sqrt{11}$

$= \boxed{-2\sqrt{11}}$

Example 10: $-4\sqrt[3]{16x} - 6\sqrt[3]{32x}$

$= -4\sqrt[3]{8\cdot 2x} - 6\sqrt[3]{8\cdot 4x}$

$= -4\cdot 2\sqrt[3]{2x} - 6\cdot 2\sqrt[3]{4x}$

$= \boxed{-8\sqrt[3]{2x} - 12\sqrt[3]{4x}}$

can't combine

Assignment:

In the following problems, simplify the radicals.

1. $\sqrt[4]{16xa^5b^2zc^6}$

2. $3\sqrt{6} - 2\sqrt{6}$

3. $\sqrt[3]{8x^5y^4}$

4. $\sqrt[4]{3x^2y^3}\ \sqrt[4]{11x^3y^6}$

5. $\sqrt[3]{\frac{1000}{54}}$

6. $\sqrt[5]{64x^{10}}$

7. $7\sqrt{11} + 16\sqrt{44}$

8. $\sqrt{72}\,\sqrt{12}$

9. $x\sqrt[3]{4x}\,\sqrt[3]{4x^2}$

10. $\dfrac{\sqrt{25p}}{\sqrt{5p}}$

11. $\sqrt{\frac{2}{3}z^3}\,\sqrt{12z}$

12. $\sqrt{72}$

13. $\sqrt{3y}\sqrt{8y^3}$

14. $\sqrt{\frac{144}{121}}$

15. $2\sqrt{72} - 3\sqrt{8} + \sqrt{3}$

16. $3\sqrt[3]{4\text{x}} - 8\sqrt[3]{32\text{x}}$

Unit 5: Lesson 05 Fractional exponents

Consider $\sqrt{x}$. It could also be written as $\sqrt[2]{x}$ and finally as …

$$\sqrt{x} = x^{1/2}$$

Similarly, $\sqrt[3]{x} = x^{1/3}$

The expression $\sqrt[r]{x^n}$ can be rewritten as $(\sqrt[r]{\boldsymbol{x}})^{\boldsymbol{n}}$

In examples 1-3 rewrite each radical using exponents. In 4-6, rewrite using radicals.

Example 1: $\sqrt{7}$

$= \boxed{7^{1/2}}$

Example 2: $\left(\sqrt[3]{5\text{y}}\right)^2$

$= \boxed{(5y)^{2/3}}$

Example 3: $\sqrt[4]{17^3}$

$= (17^3)^{1/4}$

$= \boxed{17^{3/4}}$

Example 4: $11^{4/5}$

$= \boxed{\sqrt[5]{11^4} \text{ or } (\sqrt[5]{11})^4}$

Example 5: $(3\text{a}^2\text{b})^{2/3}$

$\boxed{\sqrt[3]{(3a^2b)^2} \text{ or } (\sqrt[3]{3a^2b})^2}$

Example 6: $6^{1/2}$

$= \boxed{\sqrt{6}}$

Simplify the following problems without using a calculator.

Example 7: $27^{-2/3}$

$= \left(27^{1/3}\right)^{-2} = \left(\sqrt[3]{27}\right)^{-2}$

$= 3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

Example 8: $9^{3/2}$

$= \left(9^{1/2}\right)^3 = (3)^3$

$= \boxed{27}$

Example 9: $3^{1/4}3^{3/4}$

$= 3^{1/4+3/4} = 3^{4/4} = 3^1$

$= \boxed{3}$

Example 10: $\left(\frac{16x^8}{81y^{12}}\right)^{.5}$

$= \sqrt{\frac{16\left(x^4\right)^2}{81\left(y^6\right)^2}}$

$= \boxed{\frac{4x^4}{9y^6}}$

Example 11: $\left(x^{\frac{1}{4}}y^{\frac{1}{2}}\right)^8$

$= x^{8/4}y^{8/2} = \boxed{x^2y^4}$

Example 12: $a^{\frac{1}{3}}\,a^{\frac{2}{3}}\,a^{-\frac{5}{3}}$

$= a^{1/3+2/3-5/3}$

$= a^{-2/3} = \frac{1}{a^{2/3}} = \boxed{\frac{1}{\sqrt[3]{a^2}}}$

Example 13: $\dfrac{-4a^{\frac{3}{5}}}{12a^{-\frac{7}{5}}}$

$= \frac{-1a^{3/5}a^{7/5}}{3} = \frac{-1a^{3/5+7/5}}{3}$

$= \frac{-1}{3}a^{10/5} = \boxed{\frac{-1}{3}a^2}$

Example 14: $\dfrac{1}{9^{-.5}}$

$= \frac{1}{9^{-1/2}} = 9^{1/2} = \sqrt{9}$

$= \boxed{3}$

Assignment:

Rewrite the following problems in radical form.

1. $(5xy^3)^{3/4}$	2. $8^{3/5}$
3. $(4z^3)^{1/4}$	4. $11^{1/3}$

Rewrite the following problems with fractional (rational) exponents.

5. $\sqrt[3]{9.5}$	6. $\left(\sqrt[8]{13}\right)^2$
7. $\sqrt[3]{4q^5}$	8. $\left(\sqrt[5]{36y^3}\right)^2$

Evaluate and simplify without using a calculator. Leave only positive exponents in the answer.

9. $27^{-2/3}$	10. $(16x^4)^{3/4}$
11. $9^{3/2}$	12. $81^{-3/4}$
13. $7^{1/3}7^{2/3}$	14. $x^{3/4}x^{1/2}x^{-1/4}$

15. $(27x^9y^6)^{1/3}$

16. $\left(\dfrac{b^{\frac{1}{3}}}{b^{\frac{4}{3}}}\right)^4$

17. $\dfrac{-15x^{\frac{-1}{6}}}{5x^{\frac{7}{6}}}$

18. $(-3p^{-1/3}p^{2/3})^2$

19. $(16z)^{1/4}$

20. $\dfrac{x^{2/3}d^{1/4}}{x^{-5/3}d^{3/4}}$

Unit 5: Lesson 06

* Solving equations having rational & variable exponents

Many times it is desirable to "get rid" of an exponent in the process of solving an equation. For example, consider:

$(x - 2)^{1/3} = 5$

Simply raise both sides to the 3 power and x – 2 will be left with no exponent. From that point it is easy to solve for *x*.

$$\left[(x-2)^{1/3}\right]^3 = [5]^3$$
$$(x-2)^1 = 125$$
$$x - 2 = 125$$
$$x = 125 + 2 = \boxed{127}$$

Rule: When raising both sides of an equation to an **even power** (or if the numerator of a fractional power is even), extraneous roots (answers) can be produced. In this case substitute prospective roots back into the **original equation** to see if they really work.

Example 1: $x^{3/4} = 8$

$$\left[x^{3/4}\right]^{4/3} = 8^{4/3}$$
$$x = \left[8^{1/3}\right]^4 \leftarrow \text{even}$$
$$x = 2^4 = \boxed{16}$$

Check:

$$16^{3/4} = 8$$
$$(16^{1/4})^3 = 8$$
$$2^3 = 8$$
$$8 = 8 \checkmark$$

Example 2: $(4x)^{1/2} + 5 = 0$

$$(4x)^{1/2} = -5$$
$$\left[(4x)^{1/2}\right]^2 = [-5]^2 \leftarrow \text{even}$$
$$4x = 25$$
$$x = 25/4$$

Check:

$$\left(4 \cdot \tfrac{25}{4}\right)^{1/2} + 5 = 0$$
$$5 + 5 \neq 0$$

No Solution

If in the process of solving a problem a quantity is raised to a fractional power and the denominator is an even number (for example ½, ¼, etc.), we must precede that quantity with ±. Recall that, for example, the ½ power indicates a square root.

Take particular note of this in example 3 below.

Example 3: $x^{2/5} + 2 = 11$

$x^{2/5} = 11 - 2$

$x^{2/5} = 9$

$\left[x^{2/5}\right]^{5/2} = 9^{5/2}$ ← odd

$x = \left[\pm 9^{1/2}\right]^5$

$x = [\pm 3]^5 = \boxed{\pm 243}$

Example 4: $-2(3x - 4)^{3/4} + 5 = 59$

$-2(3x-4)^{3/4} = 59 - 5$

$-2(3x-4)^{3/4} = 54$

$(3x-4)^{3/4} = -27$

even

$\left[(3x-4)^{3/4}\right]^{4/3} = [-27]^{4/3}$

$3x - 4 = \left[(-27)^{1/3}\right]^4$

$3x - 4 = (-3)^4$

$3x - 4 = 81$

$3x = 81 + 4$

$x = 85/3$

Check!

$-2\left(3\,\frac{85}{3} - 4\right)^{3/4} + 5 = 59$

$(81)^{3/4} = \frac{54}{-2}$

$\left(81^{1/4}\right)^3 = -27$

$(3)^3 = -27$

$27 \neq -27$

$\boxed{\text{No Solution}}$

If bases are the same on both sides of an equation as in $\mathbf{3^a = 3^b}$, then drop the bases and set exponents equal… **a = b**.
. . . (This is illustrated in the following examples.)

Example 5. $2^{x-6} = 2^{4x}$

Since bases are the same, set exponents equal.

$x - 6 = 4x$

$x - 4x = 6$

$-3x = 6$

$x = \boxed{-2}$

Example 6. $(3^x)^2\, 3^4 = 9$

$3^{2x} \cdot 3^4 = 3^2$

$3^{2x+4} = 3^2$

Set exponents equal

$2x + 4 = 2$

$2x = -4 + 2$

$2x = -2$

$x = \boxed{-1}$

Assignment:

Solve for the variable and check for extraneous roots when appropriate.

1. $x^{2/3} - 18 = 14$

2. $(2x - 1)^{1/3} - 2 = 0$

3. $(2x - 5)^{2/3} = 16$

4. $5x^{2/3} = 125$

5. $(4x + 2)^{1/2} + 5 = 0$

6. $2x^{3/5} = 16$

7. $(x + 1)^{3/2} + 5 = 69$

8. $2(x - 1)^{1/2} + 6 = 12$

9. $2(x - 1)^{1/4} = 2$

*10. $(1/3)(x + 1)^{5/4} + 23 = 101/3$

11. $3^{x-2} = 3$

12. $5^x 5^2 = 5^{3x+1}$

13. $4^x 4^2 = 16$

*14. $5^{2x-3}5^{x^2} = 1$

Unit 5: Lesson 07 ***Solving radical equations**

To solve an equation containing a radical, follow these steps:

- Isolate the term containing the radical.
- Raise both sides of the equation to a power that eliminate the radical. For example, if the radical is a square root, raise both sides to the 2 power.
- Solve for the variable.

Example 1: $\sqrt{2x-5}=3$

$$(\sqrt{2x-5})^2=(3)^2$$
$$2x-5=9$$
$$2x=14$$
$$x=\boxed{7}$$

Check:

$$\sqrt{2\cdot 7-5}=3$$
$$\sqrt{9}=3$$
$$3=3\ \checkmark$$

Example 2: $\sqrt[3]{2x+3}-4=0$

$$\sqrt[3]{2x+3}=4$$
$$(\sqrt[3]{2x+3})^3=(4)^3$$
$$2x+3=64$$
$$2x=64-3$$
$$2x=61$$
$$x=\boxed{\frac{61}{2}}$$

Not necessary to check since we raised both sides to an <u>odd</u> power.

Example 3: $x - \sqrt{x + 24} = -4$

$x + 4 = \sqrt{x+24}$

$(x+4)^2 = (\sqrt{x+24})^2$

$x^2 + 8x + 16 = x + 24$

$x^2 + 8x - x + 16 - 24 = 0$

$x^2 + 7x - 8 = 0$

$(x+8)(x-1) = 0$

$x + 8 = 0 \quad x - 1 = 0$

$x = -8 \quad x = \boxed{1}$

reject

Check -8:

$-8 - \sqrt{-8+24} = -4$

$-8 - \sqrt{16} = -4$

$-8 - 4 = -4$

$-12 \neq -4$ reject

Check 1:

$1 - \sqrt{1+24} = -4$

$1 - \sqrt{25} = -4$

$1 - 5 = -4$

$-4 \stackrel{\checkmark}{=} -4$

Example 4: $\sqrt{x + 1} - 2 = \sqrt{x - 3}$

$(\sqrt{x+1} - 2)^2 = (\sqrt{x-3})^2$

$x + 1 - 4\sqrt{x+1} + 4 = x - 3$

$-4\sqrt{x+1} = -3 - 1 - 4$

$-4\sqrt{x+1} = -8$

$\sqrt{x+1} = 2$

$(\sqrt{x+1})^2 = 2^2$

$x + 1 = 4$

$x = \boxed{3}$

Check:

$\sqrt{3+1} - 2 = \sqrt{3-3}$

$\sqrt{4} - 2 = 0$

$2 - 2 = 0$

$0 \stackrel{\checkmark}{=} 0$

Assignment: Solve for the variable in each problem. Be sure to check your solution when necessary.

1. $\sqrt[3]{x+5} = -2$

2. $4\sqrt{x-2} + 6 = -2$

3. $11 + x = \sqrt{x+52} + 1$

4. $-1 - x + 3\sqrt{x+1} = 0$

---

5. $\sqrt[4]{m-3} + 9 = 11$

---

6. $\sqrt{q-4} = 30$

7. $2 - \sqrt[5]{p-2} = 3$

---

*8. $\sqrt{x+3} = 1 + \sqrt{x-2}$

---

9. $4\sqrt{x-6} - \sqrt{x} = 0$

**10. $\sqrt{x-1} = \frac{1+\sqrt{x+6}}{\sqrt{2}}$

Unit 5: Lesson 08 Rationalizing denominators

It is generally not considered simple to leave a **fraction under a radical** or to have a **radical in the denominator** of a fraction:

$$\sqrt{\frac{3}{x}} \quad \text{or} \quad \frac{6x}{\sqrt{5y}}$$

In both cases above the expressions can be simplified so as to neither have a fraction under a radical nor to have a radical in a denominator. This is known as **rationalizing the denominator**:

Example 1: $\sqrt{\frac{2x}{3q}}$

$$\sqrt{\frac{2x}{3q}\cdot\frac{3q}{3q}} = \sqrt{\frac{6xq}{9q^2}}$$

$$= \boxed{\frac{\sqrt{6xq}}{3q}}$$

Example 2: $\sqrt[5]{\frac{2}{x^2}}$

$$\sqrt[5]{\frac{2}{x^2}\cdot\frac{x^3}{x^3}}$$

$$= \sqrt[5]{\frac{2x^3}{x^5}} = \boxed{\frac{1}{x}\sqrt[5]{2x^3}}$$

Example 3: $\frac{4z}{\sqrt{6z}}$

$$\frac{4z}{\sqrt{6z}}\cdot\frac{\sqrt{6z}}{\sqrt{6z}} = \frac{4z\sqrt{6z}}{\sqrt{36z^2}}$$

$$= \frac{4\cancel{z}\sqrt{6z}}{6\cancel{z}} = \boxed{\frac{2\sqrt{6z}}{3}}$$

Example 4: $\frac{\sqrt{8}}{\sqrt[3]{x}}$

$$= \frac{\sqrt{8}}{\sqrt[3]{x}} = \frac{\sqrt{4\cdot 2}}{\sqrt[3]{x}}\cdot\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

$$= \frac{2\sqrt{2}\,\sqrt[3]{x^2}}{\sqrt[3]{x^3}}$$

$$= \boxed{\frac{2\sqrt{2}\,\sqrt[3]{x^2}}{x}}$$

Example 5: $\frac{x}{\sqrt{3x}-2}$

$$\underbrace{\frac{x}{\sqrt{3x}-2}}_{a-b}\;\underbrace{\frac{\sqrt{3x}+2}{\sqrt{3x}+2}}_{a+b}=\frac{x(\sqrt{3x}+2)}{\underbrace{(\sqrt{3x})^2-2^2}_{a^2-b^2}}=\boxed{\frac{x\sqrt{3x}+2x}{3x-4}}$$

Assignment: Simplify and arrange each expression so as to not have a radical in the denominator or a fraction under a radical:

1. $\sqrt{\frac{x}{2}}$	2. $\sqrt{\frac{5}{4}}$
3. $\sqrt{\frac{y^2}{x^5}}$	4. $\sqrt{\frac{3x^5}{5q}}$
5. $\frac{18}{\sqrt{8x^3}}$	6. $\frac{q^2+3}{\sqrt[3]{q^2}}$

7. $\frac{1}{\sqrt{x-3}}$

8. $\frac{\sqrt{x+3}}{2}$

9. $\frac{1}{\sqrt{x^2-6x+9}}$

10. $\left(\sqrt{2x}\right)^{-1}$

*11. $\frac{1}{\sqrt{x}+2}$

12. $\frac{2x}{5(x+7)}$

13. $\sqrt{\left(\frac{3}{x}\right)^{-1}}$

**14. $\frac{x}{\sqrt{8x^2}+1}$

Unit 5:
Cumulative Review

1. Find the equation of the line parallel to the x-axis and passing through the point (0, 12).

2. Find the equation of the line with x-intercept 3 and y-intercept –7.

3. A total of 1806 tickets to a concert were sold which amounted to \$20,040. The price of a regular ticket was \$12 while a balcony ticket was \$10. How many of each type ticket was sold?

4. Produce the solution to the following system of equations on a graphing calculator. Graph the lines and find their intersection point.
$.33x - 5y = 16.071$ and $x + .908y = 4.1$

5. Multiply $(3x - 5z)(2x + 7z)$

6. Factor $x^2 - 81y^2a^2$

7. Multiply $(5x - 2y^2)^2$

8. Factor $a^4 - b^4$

9. $\sqrt{x+1} + 3 = 0$

10. Simplify $\sqrt[5]{16x^6y^{10}}$

11. Vertical line A passes through (18, 22) while horizontal line B passes through (5.7, –.3). What is the intersection point of these two lines?

Unit 5: Review

1. Simplify $(3x^4y^3)^3$

2. Simplify $4^{x+3}4^{2x+1}$

3. Simplify $\left(\frac{4x^{-2}}{yx^5}\right)^{-3}$

4. Simplify $\frac{6y^2}{4y^{-3}}\left(\frac{y^4}{y^{-2}}\right)^{-3}$

5. Simplify $(xy^{15})^0 4x^2x^3$

6. Simplify $-\sqrt[3]{-96}$

7. Simplify $\frac{\sqrt[3]{8x}}{\sqrt[3]{x^4}}$

8. Simplify $-7y\sqrt[3]{16y} - \sqrt[3]{128y^4}$

9. Simplify $\sqrt[5]{64x^2y^3}\,\sqrt[5]{2x^6y^3}$

10. Simplify $\sqrt{50}\,\sqrt{32}$

11. Express $13^{3/5}$ in radical form.

12. Express $\left(\sqrt[3]{2y}\right)^4$ in exponential form.

13. Simplify $\sqrt[5]{64x^{15}y^3}$ and leave in radical form.

14. Simplify $(16x^8)^{.75}$ and leave in exponential form.

15. $\sqrt{2x-3}+5=0$

16. Simplify $\dfrac{z}{\sqrt[3]{10z}}$

17. Simplify $\sqrt{\dfrac{27}{12p}}$

Alg II, Unit 6

Completing the square, The quadratic formula

Unit 6: Lesson 01 — Solving equations by taking the square root

There are some situations in which an equation must be solved by taking the square root of both sides.

When creating an **even** root, be sure to put a ± in front of the radical.

Solve for *x* by taking the square root.

Example 1: $x^2 - 9 = 0$

$$x^2 = 9$$
$$\sqrt{x^2} = \pm\sqrt{9}$$
$$x = \pm 3$$
$$x = \boxed{3, -3}$$

Example 2: $x^2 - 50 = 0$

$$x^2 = 50$$
$$\sqrt{x^2} = \pm\sqrt{50}$$
$$x = \pm\sqrt{25 \cdot 2}$$
$$x = \pm 5\sqrt{2}$$
$$\boxed{x = 5\sqrt{2}, -5\sqrt{2}}$$

Example 3: $7x^2 - 1400 = 0$

$$7x^2 = 1400$$
$$x^2 = 200$$
$$\sqrt{x^2} = \pm\sqrt{200} = \pm\sqrt{100 \cdot 2}$$
$$x = \boxed{\pm 10\sqrt{2}}$$

Example 4: $x^2 - 6x + 9 = 2$

$$(x-3)^2 = 2$$
$$\sqrt{(x-3)^2} = \pm\sqrt{2}$$
$$x - 3 = \pm\sqrt{2}$$
$$x = \boxed{3 \pm \sqrt{2}}$$

Assignment:

Solve by taking the square root.

1. $x^2 - 3 = 0$	2. $x^2 - 3 = 2$
3. $x^2 - 25 = 0$	4. $(5/2)x^2 - 7/5 = 0$
5. $x^2 - 8x + 16 = 0$	*6. $(x - 4)^2 - 2x = -9$

7. $(x - 2)^2/4 - 4 = 0$

**8. $x^2 - 22x = -117$

9. $(x + 2)^2 - 36 = 0$

*10. $(11x - 4/3)^2 = 17$

11. $(x - 22)^2 = 11$

12. $(x + .101)^2 = 9$

Unit 6: Lesson 02 Completing the square

Consider the following perfect squares. Notice the pattern:

$(x + 3)^2 = x^2 + 6x + 9 \quad \rightarrow (\text{half of } 6)^2 \text{ is } 9$

$(x - 4)^2 = x^2 - 8x + 16 \quad \rightarrow (\text{half of } -8)^2 \text{ is } 16$

$(x + 5)^2 = x^2 + 10x + 25 \quad \rightarrow (\text{half of } 10)^2 \text{ is } 25$

Example 1: What would we need to add to $x^2 + 8x$ in order to make it a perfect square?

$$\left(\frac{8}{2}\right)^2 = (4)^2 = \boxed{16}$$

Example 2: What would we need to add to $x^2 + 9x$ in order to make it a perfect square?

$$\left(\frac{9}{2}\right)^2 = \boxed{\frac{81}{4}}$$

Quadratics can be solved by adding the appropriate quantity to both sides so as to complete the square:

$$x^2 + 6x = 2$$
$$x^2 + 6x + 9 = 2 + 9$$
$$\boxed{(x+3)^2 = 11}$$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

So add 9 to both sides

Now take the square root of both sides and solve for *x*. Be sure to **put a ± in front** of the numerical square root.

$$\sqrt{(x+3)^2} = \pm\sqrt{11}$$
$$x + 3 = \pm\sqrt{11}$$
$$x = \boxed{-3 \pm \sqrt{11}}$$

Example 3: Solve by completing the square. $x^2 + 12x = 13$

$\left(\frac{12}{2}\right)^2 = 36$

$x^2 + 12x + 36 = 13 + 36$

$(x+6)^2 = 49$

$\sqrt{(x+6)^2} = \pm\sqrt{49}$

$x + 6 = \pm 7$

$x = -6 \pm 7$

$x = -6 + 7 = \boxed{1}$

$x = -6 - 7 = \boxed{-13}$

Example 4: Solve by completing the square. $x^2 = 11x - 2$

$x^2 - 11x = -2 \qquad \left(\frac{-11}{2}\right)^2 = \frac{121}{4}$

$x^2 - 11x + \frac{121}{4} = -2 + \frac{121}{4}$

$\left(x - \frac{11}{2}\right)^2 = \frac{-2}{1}\cdot\frac{4}{4} + \frac{121}{4}$

$\sqrt{\left(x - \frac{11}{2}\right)^2} = \pm\sqrt{\frac{113}{4}}$

$x - \frac{11}{2} = \pm\frac{\sqrt{113}}{2}$

$x = \frac{11}{2} \pm \frac{\sqrt{113}}{2}$

$x = \boxed{\frac{11 \pm \sqrt{113}}{2}}$

The "completing the square" rule of taking adding half of the linear coefficient and squaring **only works** when the **coefficient of the squared term is 1**. If that coefficient is other than one, then begin by **dividing by that coefficient**:

Example 5: Solve $2x^2 + 12x = 2$

$\frac{2x^2}{2} + \frac{12x}{2} = \frac{2}{2} \qquad \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$x^2 + 6x = 1$

$x^2 + 6x + 9 = 1 + 9$

$(x+3)^2 = 10$

$\sqrt{(x+3)^2} = \pm\sqrt{10}$

$x + 3 = \pm\sqrt{10}$

$x = \boxed{-3 \pm \sqrt{10}}$

Assignment:

What needs to be added to the following polynomials in order to make them a perfect square?

1. $x^2 + 22x$	2. $x^2 + 100x$

Solve the following equations by completing the square.

3. $x^2 + 2x = 8$	4. $x^2 - 12x = -4$
5. $x^2 + x = 3/4$	6. $x^2 - 18x + 4 = 0$

**7. $x^2 + .5x = .1$

8. $4x^2 + 16x = 4$

9. $3 - 2x - x^2 = 0$

10. $6x + 1 = x^2$

11. $.3x^2 - 30\,x = .3$

12. $8x - 2x^2 + 14 = 0$

13. $106x^2 + 106x = 106$

**14. $ax^2 + bx + c = 0$ (Your answer will be in terms of a, b, and c.)

Unit 6: Lesson 03 ***Deriving the quadratic formula**

General form of the quadratic equation:

$$ax^2 + bx + c = 0$$

a, b, and c represent numbers.
For example, in the equation

$$3x^2 - 4x + 3 = 0$$

$$a = 3 \quad b = -4 \quad c = 3$$

Solve by **completing the square**. The first step is to divide by (*a*) so as to cause the squared coefficient to be 1.

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Move the constant term (*c/a*) to the right side:

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Next, calculate half of the linear coefficient and then square:

linear coef $\rightarrow \frac{b}{a}$

$$\frac{1}{2}\,\frac{b}{a} = \frac{b}{2a} \qquad \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

Now add this quantity to both sides. This “completes the square”.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}\,\frac{4a}{4a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Recognize that the left side is a perfect square so express it as such:

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Take the square root of both sides and simplify.

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

Solve for x and simplify:

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

This result ,$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, is known as the **quadratic formula** and **is** arguably the **most important formula in algebra**.

Memorize the following:

$ax^2 + bx + c = 0$ ← general form of the quadratic equation

$$\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$ ← quadratic formula (solution to the above)

Practice: Begin with the general form of the quadratic equation and derive the quadratic formula.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a} \qquad \left(\frac{b}{a}\cdot\frac{1}{2}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}\cdot\frac{4a}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Quadratic Formula

Assignment:

1. Begin with $ax^2 + bx + c = 0$ and derive the quadratic formula. If you have trouble, glance back at your notes occasionally as you proceed through the derivation.

2. Consider the general form of the quadratic equation, $ax^2 + bx + c = 0$. What is the coefficient of the squared term?

3. Consider the general form of the quadratic equation, $ax^2 + bx + c = 0$. What is the coefficient of the linear term?

4. In the following quadratic equation, identify a, b, and c:
$3x^2 - 6x - 17 = 0$

5. In the following quadratic equation, identify a, b, and c:
$-x^2 + 2x = 5$

6. In the following quadratic equation, identify a, b, and c:
$4x - 15x^2 - 12 = 0$

*7. In the following quadratic equation, identify a, b, and c:
$-3 = 1 + 2x + 3x^2$

*8. Identify *a*, *b*, and *c* from $2x^2 - 6x - 3 = 0$ and substitute these values into the quadratic formula to produce two solutions of this equation.

Unit 6: Lesson 04 **Using the quadratic formula**

Use the quadratic formula to solve quadratic equations such as

$3x^2 - 4x - 5 = 0$

Step 1: Identify a, b, and c

Step 2: Substitute a, b, and c into the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following quadratic equations using the quadratic formula.

Example 1: $3x^2 + 6x - 2 = 0$

$3x^2 + 6x - 2 = 0$

$a = 3 \quad b = 6 \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{36 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{36 + 24}}{6}$$

$$x = \frac{-6 \pm \sqrt{60}}{6}$$

$$x = \frac{-6 \pm \sqrt{4 \cdot 15}}{6}$$

$$x = \boxed{\frac{-6 \pm 2\sqrt{15}}{6}}$$

Example 2: $7x = -1 + 2x^2$

$7x = -1 + 2x^2$

$-2x^2 + 7x + 1 = 0$

$a = -2 \quad b = 7 \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{49 - 4(-2)(1)}}{2(-2)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{-4}$$

$$x = \boxed{\frac{-7 \pm \sqrt{57}}{-4}}$$

Example 3: $x^2 + 4x - 21 = 0$

$$x^2 + 4x - 21 = 0$$

$a = 1 \quad b = 4 \quad c = -21$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(-21)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 84}}{2}$$

$$x = \frac{-4 \pm \sqrt{100}}{2}$$

$$x = \frac{-4 \pm 10}{2}$$

$$x = \frac{-4 + 10}{2} = \frac{6}{2} = \boxed{3}$$

$$x = \frac{-4 - 10}{2} = \frac{-14}{2} = \boxed{-7}$$

Example 4: $x^2 + 2x + 8 = 0$

$a = 1 \quad b = 2 \quad c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 32}}{2}$$

$$x = \frac{-2 \pm \sqrt{-28}}{2}$$

$\boxed{\text{No Solution}}$

- Can't take the square root of a negative number.

This will be possible later when we study negative numbers.

See **Enrichment Topic J** for how to solve quadratic **systems**.

Assignment:

Solve the following quadratic equation using the quadratic formula.

1. $x^2 + x - 2 = 0$	2. $5x^2 + x - 2 = 0$
3. $-x^2 + x + 1 = 0$	4. $20x^2 + 1 = 4x$

5. $x^2 + 2x + 1 = 0$

6. $-2x + 4x = x^2$

7. $.7x^2 + 2x - .5 = 0$ (Use a calculator to produce final numerical answers.)

8. $x^2 + x = 0$

9. $x^2 - 16 = 0$

*10. $fx^2 + gx + h = 0$ where f, g, and h are constants.

Unit 6: Lesson 05

Determining the nature of the roots
The discriminant

Without actually finding the roots of a quadratic equation, $ax^2 + bx + c = 0$, it is possible to categorize the nature of the roots. To do this, look at the **discriminant** which is defined as:

$$b^2 - 4ac$$

Notice this is the quantity **under the radical** in the quadratic formula. There are three distinct possibilities for the value of the discriminant:

- $b^2 - 4ac > 0$ …which gives rise to **two different real roots**. (Two roots because of the ±.) This is the "normal" case.
- $b^2 - 4ac = 0$ …which gives rise to a **double root**. It tries to give two roots, but when we add and subtract 0, we get the same thing.
- $b^2 - 4ac < 0$ …which produces **no real roots** since we can't take the square root of a negative number. (Actually, **it is** possible to take the square root of a negative number, but it produces imaginary numbers as will be studied in Unit 14 next semester.)

It is additionally possible to classify any real roots as rational or irrational. This analysis will be explored later in Lesson 12-7.

Example 1: Without solving for the roots, determine the nature of the roots of $2x^2 + 4x - 3 = 0$.

$2x^2 + 4x - 3 = 0$

$a = 2 \quad b = 4 \quad c = -3$

$b^2 - 4ac = 4^2 - 4(2)(-3)$

$= 16 + 24 = 40 > 0$

$\boxed{\text{Two different real roots}}$

Example 2: Without solving for the roots, determine the nature of the roots of $x^2 - 4\sqrt{3}\,x + 12 = 0$.

$x^2 - 4\sqrt{3}\,x + 12 = 0$

$a = 1 \quad b = -4\sqrt{3} \quad c = 12$

$b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(1)12$

$= 16 \cdot 3 - 48$

$= 48 - 48 = 0$

$\boxed{\text{A double root}}$

Example 3: Without solving for the roots, determine the nature of the roots of $x^2 + 2x + 6 = 0$.

$x^2 + 2x + 6 = 0$

$a = 1 \quad b = 2 \quad c = 6$

$b^2 - 4ac$

$= 2^2 - 4(1)6$

$= 4 - 24 = -20 < 0$

$\boxed{\text{No real roots}}$

Example 4: Determine the value(s) of *k* so that this quadratic will have two different real roots: $-x^2 - 3x + k = 0$

$-x^2 - 3x + k = 0$

$a = -1 \quad b = -3 \quad c = k$

$b^2 - 4ac > 0$

$(-3)^2 - 4(-1)k > 0$

$9 + 4k > 0$

$4k > -9$

$\boxed{k > -9/4}$

Assignment: In the following problems, determine the nature of the roots without solving for the roots.

1. $5x^2 - 6x + 5 = 0$

2. $-5z^2 - z + 3 = 0$

3. $x^2 = -4x + 5$

4. $-3p^2 + 4p = -1$

5. $2x^2 - x = 0$

6. $3x^2 - 4x + 4/3 = 0$

7. $y^2 = 6y + 7$

8. $-7x^2 + 9x - 3 = 0$

9. $0 = 3x^2 + 6$

10. $x^2 + 7/4 = \sqrt{7}x$

11. Determine the value(s) of *k* so that this quadratic will have two different real roots: $3x^2 – x + 2k = 0$

12. Determine the value(s) of *k* so that this quadratic will have a single (double) root: $-2x^2 – 5x + k = 0$

13. Determine the value(s) of *k* so that this quadratic will have no real roots: $2kx^2 – 5x + 6 = 0$

14. What is the value of the discriminant of $11x^2 - 8x + 1 = 0$?

15. What is the value of the discriminant of $-x + 9 = 2x^2$?

Unit 6: Cumulative Review

1. What is the equation of a line with x-intercept –4 and y-intercept 2?

2. Consider the line $3x - 2y = 8$. What would be the equation of this line if it is translated vertically upward by 2 units?

3. Solve by factoring: $x^2 - 81 = 0$

4. Simplify $\sqrt[5]{64a^7b^{10}}$

5. Solve the system

$4x + 5y = 2, \quad 2x - 7y = 1$

6. Solve $\sqrt{2x+1} - 4 = 0$

7. Simplify $\left(\frac{49x^2}{y^4}\right)^{\frac{1}{2}}$

8. If the slope of a line is negative, would a line perpendicular to it have a negative, positive, zero, or no slope?

9. Factor $10x^2p - 15x + 8xp - 12$. (Hint: factor by grouping)

Unit 6: Review

*1. Beginning with the general form of the quadratic equation, derive the quadratic formula.

Solve by taking the square root.

2. $(x + 6)^2 - 47 = 0$	3. $(2x - 7)^2 = 49$

Solve by completing the square.

4. $4x^2 - 24x + 1 = 0$	5. $x^2 - 9x = 2$

Solve using the quadratic formula.

6. $4x^2 - 24x + 1 = 0$	7. $x^2 - 9x = 2$
8. $x^2 - 2x + 8 = 0$	9. $11x - 2x^2 = -4$

10. Without finding the roots, find the nature of the roots of $x^2 - 3x + 11 = 0$.

11. Without finding the roots, find the nature of the roots of $x^2 - 12x + 36 = 0$.

12. Determine the value of *k* so this quadratic will have two different real roots:
$-3kx^2 + 2x - 8 = 0$

Alg II, Unit 7

Relations and Functions

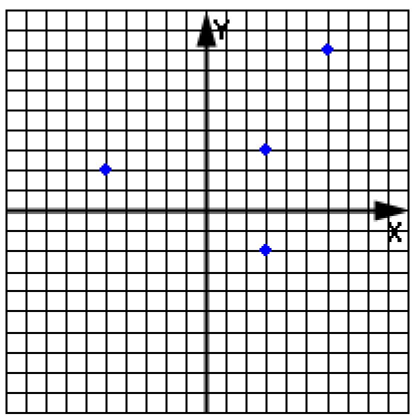
Unit 7: Lesson 01 **Representations of Relations and Functions**

Relation definition: A relation is a collection of points (officially called a set of points). There are **five ways** to show such a collection (set).

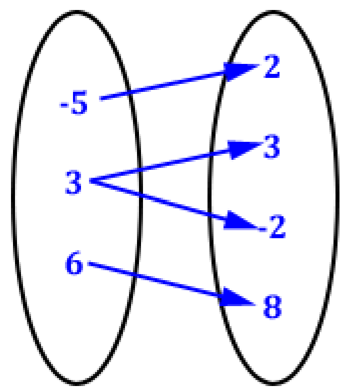
1st way: Let's begin with the most familiar:

{ (–5, 2), (3, 3), (3, –2), (6, 8) }

2nd way: Graph



3rd way: Mapping



4th way: Table

x	y
-5	2
3	3
3	-2
6	8

5th way: Formula rule (This example does not apply to the previous examples.)

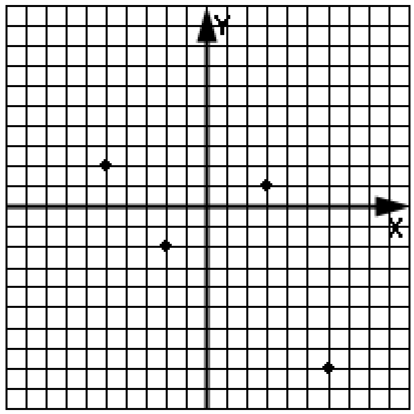
$y = 3x + 5$

Function definition: A function is a relation (a set of points) that has no two x-coordinates alike. An equivalent definition would be, "a relation that **passes the vertical line test**." The vertical line test is failed if a vertical line passes through more than one point on the graph.

Look at the table above. It is **not a function** because of the repeated 3's in the x column. Notice in the graph above that a vertical line would pass through the points (3, 3) and (3, –2); therefore, it is not a function.

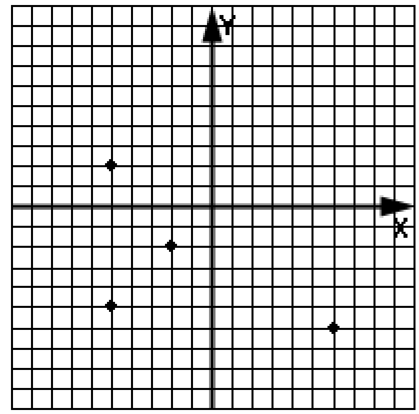
In the following examples, determine if the relation is a function or not.

Example 1:



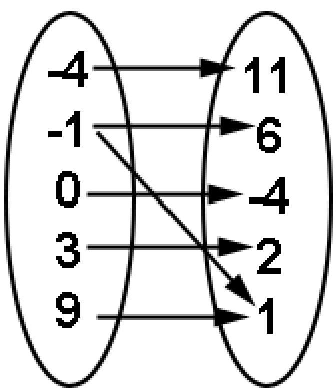
Function

Example 2:



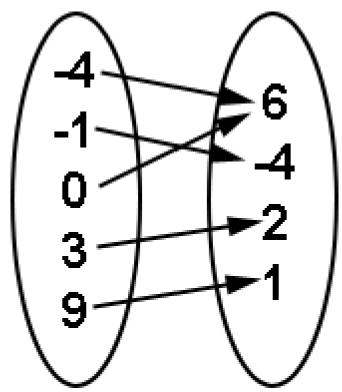
Fails vert. line test
Not a Function

Example3:



-1 is repeated
Not a function

Example 4:



Function

Example 5:

{ (11, 2), (–4, 13), (11, –1), (3, 18) }

11 is repeated.
Not a function.

Example 6:

{ (12, –2), (–4, –2), (11, –2), (16, –2) }

Function

Assignment:

1. Define a mathematical relation.

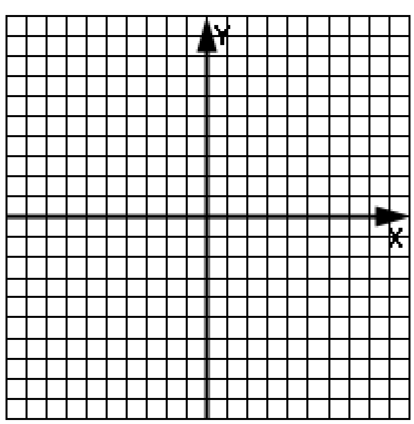
2. Define a mathematical function.

3. Draw a graph that is an example of a relation that is **not a function** because it fails the vertical line test.

4. Draw a graph that is an example of a relation that **is also a function** because it passes the vertical line test.

In the following problems, represent this relation in the form indicated:
{ (–2, 5), (5, 6), (2, –4), (–2, 4) }

5. Represent with a graph.



6. Represent with a table.

7. Represent with a mapping.

8. Is the relation above a function?

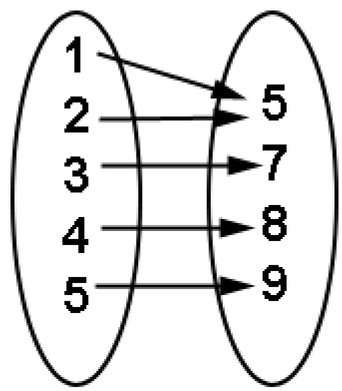
In the following problems, determine if the relation is a function or not

9. { (4, 7), (3, 3), (6, 14), (3, –11) }

10. { (1, 7), (2, 7), (3, 7), (4, 7) }

11. { (7, 1), (7, 2), (7, 3), (7, 4) }

12.



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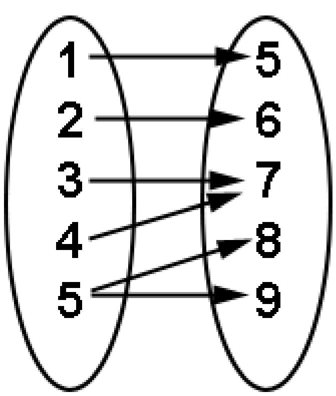

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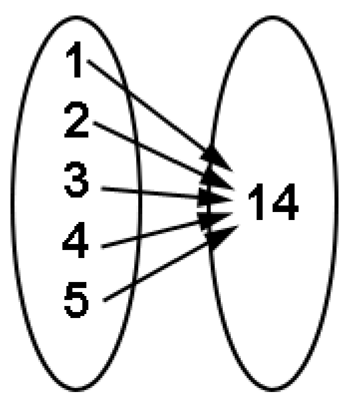

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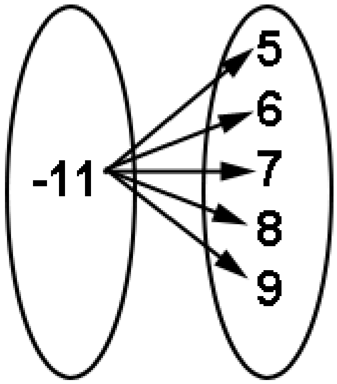

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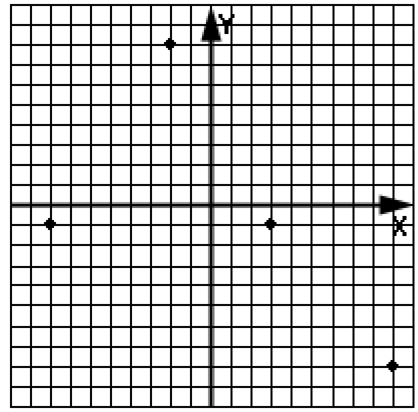

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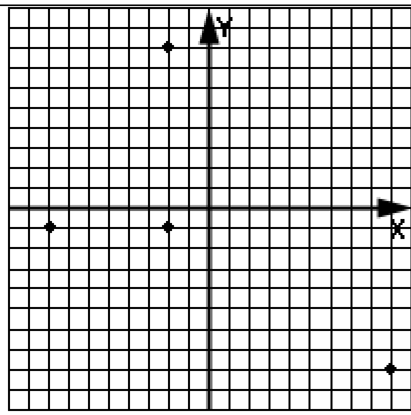

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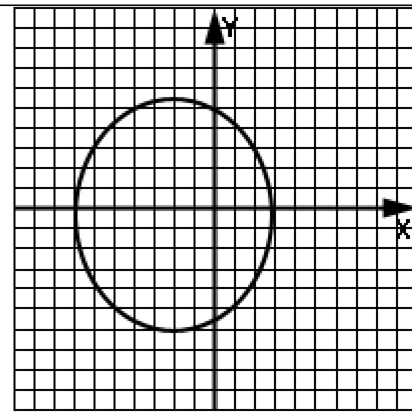

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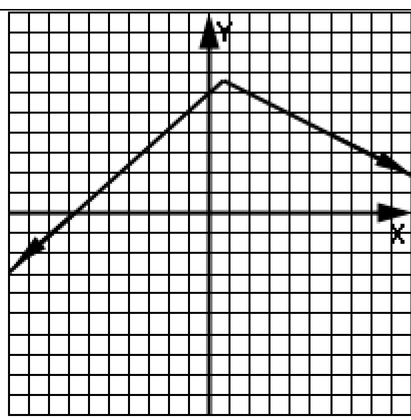

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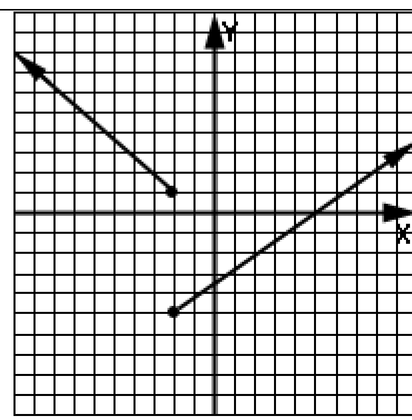
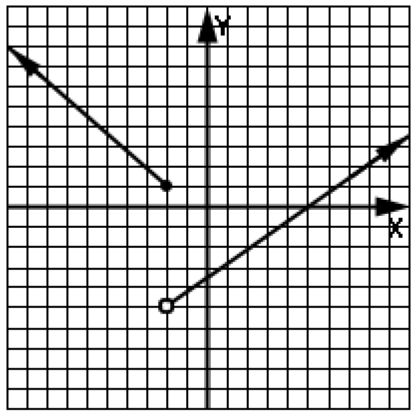


21.



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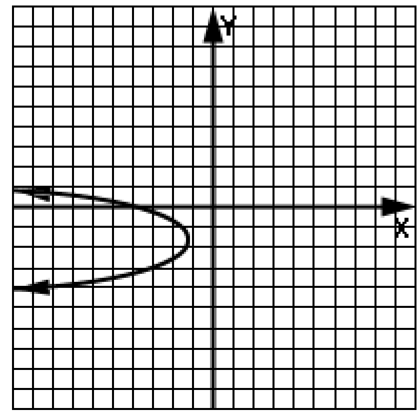


23. The line given by $y = -2x + 4$

24. The line given by $x = 6$

Unit 7: Lesson 02 Independent & dependent variables; Domain & Range

The **independent variable** (typically x) for a relation corresponds to the "input values."

The **dependent variable** (typically y) for a relation corresponds to the "output values."

Example 1:
Consider the relation (and also a function) $y = x^2$
Generate a few of the points on this relation with a table as follows:

Indep. Choose any value

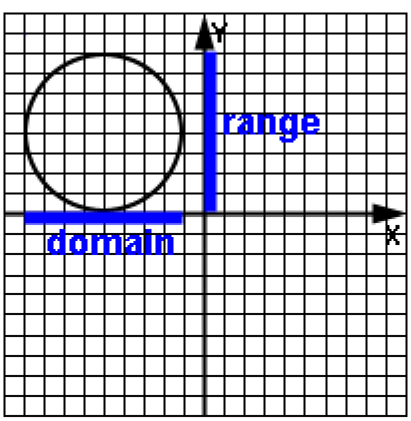
x	y
1	1
2	4
3	9

Dep. What we get here depends on x.

The **domain** of a relation is the permissible values of the independent variable (typically x).

The **range** of a relation is the values of the dependent variable (typically y) that result from using the domain values of the independent variables, *x*.

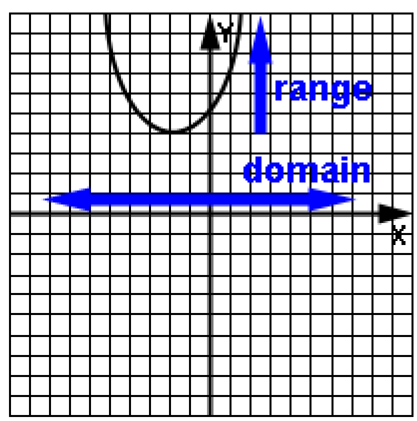
Example 2: Find the domain and range. Is it a function?



D: $-9 \le x \le -1$
R: $y \ge 0$

Not a Function

Example 3: Find the domain and range. Is it a function?



D: All real x
R: $y \ge 4$

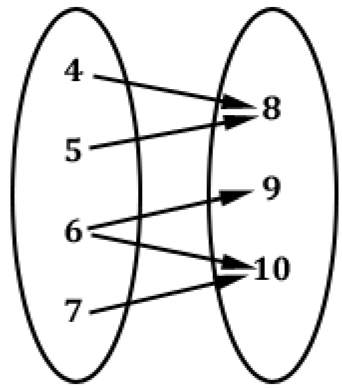
Function

Example 4: Find the domain and range. Is it a function?

{ (3, 7), (6, -2), (11, 9), (6, –7) }

D: {3, 6, 11}
R: {7, -2, 9, -7}
Not a function
(x = 6 is repeated)

Example 5: Find the domain and range. Is it a function?



D: {4, 5, 6, 7}
R: {8, 9, 10}
Not a Function
(6 is repeated)

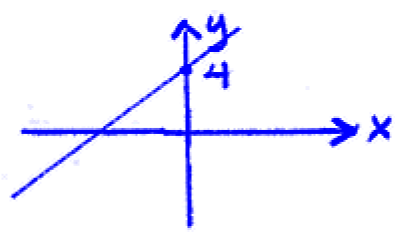
Example 6: Find the domain and range. Is it a function?

x	y
5	22
6	–2
5	17.5
2	0

D: {5, 6, 2}
R: {22, -2, 17.5, 0}
Not a function
(5 is repeated)

Example 7: Find the domain and range. Is it a function?

y = 3x + 4



D: All real x
R: All real y
Function

It is possible to restrict the domain of a function on a graphing calculator so that **only a portion** of the graph displays (that permitted by the domain).

See **Calculator Appendix F** for details on how to do this.

Assignment:

1. In the relation $z = 3w^2 - 4w + 6$ which is the independent and which is the dependent variable?

2. The dependent variable is normally graphed as the horizontal or vertical axis?

3. What is meant by the domain of a relation?

4. What is meant by the range of a relation?

5. If a relation has a domain given by $-3 \le x < 4$ and a range given by $y > -1$, which of the following points could **not** possibly be a point on the relation?

$(-3, 15)$, $(4, 12)$, $(0, 0)$, $(-3, -4)$, $(3.5, -1)$

6. Which is considered to be the **input** of a function or relation, the independent or the dependent variable?

7. What is the domain and range of the function $y = 2x - 8$? Is it a function?

8. What is the domain and range of the vertical line given by $x = -7$? Is it a function?

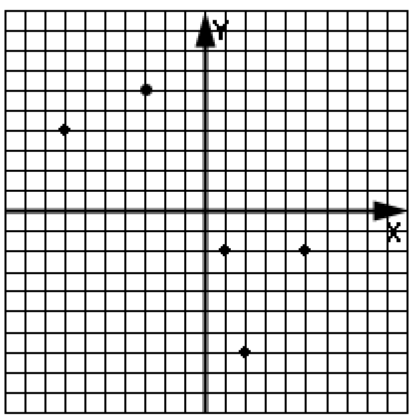
9. What is the domain and range of the relation given by this table? Is it a function?

x	y
2	–1
2	0
–4	8
6	–19
3	4

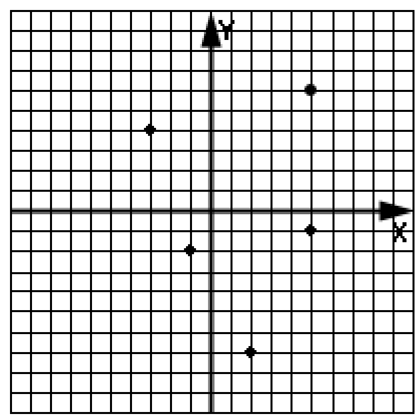
10. What is the domain and range of the relation given by this table? Is it a function?

x	y
–1	3
0	3
–11	5
19	5
105	15

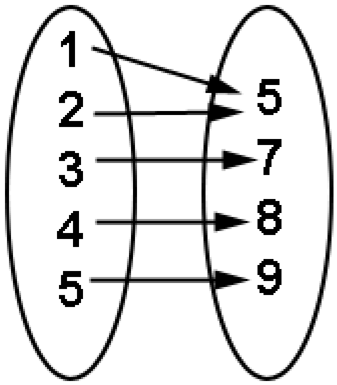
11. What is the domain and range of this relation? Is it a function?



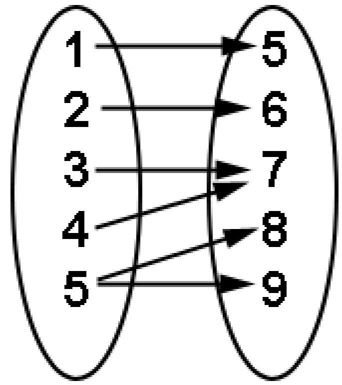
12. What is the domain and range of this relation? Is it a function?



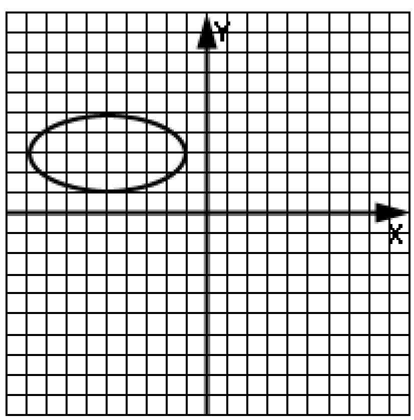
13. What is the domain and range of this relation? Is it a function?



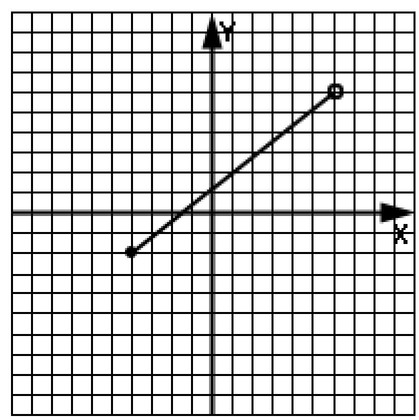
14. What is the domain and range of this relation? Is it a function?



15. What is the domain and range of this relation? Is it a function?



16. What is the domain and range of this relation? Is it a function?



17. What is the domain and range of y = 3x? Is it a function?

18. What is the domain and range of y = –x – 1? Is it a function?

Unit 7: Lesson 03 Function notation; Evaluating functions

Consider the function f = 3x + 2.
This is more conventionally written as f(x) = 3x + 2.

f(x) does **not** mean *f* times *x*. In this context we read it ***f* of *x***.

f(x) = 3x + 2 could be rewritten in a more familiar form as y = 3x + 2. Since *y* is the dependent variable and x is the independent variable, we conclude the following about the f(x) version:

f is the dependent variable
x is the independent variable

To evaluate the function $f(x) = 5x^2 + 2x - 7$ at a specific *x* value (for example, 3), we use the following notation:

$$f(3) = 5(3)^2 + 2(3) - 7 = 45 + 6 - 7 = \boxed{44}$$

Relations and functions do not have to be called *f*. Consider the following example using g(x) = 4x – 2 evaluated at x = –2:

$$g(-2) = 4(-2) - 2 = -8 - 2 = \boxed{-10}$$

Example 1: Find g(–4) where
$g(x) = 2x^2 - 5$

$$g(-4) = 2(-4)^2 - 5$$
$$= 2 \cdot 16 - 5$$
$$= 32 - 5 = \boxed{27}$$

Example 2: Find h(z+2) where
h(x) = (2x –1)/3

$$h(z+2) = [2(z+2) - 1]/3$$
$$h(z+2) = [2z + 4 - 1]/3$$
$$= \boxed{(2z + 3)/3}$$

In the following examples, use f(x) = $\sqrt{\dfrac{x^{-3}}{4}}$ and g(x) = 2x + 1

***Example 3:** Find the composite function f(g(x)).

$$f(g(x)) = \sqrt{\frac{(g(x))^{-3}}{4}}$$
$$= \sqrt{\frac{(2x+1)^{-3}}{4}}$$
$$= \frac{\sqrt{(2x+1)^{-3}}}{2}$$
$$= \frac{1}{2}\sqrt{\frac{1}{(2x+1)^3}\,\frac{2x+1}{2x+1}}$$
$$= \frac{1}{2}\sqrt{\frac{2x+1}{(2x+1)^4}}$$
$$= \boxed{\frac{1}{2}\,\frac{\sqrt{2x+1}}{(2x+1)^2}}$$

***Example 4:** Find the composite function g(f(x)).

$$g(f(x)) = 2\sqrt{\frac{x^{-3}}{4}} + 1$$
$$= \cancel{\frac{2}{2}}\sqrt{x^{-3}} + 1$$
$$= \sqrt{\frac{1}{x^3}\,\frac{x}{x}} + 1$$
$$= \sqrt{\frac{x}{x^4}} + 1$$
$$= \boxed{\frac{\sqrt{x}}{x^2} + 1}$$

Assignment:

1. In the function $h(x) = 4x^3 - 2x^2 - x + 2$ what is the independent variable and what is the dependent variable?

2. In the function $g(y) = 4y^3 - 2y^2 - y + 2$ what is the independent variable and what is the dependent variable?

3. If $f(-7)$ does **not** mean f times -7, what **does** it mean?

4. Write out in words how we would say $f(g(x))$.

In the following problems, use $f(x) = x^2 - x - 1$ and $g(x) = (x - 5)/x$

5. Evaluate $f(-3)$

6. Evaluate $f(5)$

7. Evaluate g(2)

8. Evaluate g(–1)

9. Evaluate f(0) + g(2)

10. Evaluate g(3) – g(–2)

*11. Simplify f(g(x))

*12. Simplify g(f(x))

13. $f(-2)/2$

14. $g(x/2)$

15. $f(w)$

*16. $g(\ g(x)\ )$

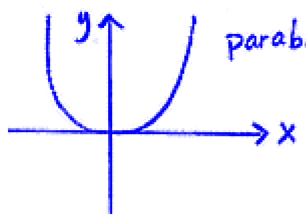
17. $f(x-1)$

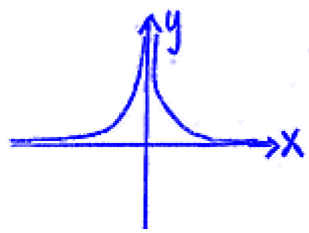
*18. $g(x)/f(x)$

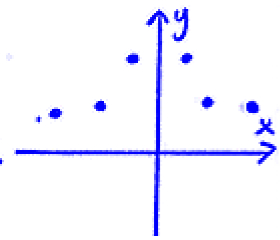
Unit 7: Lesson 04 *Even and odd functions

An **even function** is one that has symmetry ("balance") with respect to (w.r.t.) the vertical axis:

Examples:

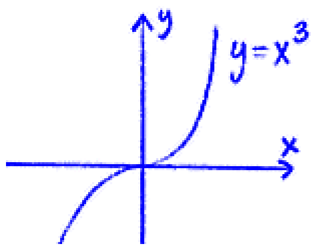


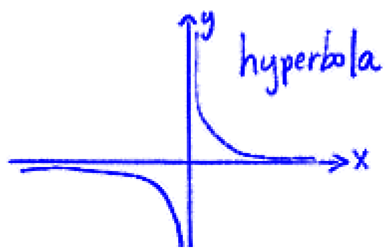


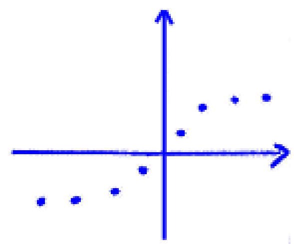


An **odd function** is one that has symmetry ("balance") w.r.t. the origin:

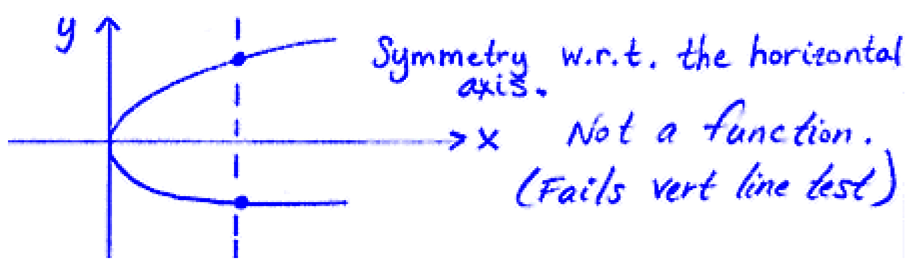
Examples:







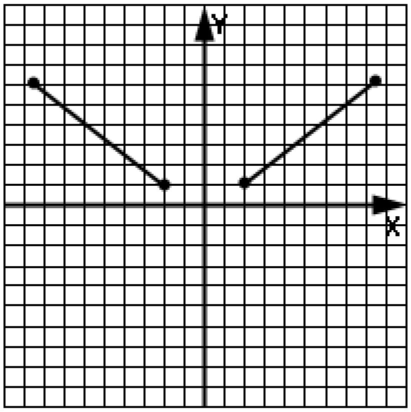
Since even functions have symmetry w.r.t. the vertical axis, it is a **common mistake** to think of odd functions as being defined as having symmetry w.r.t. the horizontal axis; however, a relation that has such symmetry is **not a function** (fails the vertical line test).



Another common mistake is to think that all functions must be either even or odd. **Functions can be neither** even nor odd.

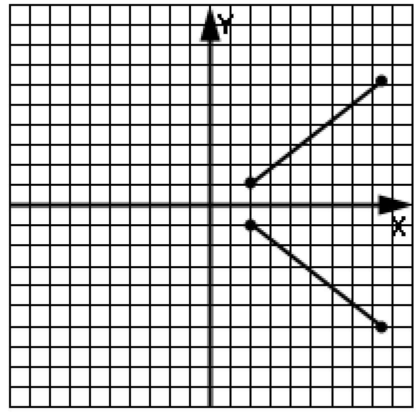
In the following examples, identify which functions are odd, even, or neither:

Example 1:



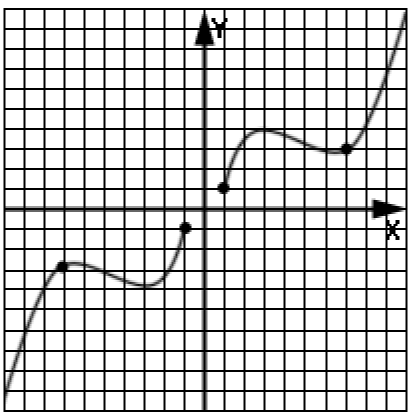
even

Example 2:



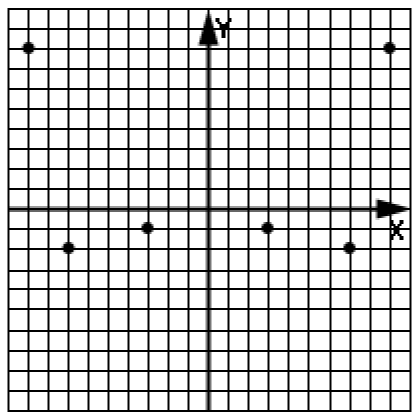
Neither,
not a function.

Example 3:



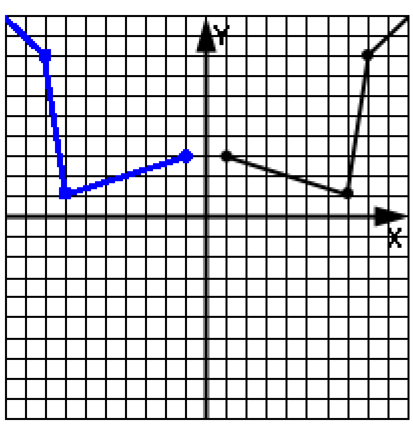
Odd

Example 4:

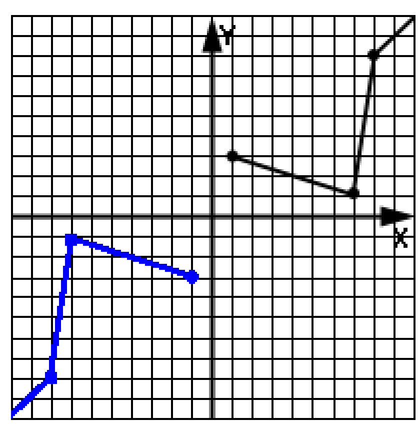


even

Example 5: Complete the drawing so as to create an even function.



Example 6: Complete the drawing so as to create an odd function.



Besides the graphical tests for even and odd functions discussed above, there are algebraic tests:

Even test: f(x) = f(–x) **Odd test: f(x) = –f(–x)**

Example 7: Show that $f(x) = 5x^4 + x^2 - 1$ is an even function.

even test

$f(x) = f(-x)$

$5x^4 + x^2 - 1 = 5(-x)^4 + (-x)^2 - 1$

$5x^4 + x^2 - 1 \overset{\checkmark}{=} 5x^4 + x^2 - 1$

it's even

Example 8: Show that $f(x) = 2x^3 - x$ is an odd function.

odd test $f(x) = -f(-x)$

$2x^3 - x = -[2(-x)^3 - (-x)]$

$= -[-2x^3 + x]$

$2x^3 - x \overset{\checkmark}{=} 2x^3 - x$

it's odd

It is often possible, but not always (be careful), to look at the exponents of a polynomial and tell if it's even or odd.

For example, $f(x) = 3x^6 - x^4$ is **even**.

… notice the exponents are **all even**.

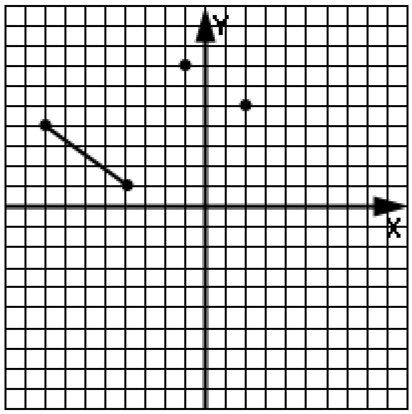
For example, $f(x) = x^5 - 4x^3$ is **odd**.

… notice the exponents are **all odd**.

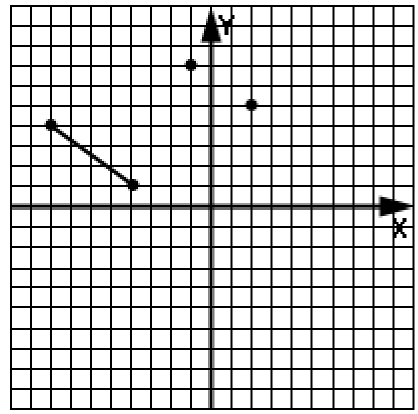
See **Calculator Appendix G** and a related video for more discussion regarding even and odd functions.

Assignment:

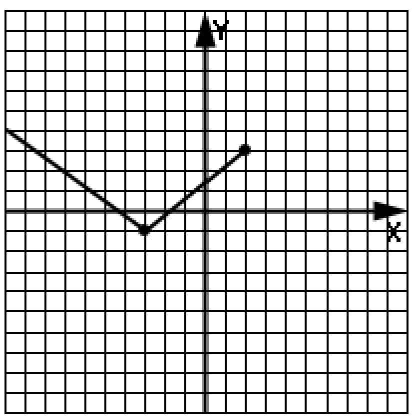
1. Complete the drawing so as to create an odd function.



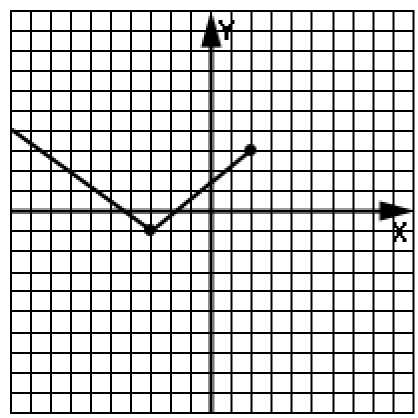
2. Complete the drawing so as to create an even function.



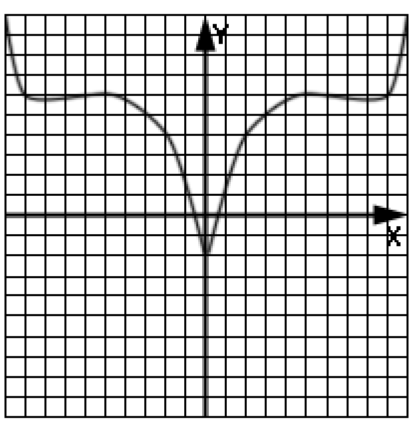
3. Complete the drawing so as to create an odd function.



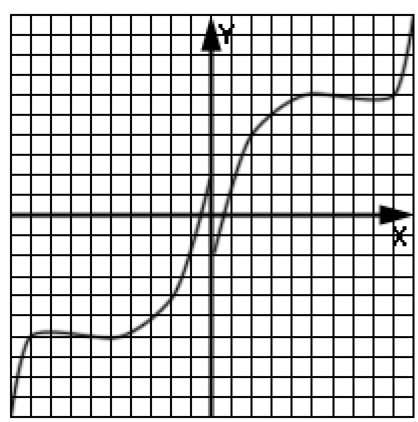
4. Complete the drawing so as to create an even function.



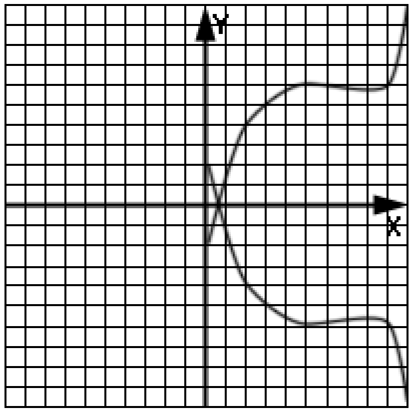
5. Is this an even or an odd function (or neither)?



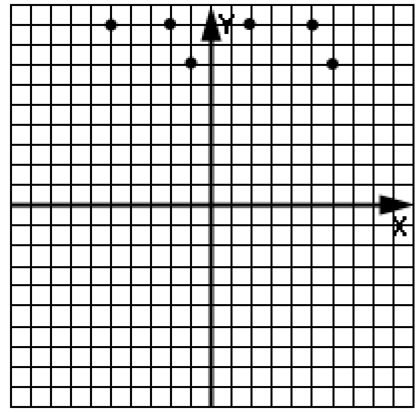
6. Is this an even or an odd function (or neither)?



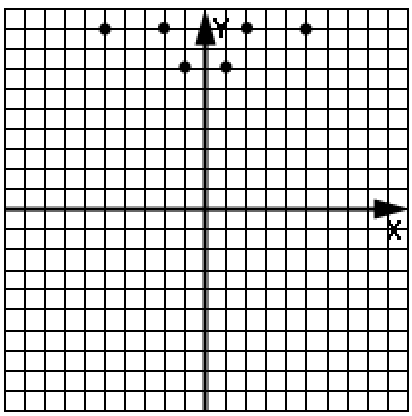
7. Is this an even or an odd function (or neither)?



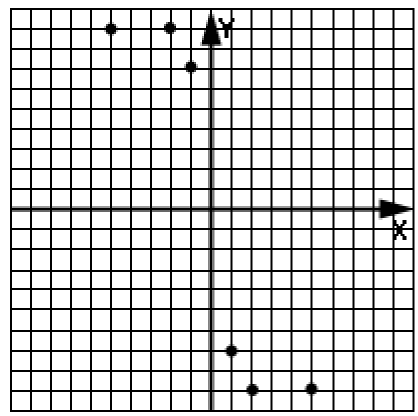
8. Is this an even or an odd function (or neither)?



9. Is this an even or an odd function (or neither)?



10. Is this an even or an odd function (or neither)?



11. Algebraically test $f(x) = x^5 - 2x$ to see if it is even, odd, or neither.

12. Algebraically test $f(x) = x^4 - 4$ to see if it is even, odd, or neither.

13. Algebraically test $f(x) = -6$ to see if it is even, odd or neither.

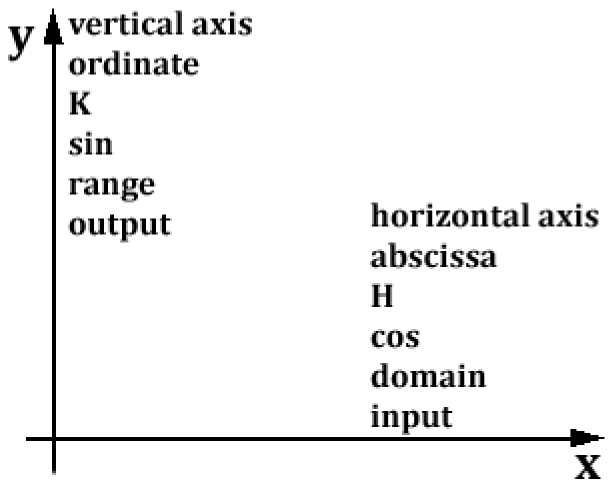
14. Algebraically test $x = 3$ to see if it is even, odd or neither.

15. Algebraically test $f(x) = x^2 - x$ to see if it is even, odd or neither.

16. Algebraically test $f(x) = 1/x^3 - 2$ to see if it is even, odd, or neither.

Unit 7: Lesson 05 **Putting it all together: x & y-axis associations**

The association of the x-axis with domain & independent variables and the association of the y-axis with range & dependent variables can be confusing. The bad news is that there are a number of other things to associate with these axes. These are summarized below:



Frankly, this doesn’t help much. What we need is some other way to memorize these associations. Consider the following chart:

x	**y**
horizontal	vertical
abscissa	ordinate
H	K
cos	sin
domain	range
input	output
independent	dependent

Several of these items have not been studied yet; however, most will come up at some point in this course. Notice that each side-by-side pair in this table **is in alphabetical order** (except for the last row) which makes them all easy to associate with *x* & *y*.

Unfortunately this alphabetical-order miracle fails with the words "independent" and "dependent." They are in **reverse alphabetical-order** when correctly placed in the table above. That's the only exception.

Assignment:

Even though you may not know the meaning of some of the following terms, it is possible to determine the answer from the alphabetical chart. Try not to look at the chart. Rather, just think of the alphabetical sequence.

1. The trig function sine (abbreviated sin) is the projection of a rotating unit vector on which axis?	2. The trig function cosine (abbreviated cos) is the projection of a rotating unit vector on which axis?
3. The x-axis is generically referred to as the abscissa or ordinate?	4. The y-axis is generically referred to as the abscissa or ordinate?
5. Relations are translated left/right by replacing *x* with x – h or x – k?	6. Relations are translated up/down by replacing *y* with y – h or y – k?
7. The domain is the permissible values for which axis: *x* or *y*?	8. The range is the permissible values for which axis: *x* or *y*?
9. For a function f(x) are the *x* values the input or output?	10. For a function f(x) are the *f* values the input or output?

11. Is the domain the permissible values for the abscissa or ordinate?	12. Is the range the permissible values for the abscissa or ordinate?
13. The x-axis is typically the vertical or horizontal axis?	14. The y-axis is typically the vertical or horizontal axis?
15. The domain is associated with *x* or *y*?	16. The range is associated with *x* or *y*?
17. The range of a relation or function is associated with the dependent or the independent variables?	18. The domain of a relation or function is associated with the dependent or the independent variables?
19. Another name for the horizontal axis is the abscissa or ordinate?	20. Another name for the vertical axis is the abscissa or ordinate?

Do the cumulative review as an additional part of the homework.

Unit 7: Cumulative Review

1. Solve $2x^2 - 12x - 9 = 0$ by completing the square.

2. Solve $x^2 + x - 56 = 0$ using the quadratic formula.

3. What would be the equation of $4x + 3y - 11 = 0$ when translated up 2 units?

4. Solve $\sqrt{2x-2} - 1 = 5$.

5. Simplify $\left(\frac{7x^{-3}}{y^5z^{-2}}\right)^{-2} x^4$ leaving only positive exponents in the answer.

6. Simplify $\sqrt[3]{4x^4}$ and express in radical form.

7. Simplify and express $\frac{\sqrt[3]{54x^4}}{x^7}$ in exponential form.

8. Solve $6^{3x-2}6^{x^2} = 36$.

9. Solve $x^{2/5} + 1 = 10$

10. Expand $(2y^3 - 6xc)^2$ and simplify.

11. Factor $4x^2 - 36$

12. Graph $-3y + 12x \le 7$

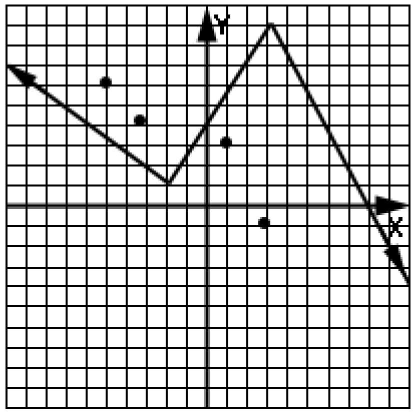
13. What is the slope of a line perpendicular to $3x - 5y = 17$?

14. A rectangle of area $28m^2$ has a width 3m shorter than its length. What are the dimensions of the rectangle?

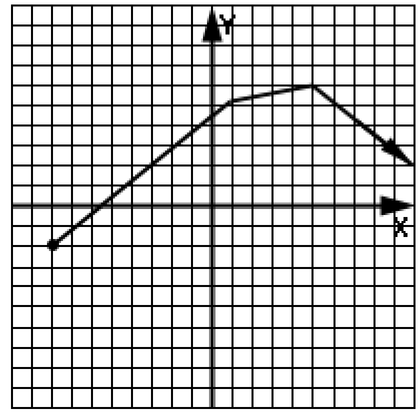
Unit 7: Review

In the following problems, identify the domain and range. Then tell whether it is a function or not.

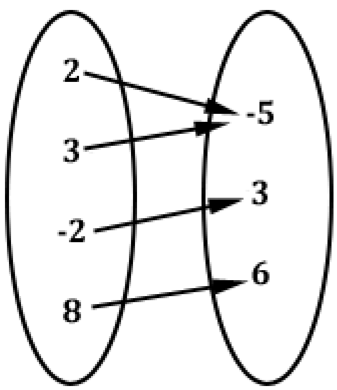
1.



2.



3.



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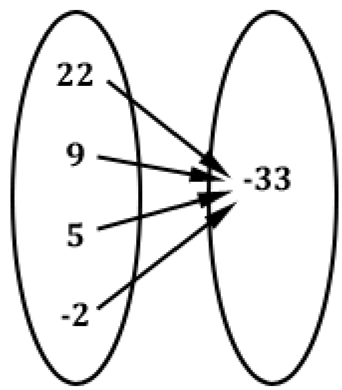


5.

x	y
–5	–5
–5	4
–5	5
–5	6
–5	10

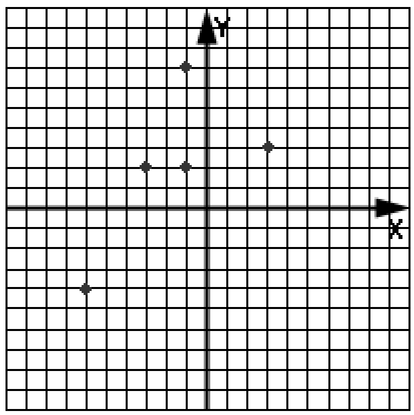
6.

x	y
2	22
101	–.5
–5	3
16	13
8	8

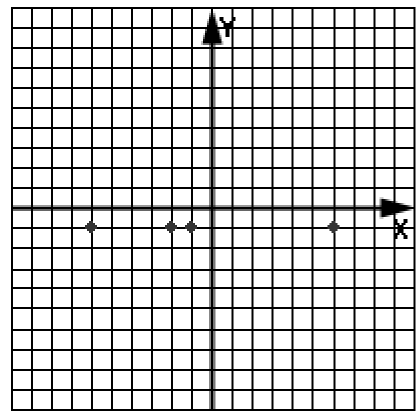
7. { (1, 4), (3, –5), (2, 6), (1, 1), (9, .3) }

8. { (11, 11), (2, 2), (0, 0), (–1, –1), (–2, –2)}

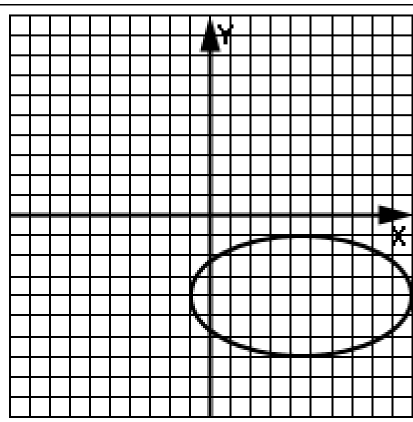
9.



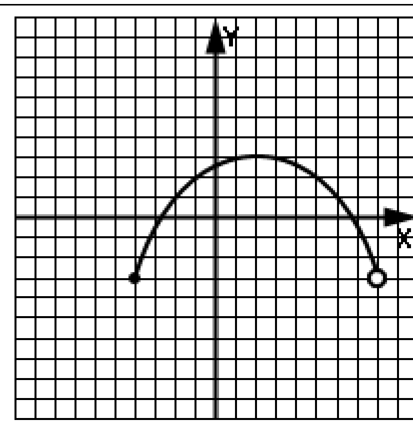
10.



11.



12.



13. $y = -4x + 3$

14. $y = 5$

In the following problems, use $f(x) = 3x^2 - 1$ and $g(x) = (x + 4)/2$

15. Evaluate $f(1)$

16. Evaluate $g(-2)$

17. Evaluate $f(2) - g(1)$

*18. Simplify $f(\, g(x) \,)$

19. Identify the dependent and independent variables in $y = -7x + 5$

20. Identify the dependent and independent variables in
$g(y) = 8y^3 - 2y - 1$

Alg II, Unit 8

Analyzing and Graphing Quadratic Functions

Unit 8: Lesson 01 Forms of quadratic functions

Quadratic functions form parabolas when graphed. A quadratic function is always recognizable because there will be only one *y* term and its exponent will be 1. The other "side" of the equation will be a polynomial with variable *x* of degree 2 (highest power is 2).

Example 1: Which of the following are quadratic functions that form parabolas?

(a) $y = 3x^2 – 2x + 6$ (b) $3y - 6x + 2x^2 = 5$ (c) $y^2 = 5x – x^2 + 1$

(d) $y = \sqrt{x^2 - 6x - 79}$ (e) $y = 1/x^2 + 2$ (f) $y = \cos(x^2 + x – 9)$

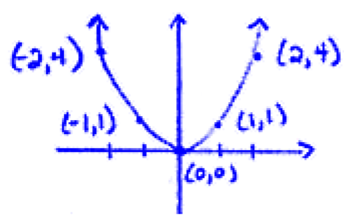
a, b

The simplest parabola is the parent function:

$y = x^2$

Make an x-y chart using *x* as the independent variable. Then graph the points to form the points of this parent function parabola:

x	y
0	0
1	1
2	4
-1	1
-2	4



General form: $y = f(x) = ax^2 + bx + c$

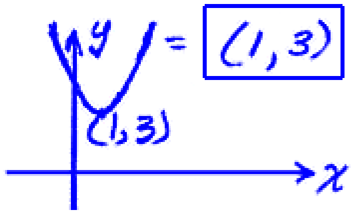
Vertex → $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

If *a* is positive the parabola goes up: $\cup$

If *a* is negative the parabola goes down: $\cap$

Example 2: Find the vertex of $f(x) = 2x^2 - 4x + 5$ and then make a rough sketch of the graph of the parabola.

Vertex $\rightarrow \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $\quad \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$

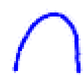


$= \boxed{(1, 3)}$

$f(1) = 2(1)^2 - 4(1) + 5$
$= 2 - 4 + 5 = 3$

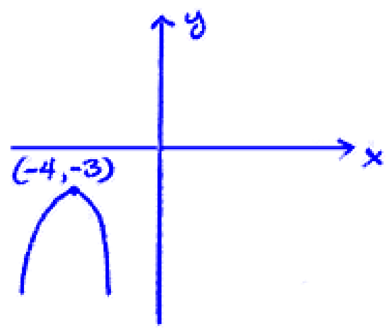
Vertex form: $y = f(x) = a(x - h)^2 + k$

Vertex → (h, k)
If *a* is positive the parabola goes up: 

If *a* is negative the parabola goes down: 

Example 3: Find the vertex of $y = -3(x + 4)^2 - 3$ and then make a rough sketch of the graph of the parabola.

$y = -3(x+4)^2 - 3$
$= -3(x - (-4))^2 - 3$
$= a(x - h)^2 + k$
$h = -4$
$k = -3$
Vertex $\rightarrow \boxed{(-4, -3)}$



Root form: $y = f(x) = a(x - r_1)(x - r_2)$

r_1 and r_2 are the roots.
If *a* is positive the parabola goes up: 

If *a* is negative the parabola goes down: 

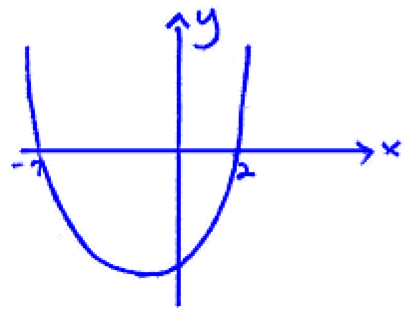
Example 4: Identify the roots for y = f(x) = 4(x –2)(x + 7) and then make a rough sketch of the graph of the parabola.

$y = 4(x-2)(x+7)$

$y = 4(x-2)(x-(-7))$

$y = a(x-r_1)(x-r_2)$

$r_1 = \boxed{2}$ $r_2 = \boxed{-7}$



Assignment:

Identify the vertex and make a rough sketch of the graphs of the parabolas.

1. $y = -3x^2 - 2x + 1$

2. $f(x) = 5x^2 - 6x + 2$

3. $f(x) = 4(x + 11)^2 - 9$

4. $y = -2(x - 7)^2 + 1$

Identify the roots and make a rough sketch of the graphs of the parabolas.

5. $f(x) = -4(x - 2)(x + 1)$	6. $y = (x + 12)(x - 6)$

In the following problems, identify the functions that are parabolas. It is possible to have more than one answer on each problem.

7. (a) $f(x) = 32x - 16x^2 + 1$ (b) $y = 6x$ (c) $y = x^2 + x + x^3$ (d) $g(x) = 1/(x^2 + 1)$ (e) $h(x) = 1/(x^{-2})$	8. (a) $f(x) = \sqrt[3]{x^2} + 22$ (b) $y = 1/(x^2 + 2x + 3)$ (c) $k(x) = x^2 + 1/x^{-1} + 10$ (d) $y = x^2 + 1/x + 10$ (e) $y = 2^x + x + 1$
9. Write the general form of a quadratic function.	10. Write the vertex form of a quadratic function having vertex (h, k).
11. Write the root form of quadratic function having roots r_1 and r_2.	12. Write the simplest equation for a parabola (it's called the quadratic parent function).

Unit 8: Lesson 02

Finding intercepts and graphing quadratic equations

Locating the x-intercepts (roots) and the y-intercepts helps establish the position of the parabola.

To find the x-intercepts, set y = 0 and solve for x. There will be two solutions.

To find the y-intercept, set x = 0 and solve for y. There will be one solution.

Example 1: Find the intercepts of $y = f(x) = 2(x - 4)^2 - 4$ and use them to graph the parabola.

Find x-intc

$0 = 2(x-4)^2 - 4$

$4 = 2(x-4)^2$

$\frac{4}{2} = (x-4)^2$

$\pm\sqrt{2} = \sqrt{(x-4)^2}$

$\pm\sqrt{2} = x - 4$

$4 \pm \sqrt{2} = x$

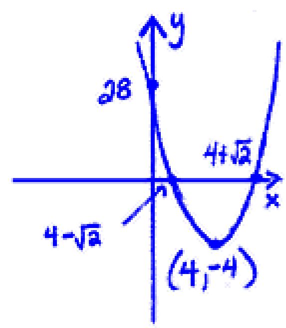
y-intc

$y = 2(0-4)^2 - 4$

$y = 2(16) - 4$

$y = 32 - 4 = 28$

$(h, k) = (4, -4)$



When given the root form of parabolic function, it is necessary to find the vertex in order to graph the function.

The x value of the vertex is the average of the two roots.

$$h = \frac{r_1 + r_2}{2}$$

To find the y value of the vertex, substitute in the x value and then solve for y. **k = f(h)**

Example 2: Find the intercepts and vertex of $y = f(x) = -4(x - 2)(x + 6)$ and use them to graph the parabola.

$r_1 = 2 \quad r_2 = -6$

$h = \frac{r_1 + r_2}{2} = \frac{2-6}{2} = \frac{-4}{2} = -2$

$k = f(h) = f(-2) = -4(-2-2)(-2+6)$

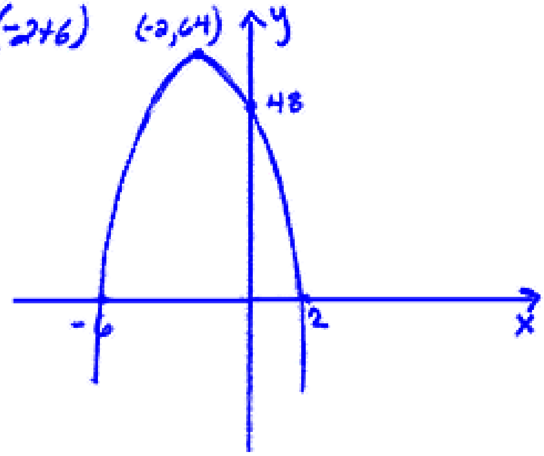
$= -4(-4)(4) = 64$

y int

$y = -4(0-2)(0+6)$

$y = -4(-2)(6)$

$y = 48$



Assignment:

In the following problems, sketch the parabola and label all intercepts and the vertex:

1. $f(x) = -(x-4)^2 + 9$

---

2. $y = (x+1)^2 + 2$

---

3. $f(x) = 5(x-4)(x-6)$

4. $.5y = (x + 2)(x + 8)$

5. $y + (x + 3)^2 = 4$

6. $f(x) = 2 - 3(x + 7)(x + 7)$

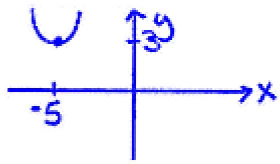
7. $y = -(x - 5)(x + 15)$

8. $f(x) = (4 + x)(-6 + x)$

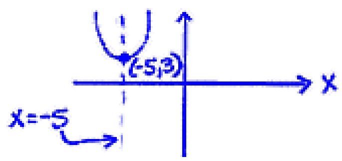
9. $y = -(x - 14)^2$

Unit 8: Lesson 03 ***Analysis of quadratic functions**

Consider a parabola with vertex (h, k) = (–5, 3):



Draw the axis of symmetry. It's a vertical line drawn through the vertex whose equation is x = h (x = –5 in this case).



Example 1: For the quadratic function $y = f(x) = x^2 + 6x + 8$ find the following information, then graph and label the parabola:

Vertex: **(–3, –1)**	Axis of Sym: **x = –3**	**Min** or Max: **–1**	Domain: **ARX**
Range: **y ≥ –1**	Y-intercept: **8**	Roots: **–2, –4**	

$a = 1 \quad b = 6 \quad c = 8$

$h = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$

$k = f(-3) = (-3)^2 + 6(-3) + 8$

$= 9 - 18 + 8 = -9 + 8$

$= -1$

Roots:

$x^2 + 6x + 8 = 0$

$(x+2)(x+4) = 0$

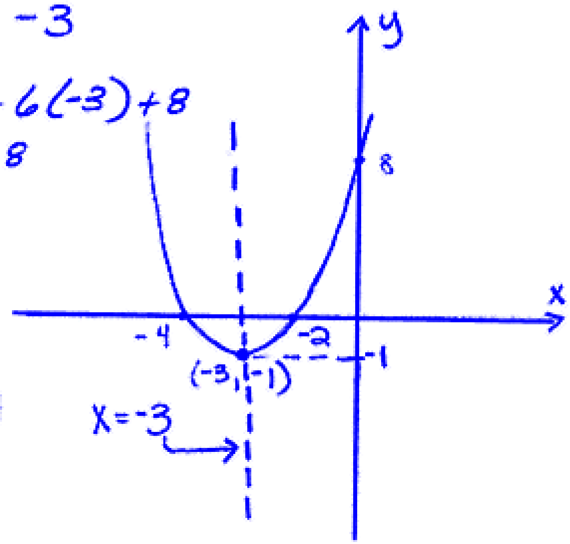
$x + 2 = 0 \qquad x + 4 = 0$

$x = -2 \qquad x = -4$

y intc:

$y = 0^2 + 6(0) + 8$

$y = 8$



Example 2: For the quadratic function $y = f(x) = -2(x - 4)^2 + 8$ find the following information, then graph and label the parabola:

Vertex: **(4, 8)**	Axis of Sym: **x = 4**	Min or **Max**: **8**	Domain: **ARX**
Range: **y ≤ 8**	Y-intercept: **–24**	Roots: **2, 6**	

$(h, k) = (4, 8)$

Roots:

$0 = -2(x-4)^2 + 8$

$-8 = -2(x-4)^2$

$4 = (x-4)^2$

$\sqrt{(x-4)^2} = \pm\sqrt{4}$

$x - 4 = \pm 2$

$x = 4 \pm 2$

$x = 4 + 2 = 6$

$x = 4 - 2 = 2$

y intc:

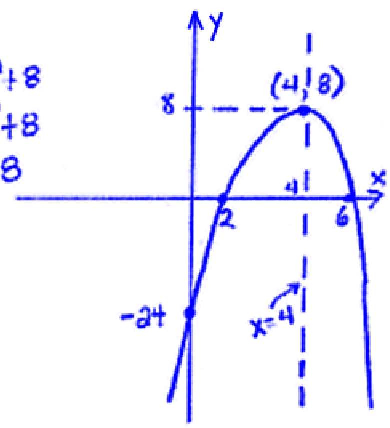
$y = -2(0-4)^2 + 8$

$y = -2(-4)^2 + 8$

$y = -2(16) + 8$

$y = -32 + 8$

$y = -24$



Example 3: For the quadratic function $y = f(x) = -2(x + 6)(x - 2)$ find the following information, then graph and label the parabola:

Vertex: **(–2, 32)**	Axis of Sym: **x = –2**	Min or **Max**: **32**	Domain: **ARX**
Range: **y ≤ 32**	Y-intercept: **24**	Roots: **–6, 2**	

$r_1 = -6 \quad r_2 = 2$

$h = \frac{-6+2}{2} = \frac{-4}{2} = -2$

$K = f(h) = f(-2) = -2(-2+6)(-2-2)$

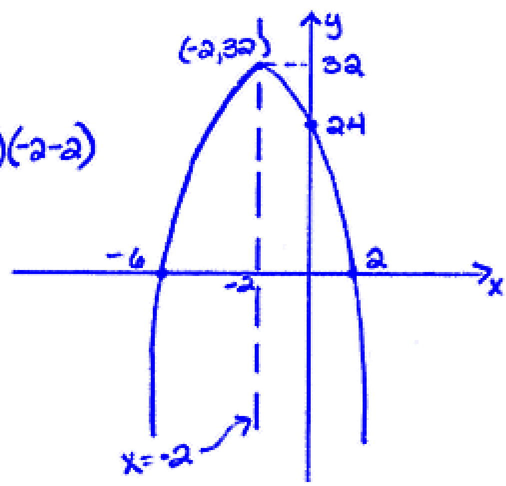
$= -2(4)(-4) = 32$

y intc:

$y = -2(0+6)(0-2)$

$y = -2(6)(-2)$

$y = 24$



Assignment:

1. For the quadratic function y = f(x) = (x + 5)(x + 11) find the following information, then graph and label the parabola:

Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

---

2. For the quadratic function y = f(x) = $3(x – 5)^2$ find the following information, then graph and label the parabola:

Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

3. For the quadratic function $y = f(x) = x^2 - 3x + 5$ find the following information, then graph and label the parabola:

Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

---

*4. For the quadratic function $y = f(x) = -2x(12 - x)$ find the following information, then graph and label the parabola:

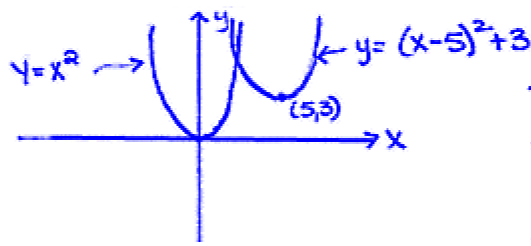
Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

Unit 8: Lesson 04 Using graphs to analyze quadratic transformations

By simultaneously graphing a parent quadratic function and then another transformed quadratic we can visually observe:

Shifting left or right (a **translation**)
Shifting up or down (a **translation**)

Example 1: Use a graphing calculator to simultaneously graph $y = x^2$ (the parent function) and $y = (x - 5)^2 + 3$. Sketch the calculator graph-display. How has the second function been transformed? (See **Calculator Appendix A** for details.)

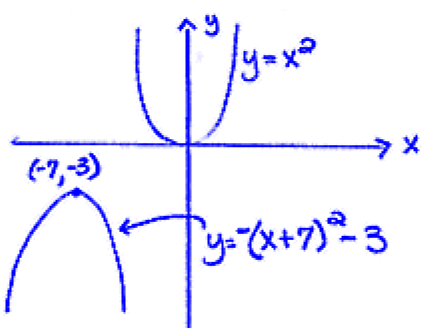


Translated (shifted) right 5 units, up 3 units

By simultaneously graphing a parent quadratic function and then another transformed quadratic we can visually observe:

Upside down (**reflection** across the x-axis)

Example 2: Use a graphing calculator to simultaneously graph $y = x^2$ (the parent function) and $y = -(x + 7)^2 - 3$. Sketch the calculator graph-display. How has the second function been transformed?



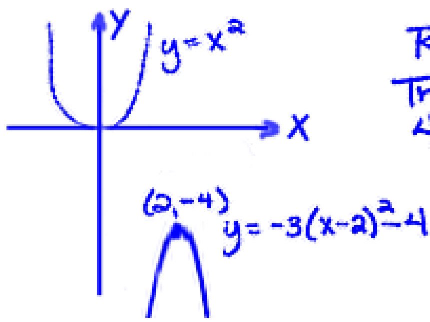
Reflected across the x-axis, translated (moved) 7 units left, 3 units down.

By simultaneously graphing a parent quadratic function and then another transformed quadratic we can visually observe:

Multiplication by a factor whose **absolute value** is greater than 1 (**stretch** in the vertical direction)

Multiplication by a factor whose **absolute value** is less than 1 (**shrink** in the vertical direction)

Example 3: Use a graphing calculator to simultaneously graph y = x^2 (the parent function) and y = –3(x –2)^2 – 4. Sketch the calculator graph-display. How has the second function been transformed?



Reflected across the X-axis, Translated 2 units to the right, 4 units down. Vertically stretched by a factor of 3.

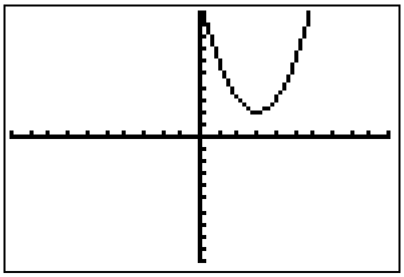
Example 4: <u>Without graphing</u> the function y = (–1/5)(x + 6)^2 –17 describe the transformations on its parent function.

Reflected across the X-axis, Vertically shrunk by a factor of 5 (equivalent to a stretch of 1/5). Shifted 6 units left and 17 units down.

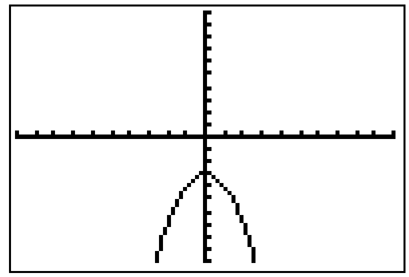
Assignment:

In the following two problems, state the transformation on the parent function $f(x) = x^2$. The tic-marks on both axes are in increments of 1. Write the equation of the function.

1.



2.



In the following problems, describe the transformations on the parent function ($f(x) = x^2$) that produces these functions. Sketch the function and label the vertex.

3. $y = (x + 3)^2 + 79$

4. $f(x) = -(x - 2)^2 + 2$

5. $f(x) = -(x + 4)^2$

6. $y = x^2 + 6$

7. $y = -x^2 - 2$

8. $f(x) = (4/5)(x - 2)^2 + 1$

9. $w(x) = 3(x + 10)^2 + 5$

10. $g(x) = -4x^2$

In the following problems, transformations on the parent function, $y = x^2$, are described. Write the new, transformed function. Sketch the function and label the vertex.

11. Translate right 2 and down 1.

12. Translate left 6 and up 4.

13. Reflected across the x-axis and shifted down 2.	14. Reflected across the x-axis, vertically stretched by a factor of 2, and translated left 3 units.
15. Vertically shrunk by a factor of 2 and translated down 2.	16. Reflected across the x-axis, vertically stretched by a factor of 3, shifted right 11 units, 2 units up.

In the following problems, describe the transformations on the parent function ($f(x) = x^2$) that produces these functions. Sketch the function and label the vertex.

17. $y – x^2 – 3 = 0$	18. $(x – 4)^2 = y + 2$

19. $4y - 16x^2 + 40 = 0$

*20. $y = 2x^2 - 12x + 29$

*21. Use a graphing calculator to find the two places where the graph of $y = 3x - 2$ intersects the graph of $y = 2(x - 3)^2 + 6$. Make a sketch of the calculator display.

Unit 8: Lesson 05 ***Writing quadratic functions**

Decide which of the following two forms of quadratic function can be used based on the given information in a problem:

Vertex form: $f(x) = a(x - h)^2 + k$
Root form: $f(x) = a(x - r_1)(x - r_2)$

Example 1:
Write the quadratic function having vertex (–7, 2) and passing through the point (1, 5).

$(h, k) = (-7, 2)$
$y = a(x-h)^2 + k$
$y = a(x+7)^2 + 2$
now sub in (1, 5)
$5 = a(1+7)^2 + 2$
$5 = a\,64 + 2$
$3 = 64a$
$\frac{3}{64} = a$

$y = a(x-h)^2 + k$
$\boxed{y = \frac{3}{64}(x+7)^2 + 2}$

Example 2:
Write the equation for the parabola having roots 1 & –6 and passing through the point (0, 4).

$r_1 = 1 \quad r_2 = -6$
$y = a(x-r_1)(x-r_2)$
$y = a(x-1)(x+6)$
now sub in (0, 4)
$4 = a(0-1)(0+6)$
$4 = a(-6)$
$-\frac{2}{3} = a$

$y = a(x-r_1)(x-r_2)$
$\boxed{y = \frac{-2}{3}(x-1)(x+6)}$

Assignment:

In each problem below, write the equation of the parabola (quadratic function) that has the given characteristics.

1. Vertex (–8, 2) and passing through (5, –22).	2. Roots 4 & –7 and passing through (–1, 6).
3. Vertex down three units from the origin and passing through (–11, 3).	4. Roots 0 & 4 and passing through (1, –1).

5. Vertex (–6, 4) and having a root at x = 8.

6. Having two roots, y-intercept –1, and with vertex (–1, 22).

7. Having roots 16 & 1, f(2) = –8.

*8. f(–2) = 1, f(–6) = 0, and having a root at x = 1.

9. Double root at $x = 7$ and passing through (3, 2).

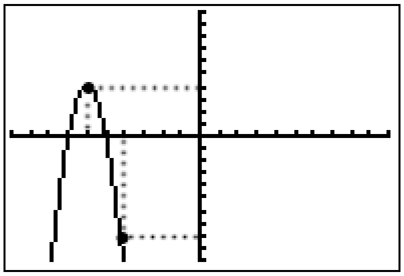
10. Vertex at (2, 5) and passing through the y-intercept of $5x – 5y = 15$.

*11. The line $y = –2x + 1$ intersects the parabola at $x = –2$. The vertex of the parabola is at the origin.

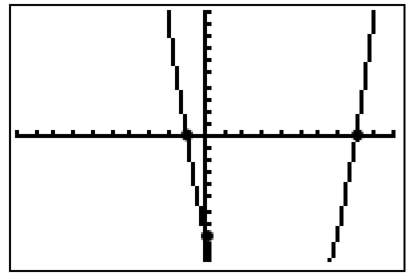
*12. A horizontal line at $y = 6$ intersects the parabola at (5, 6) & (9, 6). The vertex is 4 units above this line.

In the following two problems, use the information from the graphs to write the corresponding quadratic functions. The tic marks represent 1 unit.

13.



14.

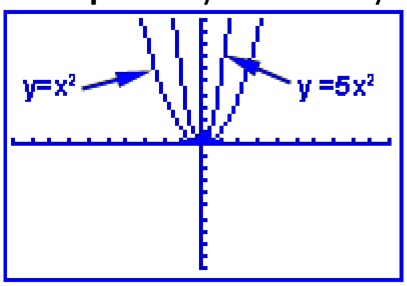


Unit 8: Lesson 06 Analyzing quadratic functions with a graphing calculator

Simultaneously graph the following pairs on a graphing calculator. (Use zoom "6. ZStandard.")

Make a sketch of the calculator display and comment about the differences between the two curves. (For more details on this, see **Calculator Appendix A** and a related video.)

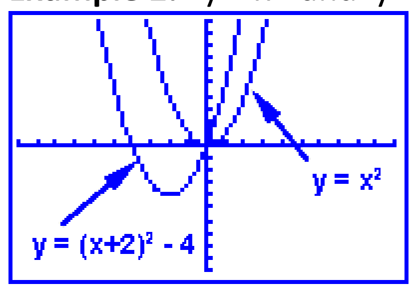
Example 1: $y = x^2$ and $y = 5x^2$



$Y_1 = x^2$
$Y_2 = 5x^2$

$Y = 5x^2$ is 5 times steeper

Example 2: $y = x^2$ and $y = (x + 2)^2 - 4$



$Y_1 = x^2$
$Y_2 = (x+2)^2 - 4$

$y = (x+2)^2 - 4$ is translated 2 units to the left and 4 down

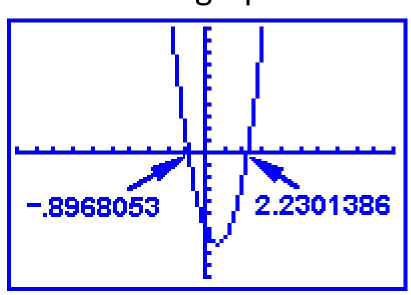
Find the roots of a quadratic equation (for example, $3x^2 - 4x - 6 = 0$) with the following technique:

- Replace the 0 with *y*
- Graph the function (a parabola)
- Find where the parabola crosses the x-axis using **2nd Calc | 2. zero** on the calculator.

For details (and a video) on finding roots, see **Calculator Appendix I**.

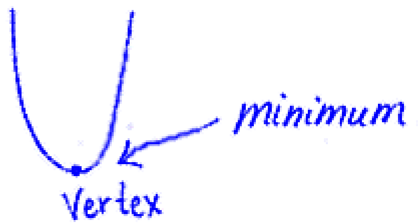
Example 3: Find the roots of $3x^2 - 4x - 6 = 0$. Sketch the graph.

$Y_1 = 3x^2 - 4x - 6$
use 2nd calc | 2. zero
two times to find the two roots.



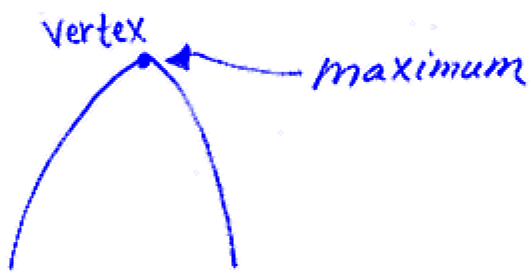
The vertex of a parabola can be located by realizing that the **minimum point** (if the parabola is "going up") **is the vertex**.

On a graphing calculator use **2nd calc | 3. minimum**



The vertex of a parabola can be located by realizing that the **maximum point** (if the parabola is "going down") **is the vertex**:

On a graphing calculator use **2nd calc | 4. maximum**

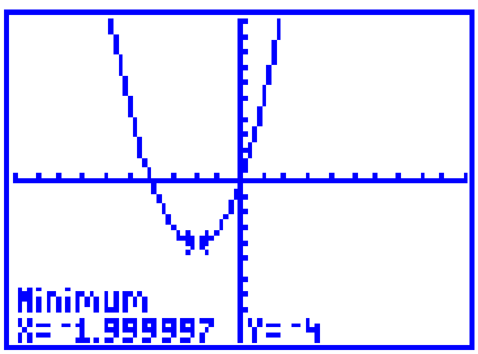


For details (and a video) on finding maximum and minimum points, see **Calculator Appendix J**.

Example 4: Use a calculator to find the vertex of y =(x +2)^2 – 4. Do this by finding the minimum or maximum point (which ever is appropriate).

Parabola "goes up" so find minimum with
2nd calc | 3. minimum

Notice the "round off" error
$-1.999997 \approx -2$



Assignment:

Graph the following pairs in problems 1 - 4 simultaneously on a calculator and sketch the display. Identify each curve by labeling and comment on the differences.

1. $y = -.3x^2$ and $y = 6x^2 + 5$	2. $y = x^2$ and $y = (x-3)^2 + 2$
3. $y = x^2 + 5$ and $y = -x^2 + 5$	4. $y = (1/5)x^2$ and $y = (9/10)x^2$

5. Find the roots of $4x^2 - 11x - 2 = 0$ by graphing the corresponding parabola on a calculator and then finding the zero's. Sketch the calculator display(s).

6. Find the roots of $3x^2 - 10x + 12 = 0$ by graphing the corresponding parabola on a calculator and then finding the zero's. Sketch the calculator display(s).

---

7. Use a calculator to find the vertex of $y = -4x^2 + 5x + 4$. Do this by finding the minimum or maximum point (which ever is appropriate). Sketch the display.

---

8. Use a calculator to find the vertex of $y = 3x^2 + 11x + 4$. Do this by finding the minimum or maximum point (which ever is appropriate). Sketch the display.

Unit 8: Lesson 07 ***Quadratic inequalities**

Quadratic inequalities are most easily solved by:

- first getting 0 on the right side of the inequality,
- graphing the resulting quadratic on the left side, and then
- noticing the x-region(s) over which the graph lies either above or below the x-axis as appropriate according to the direction of the inequality symbol.

The graph of the parabola produced by the quadratic mentioned above is easily produced by:

- finding the roots or the vertex, and then
- sketching the parabola either going “up” or “down” as appropriate according to the sign in front of the x^2 term.

Example 1: Solve $(x - 3)(x + 5) < 0$

$x - 3 = 0 \qquad x + 5 = 0$

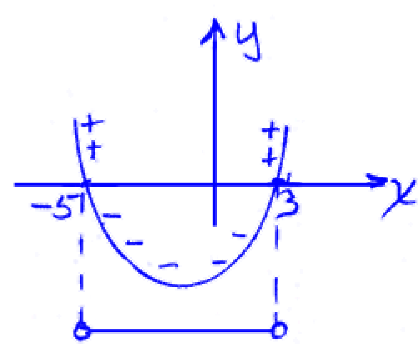
$x = 3 \qquad x = -5$

roots

$(x-3)(x+5) < 0$

$+x^2 + 2x - 15 < 0$

↳ ∪ ↳ neg.



$\boxed{-5 < x < 3}$

Example 2: Solve $x^2 - 10x + 18 > 2$

$x^2 - 10x + 18 > 2$

$x^2 - 10x + 18 - 2 > 0$

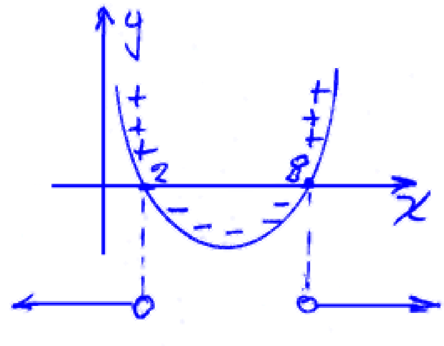
$+x^2 - 10x + 16 > 0$ pos.

$(x-8)(x-2) > 0$

$x - 8 = 0 \quad x - 2 = 0$

$x = 8 \quad x = 2$

roots

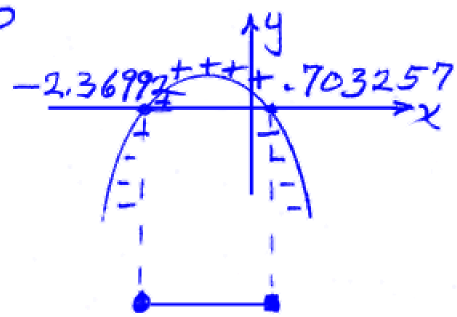


$\boxed{x < 2 \text{ or } x > 8}$

Example 3: Solve $-.6x^2 - x + 1 \geq 0$ by finding the roots with a graphing calculator.

$-.6x^2 - x + 1 \geq 0$

pos.



$\boxed{-2.36992 \leq x \leq .703257}$

Assignment: Make a sketch of the parabola and then solve these inequalities.

1. $(x - 8)(x + 5) < 0$

2. $-(x - 1)(x + 10) < 0$

3. $x^2 - 9x \geq 0$

4. $-(x - 2)^2 + 1 > -4$ (Use a calculator to find the roots.)

5. $x^2 + 12x + 36 \leq 0$

6. $-(x - 4)^2 - 7 > 0$

7. $x(x + 10) - 9 \leq 2$

8. $x^2 + 4x + 1 > 0$ (Find the roots by manually solving the quadratic formula.)

9. $-x^2 \geq 6x + 5$ (Find the roots using a graphing calculator.)

10. $3x^2 + x + 4 < 2x^2 + x + 9$

Unit 8: Cumulative Review

1. Multiply $(5x - 2y)(5x + 2y)$

2. Factor $3z^4 - 27x^2$

3. Find the intersection point of these two lines: $2x + 8y = 9$ and $3x - 2y = -1/2$ using elimination.

4. Find the equation of a line that passes through the vertex of $y = 4(x - 7)^2 + 2$ and is parallel to $y + x = 7.2$.

5. Simplify $xy\left(\sqrt[3]{x^5y^{14}}\right)$ and leave in radical form.

6. Determine if (–7, 8) is part of the solution of $3x - 3y > 4$.

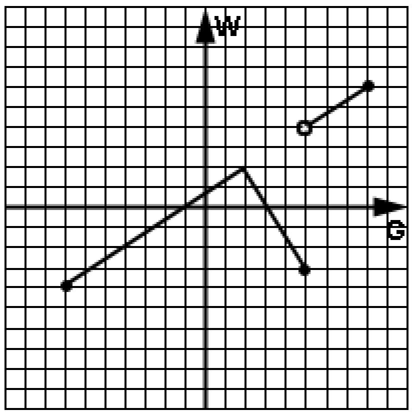
7. Use the quadratic formula to find the roots of $5x^2 - 2x - 7 = 0$.

8. Use completing-the-square to solve $x^2 - 6x - 7 = 0$.

9. Use factoring to solve $x^2 - 6x - 7 = 0$.

10. What would be the new equation of $y = 3x - 2$ if translated up 5 units?

11. What is the domain and range of this relation? Which variable is the dependent and which is the independent? Is it a function?



12. Simplify $4(2x^3\ y^4)^2(3xy)^3$

Unit 8: Review

No calculators on this review or the test that follows.

1. Give the following forms of the equation for a parabola in terms of a, h, k, r_1, and/or r_2:

General form:

Vertex form:

Root form:

2. Is the following equation a quadratic function?

$y + x^2 = 3x - 8$

3. Does the following equation produce a parabola when graphed?

$y - 3/x^2 = 4x - 2/5$

4. What is the (h, k) vertex point of the parabola produced by the following function?

$f(x) + 2 = (x + 3)(x + 3) - 6$

5. What are the roots of the following quadratic function?

$y = -22(x - 13)(x + 2)$

6. What is the vertex of the parabola produced by the following function?

$f(x) = 4x - x^2 + 6$

In the following two problems, give the transformations on the parent function, $f(x) = x^2$, that produces these functions.

7. $f(x) = -3(x - 7)^2 - 5$

*8. $y = 2(x + 1)(x - 7)$

In the following two problems, the given transformation is applied to the parent function, $f(x) = x^2$. What is the new, resulting function?

9. Reflect across the x-axis, translate left 5 and up 2.	10. Vertically stretch by a factor of 3, shift up 21.

11. For the quadratic function $y = f(x) = -4(x - 1)(x + 9)$ find the following information and graph & label the parabola:

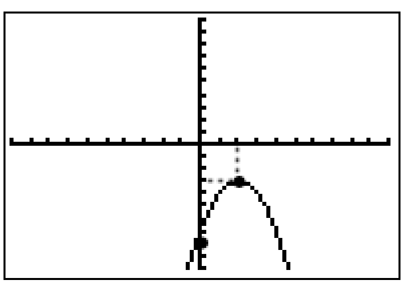
Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

12. For the quadratic function $y = f(x) = (6 + x)^2 + 4$ find the following information and graph & label the parabola:

Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

In the following two problems, use the given information to find the equation of the parabola.

13.



14. Roots at 5 and –2, passing through (–1, 3).

15. Solve $(x - 6)(x + 7) < 0$.

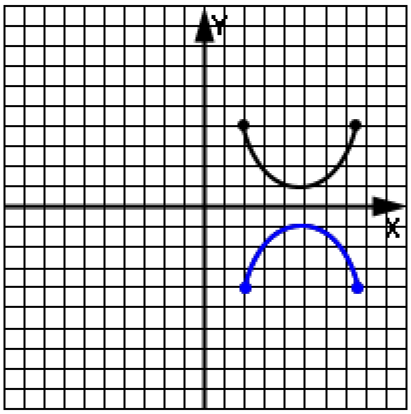
16. Solve $x^2 - 8x \geq -7$.

Alg II, Unit 9

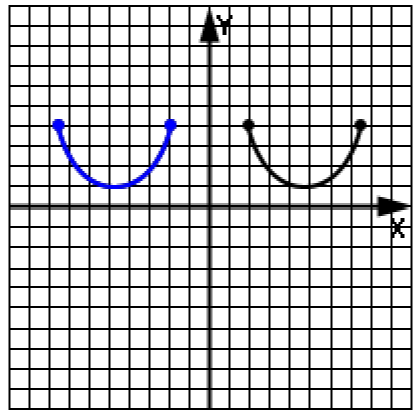
Reflections, translations, and inverse functions

Unit 9: Lesson 01 **Reflection fundamentals**

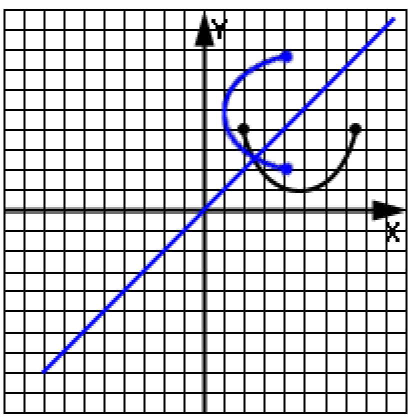
Example 1:
Graphically reflecting a function across the x-axis:



Example 2:
Graphically reflecting a function across the y-axis:



Example 3:
Graphically reflecting a function across the line y = x   (45^o line):



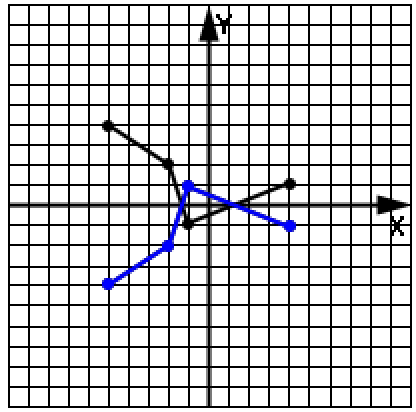
Reflections of the type done in example 3 above are important in the study of **inverse functions**.

Notice that all three examples above produce **reflections** of the original function **across a line**.

Example 4: Using a table of points, produce a **reflection across the x-axis.** Make a new table in which we **change the signs of the *y* values**.

x	y
-5	4
-2	2
-1	-1
4	1

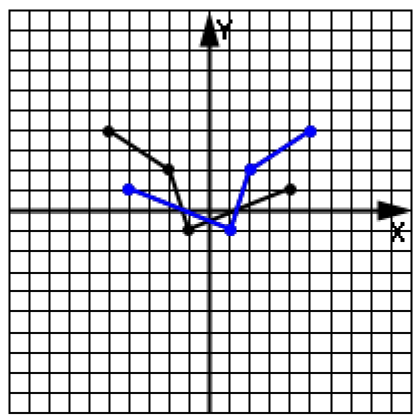
x	y
–5	**–4**
–2	**–2**
–1	**1**
4	**–1**



Example 5: Using a table of points, produce a **reflection across the y-axis.** Make a new table in which we **change the signs of the *x* values**.

x	y
–5	4
–2	2
–1	–1
4	1

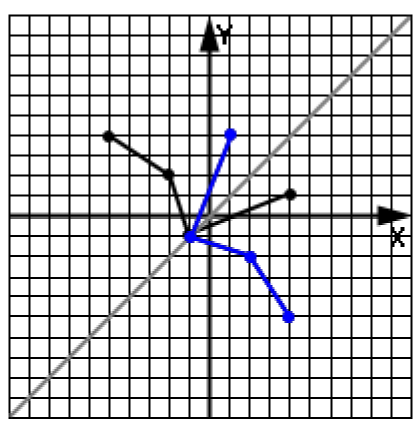
x	y
5	**4**
2	**2**
1	**–1**
-4	**1**



Example 6: Using a table of points, produce a reflection across the line y = x. Make a new table in which we **interchange the *x* and *y* values**.

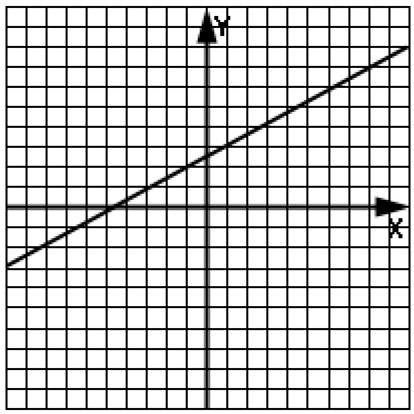
x	y
–5	4
–2	2
–1	–1
4	1

x	y
4	**–5**
2	**–2**
-1	**–1**
1	**4**

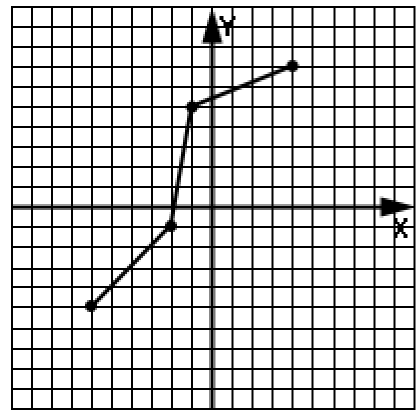


Assignment:

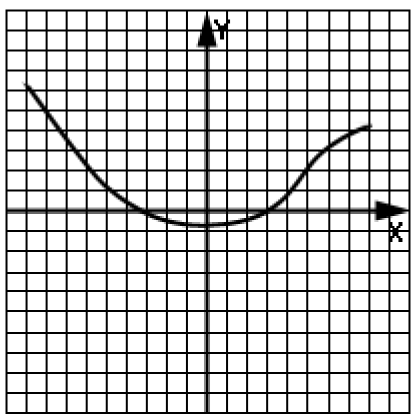
1. Sketch the reflection of the function across the x-axis:



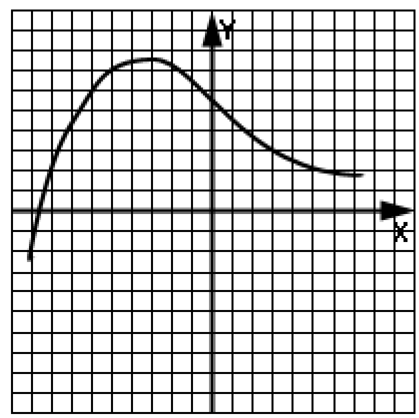
2. Sketch the reflection of the function across the x-axis:



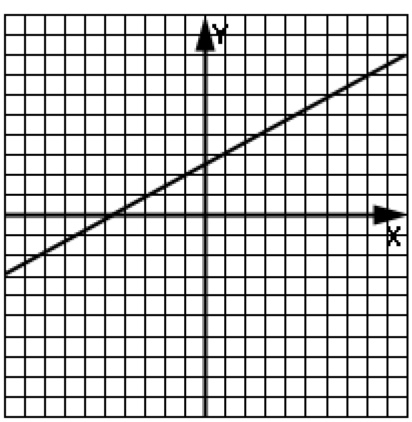
3. Sketch the reflection of the function across the x-axis:



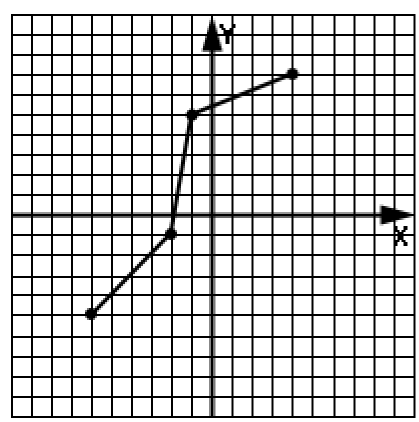
4. Sketch the reflection of the function across the x-axis:



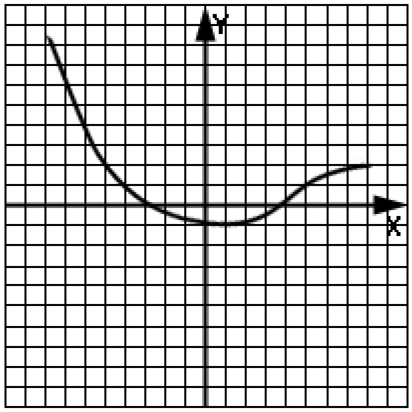
5. Sketch the reflection of the function across the y-axis:



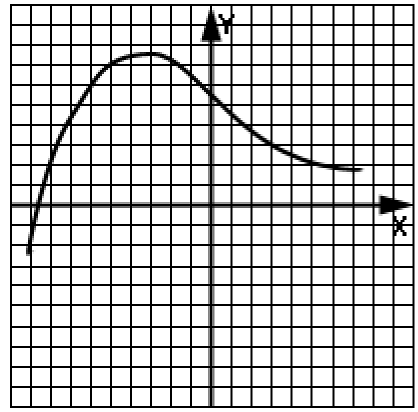
6. Sketch the reflection of the function across the y-axis:



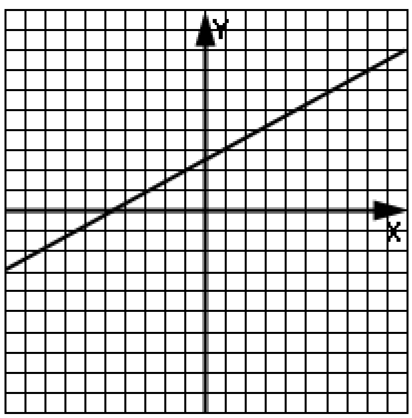
7. Sketch the reflection of the function across the y-axis:



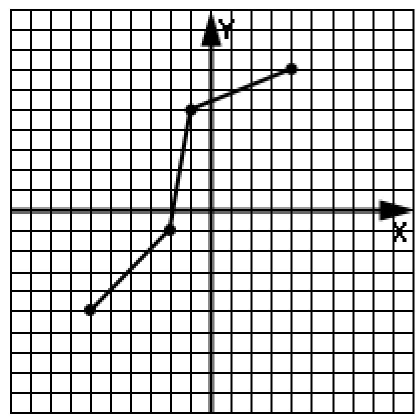
8. Sketch the reflection of the function across the y-axis:



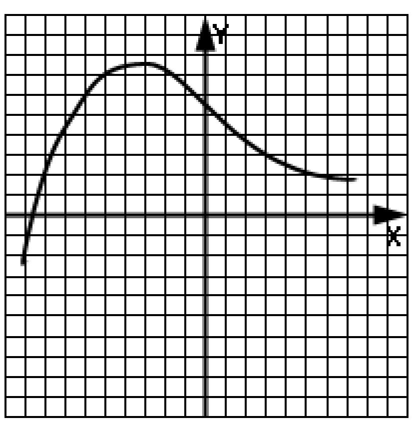
9. Sketch the reflection of the function across the line given by y = x:



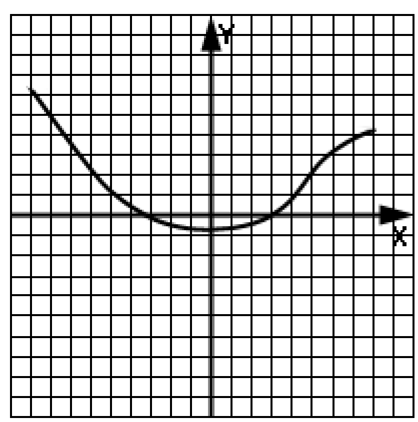
10. Sketch the reflection of the function across the line given by y = x:



11. Sketch the reflection of the function across the line given by y = x:



12. Sketch the reflection of the function across the line given by y = x:

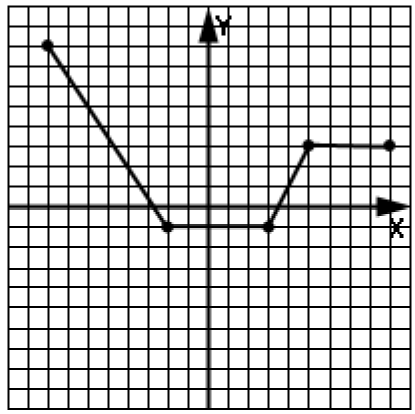


Fill in the blanks of the second table so as to produce the indicated reflection. Sketch the reflection alongside the graph of the provided graph of the original function.

13. Reflection across the x-axis

x	y
–8	8
–2	–1
3	–1
5	3
9	3

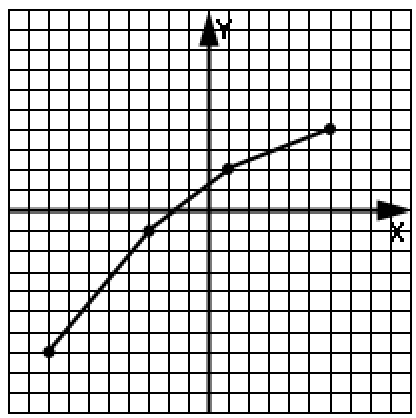
x	y



14. Reflection across the y-axis

x	y
–8	–7
–3	–1
1	2
6	4

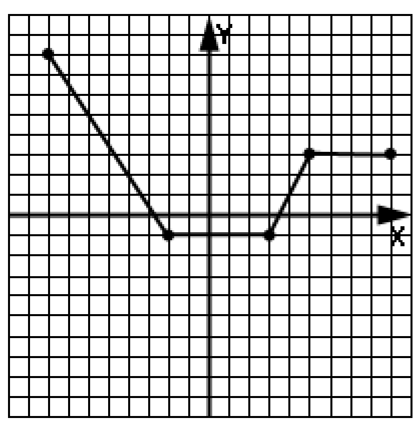
x	y



15. Reflection across the line y = x

x	y
–8	8
–2	–1
3	–1
5	3
9	3

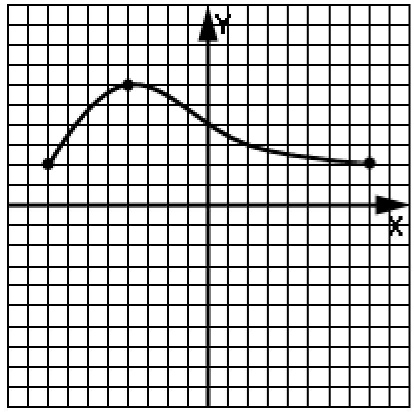
x	y



16. Reflection across the x-axis

x	y
–8	2
–4	6
8	2

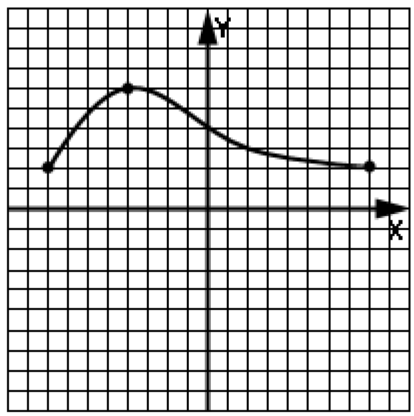
x	y



17. Reflection across the y-axis

x	y
–8	2
–4	6
8	2

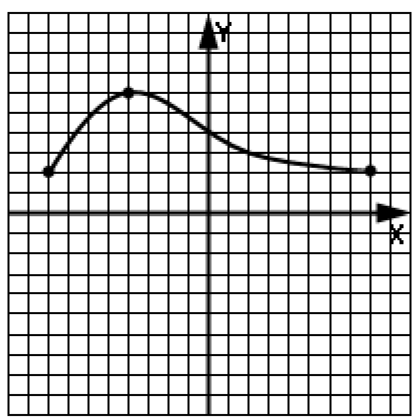
x	y



18. Reflection across the line y = x

x	y
–8	2
–4	6
8	2

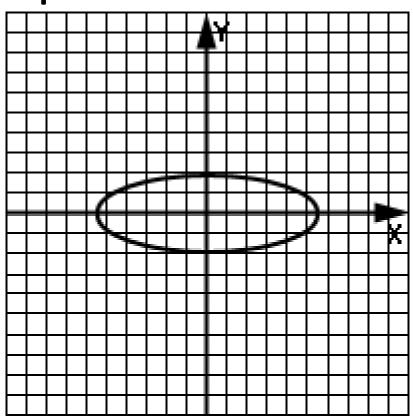
x	y

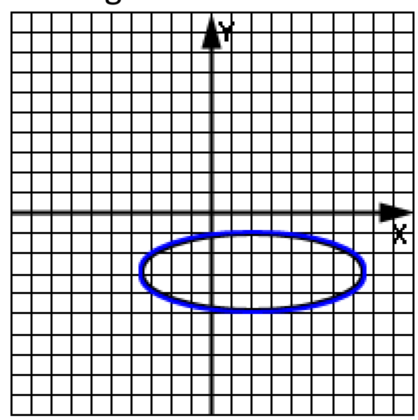


Unit 9: Lesson 02 — Translations and reflections of relations

Translating a relation or function means to **move (shift)** all the points in the indicated direction.

Example 1: Sketch the relation that's translated right 2 units and down 3.

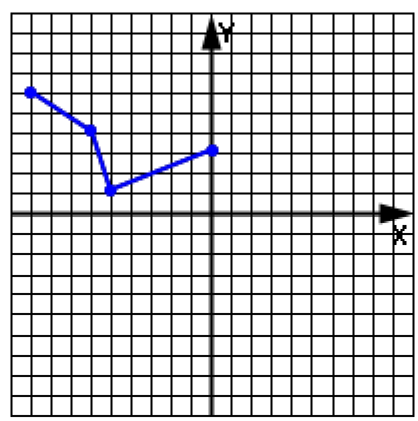




Example 2: A function is described by line segments connecting the points in the table below. Create a new table in which the points have been translated left 4 units and up 2. Sketch the translated function.

x	y
–5	4
–2	2
–1	–1
4	1

x	y
–9	**6**
–6	**4**
–5	**1**
0	**3**



Consider a relation described by a formula involving *x* & *y*. It is algebraically translated left/right *h* units by **replacing all *x's* in the function with *x – h*** and up/down by **replacing all *y's* with *y – k***.

Positive values of *h* and *k* result in a translation right and up. Negative values will result in left and down shifts.

***Example 3:** Write a relation that is the result of translating $xy - 5x^2 = 23$ left 3 units and up 14.

$$(x+3)(y-14) - 5(x+3)^2 = 23$$

To simultaneously reflect a function, f(x) across the x-axis and translate left/right *h* and up/down *k* units do the following:

- Multiply f(x) by –1.
- Replace *x* in the function with *x – h.*
- Replace *y* (also called f(x)) with *y – k.*
- Solve for *y.*

Example 4: Consider the function $f(x) = 24x^3 - 2x + \sqrt{x-3}$. Write the equation of a function, g(x), that reflects this function across the x-axis and then move it left 4 units and up 101 units.

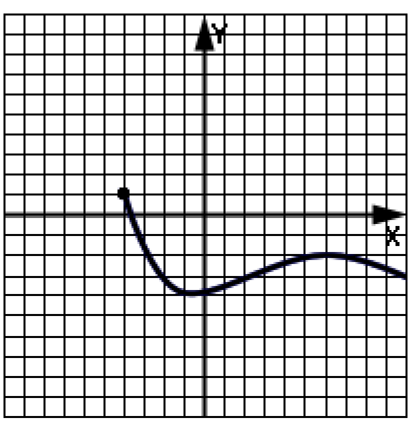
First reflect, $y = -(24x^3 - 2x + \sqrt{x-3})$

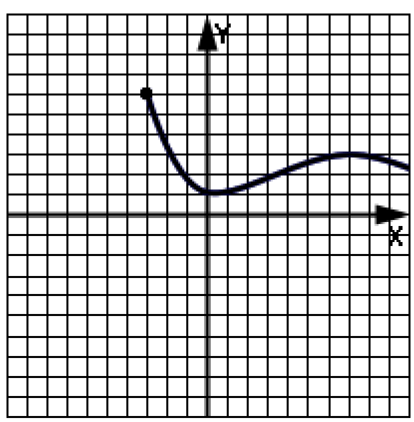
Shift left with $x-(-4) = x+4$, up with $y-101$

$$y - 101 = -(24(x+4)^3 - 2(x+4) + \sqrt{x+4-3})$$

$$g(x) = \boxed{101 - (24(x+4)^3 - 2(x+4) + \sqrt{x+4-3})}$$

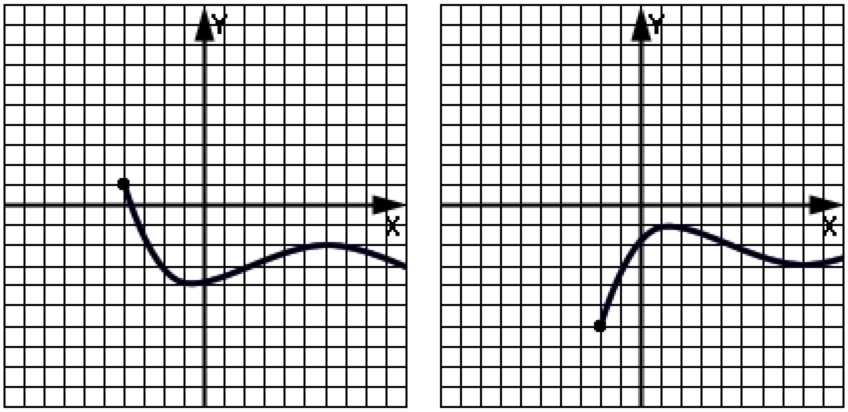
Example 5: The left graph below describes the original function while the right graph shows it translated. Describe the translation.





Translate 1 right, up 5.

Example 6: The left graph below describes the original function while the right graph shows a transformation. Completely describe the transformation.

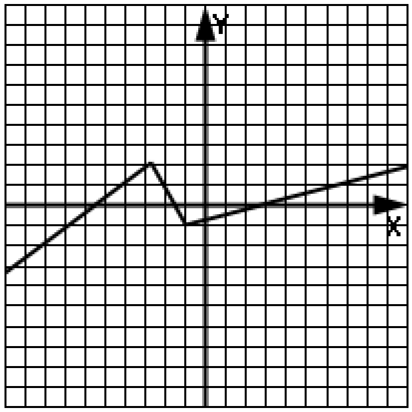


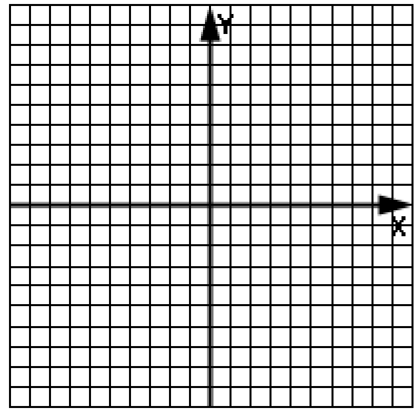
Reflected across the x-axis, translated down 5, right 2.

See **Enrichment Topic F** for a related topic, "Rotations."

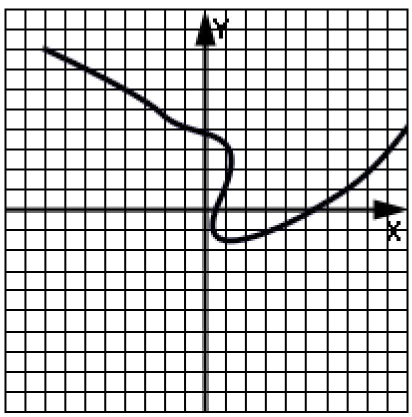
Assignment:

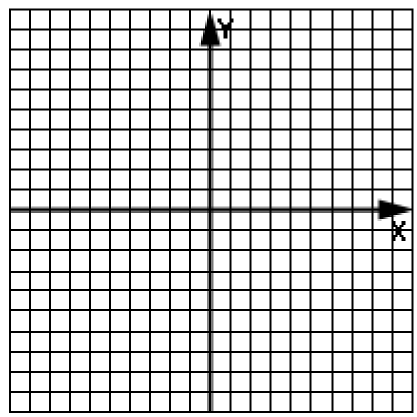
1. Sketch the relation that is translated 5 units left and 3 units up.





2. Sketch the relation that is translated 4 units right and 7 units down.

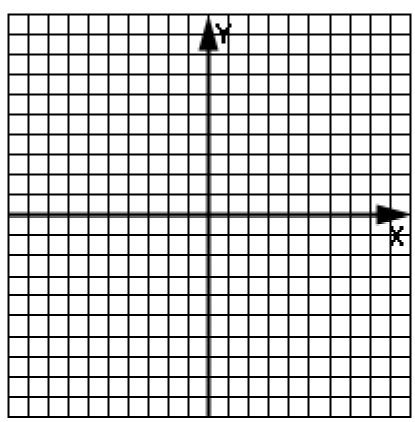




3. A function is described by line segments connecting the points in the table below. Create a new table in which the points have been translated left 2 units and up 4. Sketch the translated function.

x	y
–5	4
–2	2
–1	–1
4	–9

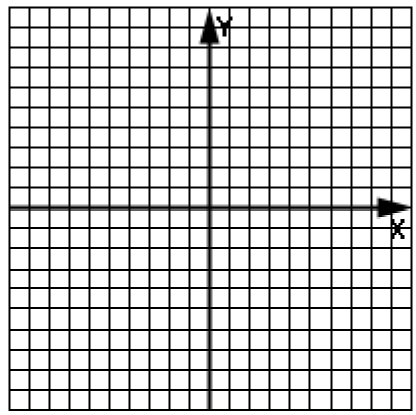
x	y



*4. The end points of a horizontal diameter of a circle are given in the table. Create a new table in which the points have been translated so the new center is at the old circle's right end point of its horiz diameter. Sketch the translated circle.

x	y
–5	3
1	3

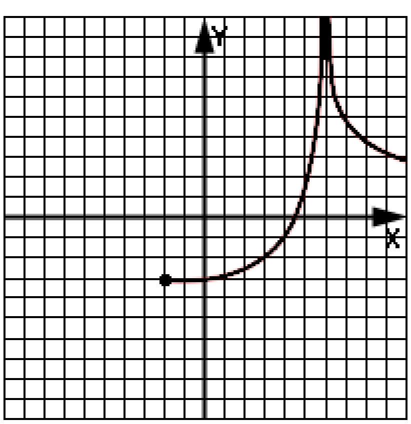
x	y

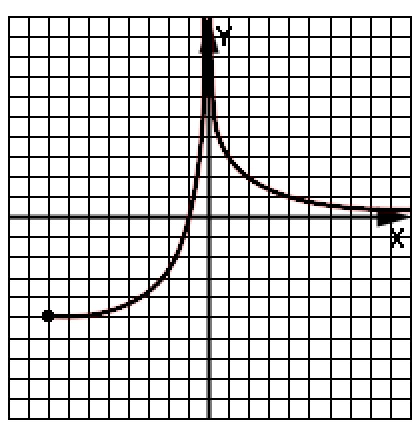


5. $f(x) = 2x^2 - 2x + 1/x$. Write the equation of a function, g(x), that reflects this function across the x-axis and then moves it right 1 unit and down 6 units.

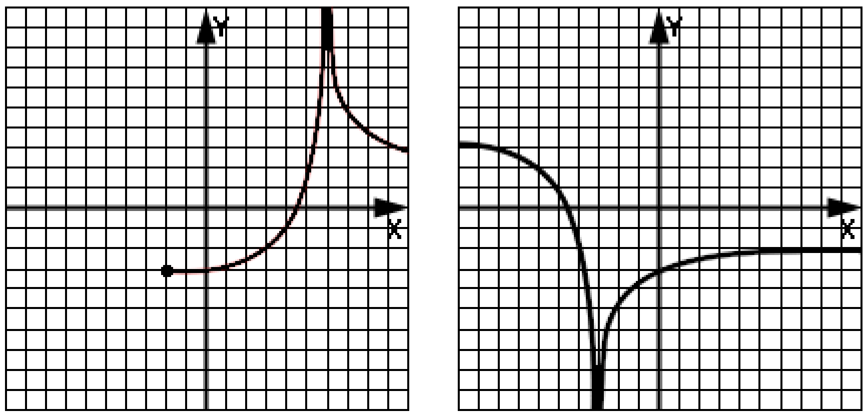
6. $f(x) = 2x^{\sin x} - \frac{1}{x+3}$. Write the equation of a function, h(x), that translates this function 7 units left.

7. The left graph below describes the original function while the right graph shows a transformation. Completely describe the transformation.





8. The left graph below describes the original function while the right graph shows a transformation. Completely describe the transformation.



9. Generally describe the technique for translating a relation with variables *x* and *y* so that the new “center” is at coordinates (h, k).

10. Generally describe the technique for reflecting a function about the x-axis.

Unit 9: Lesson 03 *Inverse function fundamentals

Plot the points given in the left table and connect with a smooth curve.

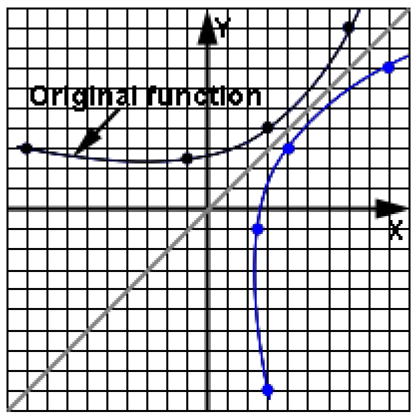
Fill in the second table with the x and y values of the first table **interchanged**. Plot these new points and connect with a smooth curve.

Draw the line y = x (45° line). What notable thing do you notice about the two curves?

They are reflections of each other across the 45° line.

x	y
–9	3
–1	2.5
3	4
7	9

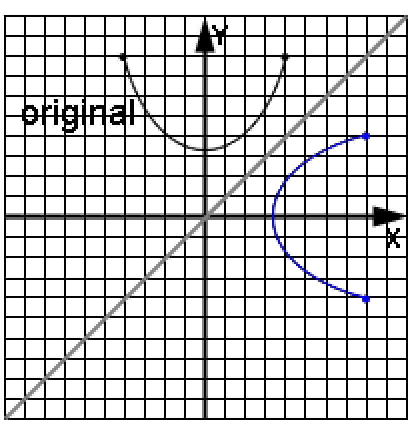
x	y
3	**–9**
2.5	**–1**
4	**3**
9	**7**



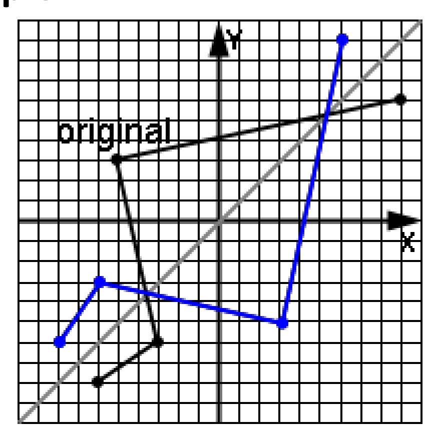
In the above example, we say that each relation is the **inverse** of the other.

Graphically (using a picture) determine the inverse of the following relations by reflecting each across the line y = x (45° line).

Example 1:



Example 2:



Algebraic determination of inverse relations:

- Exchange x & y in the equation
- Solve for y

Example 3: Find the inverse relation of $f(x) = -5x + 2$.

$$y = -5x + 2$$
$$x = -5y + 2$$
$$5y = -x + 2$$
$$y = \boxed{-\frac{1}{5}x + \frac{2}{5}}$$

Example 4: Find the inverse relation of $y = 4x^2 - 6$.

$$x = 4y^2 - 6$$
$$x + 6 = 4y^2$$
$$\frac{x+6}{4} = y^2$$
$$\boxed{\pm\sqrt{\frac{x+6}{4}} = y}$$

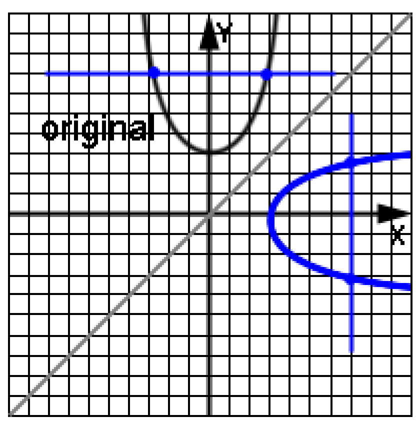
One-to-one functions (horizontal line test):

A one-to-one function is one that passes the **horizontal line test** (a horizontal line passes through no more than one point).

The reason this is important is that when a function that does **not** pass this test and is reflected across the y = x line, the resulting inverse is **not a function** because it now **fails the vertical line test**. The following example demonstrates this.

Example 5: Draw a horizontal line that demonstrates that the function is not one-to-one.

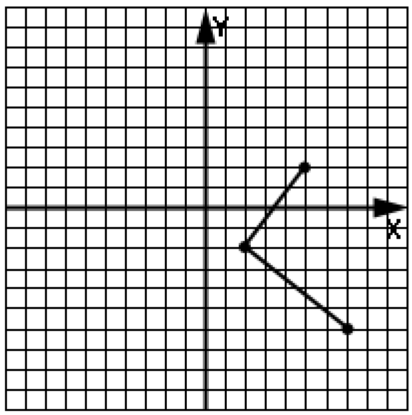
Reflect the function across the y = x line and demonstrate with a vertical line that this resulting inverse of the original is not a function.



In the following two examples, a relation is drawn.

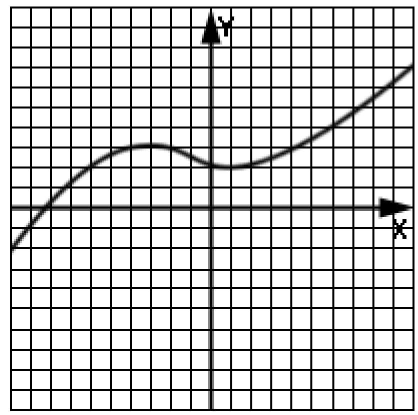
- Decide if the original is one-to-one.
- Decide if the original is a function.
- Without drawing the inverse, decide if the inverse would be a function or not.

Example 6.



Is original one-to-one? ___Yes___
Is original a function? ___No___
Is inverse a function? ___Yes___

Example 7.



Is original one-to-one? ___No___
Is original a function? ___Yes___
Is inverse a function? ___No___

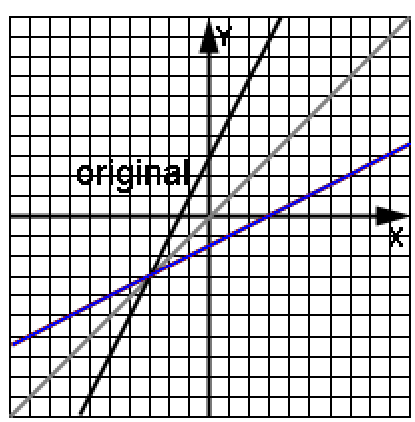
The inverse of f(x) is symbolically written as $\mathbf{f^{-1}(x)}$

Example 8. Write the equation for the inverse of y = f(x) = 2x + 3. Then graph the original function, $f^{-1}(x)$, and the line y = x.

Is the original relation a function? yes

Is the inverse a function? yes

$y = 2x + 3$ original
$x = 2y + 3$ inverse
$x - 3 = 2y$
$\boxed{\frac{x-3}{2} = y} = f^{-1}(x)$



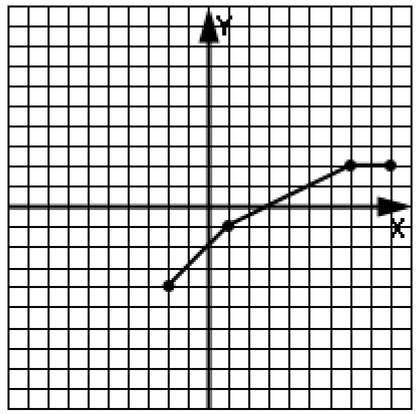
Assignment:

In the following two problems, fill in the second table by interchanging the x & y coordinates values of the first table. Plot and connect the points of the first table with line segments. Similarly, plot and connect the points of the second table with **line segments**. Draw the line described by y = x.

1.

x	y
–2	–4
1	–1
7	2
9	2

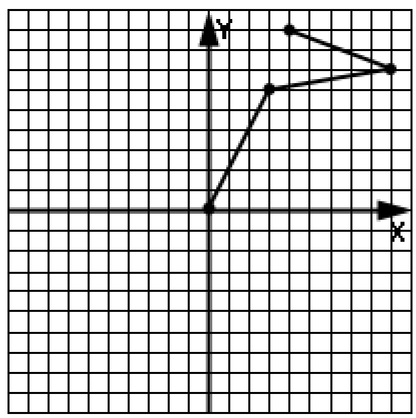
x	y



2.

x	y
0	0
3	6
9	7
4	9

x	y

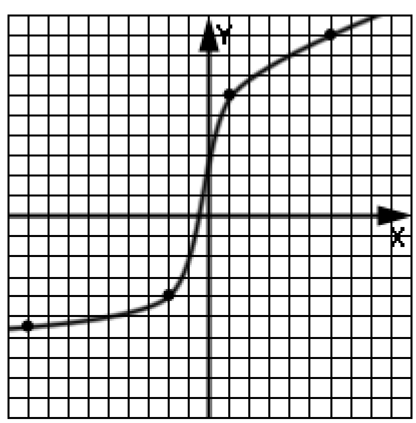


For problems 3 & 4, same instructions as above but connect points with a **smooth curve**.

3.

x	y
–9	–5.5
–2	–4
1	6
6	9

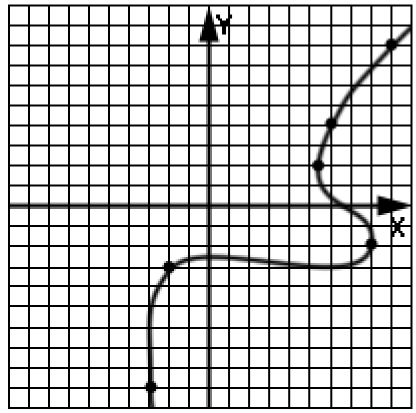
x	y



4.

x	y
–3	–9
–2	–3
8	–2
5.5	2
6	4
9	8

x	y



5. In the above four problems, what significant thing can be said about a relation and its inverse relative to the line y = x?

6. If a relation crosses the line y = x at a particular location, what can be said about its inverse concerning that point? (Look at problems 2 & 3 above and the answer should be obvious.)

7. Find $f^{-1}(x)$ where f(x) = 5x – 6.	8. Find the inverse of $y = x^2 - 4$.
9. Find the inverse of $f(x) = 2x^3 + 1$.	10. Find the inverse of y = x.

*11. Find the inverse of the linear function that passes through (1, 4) and is parallel to the line given by $y = x - 11$.

*12. Find $f^{-1}(x)$ where $f(x) = 1/x - 6$.

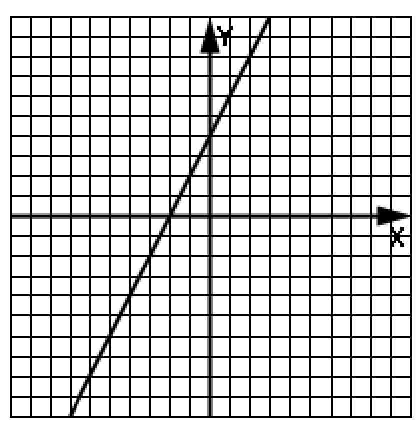
13. Find the inverse of $y = 22x - 19$.

14. Find the inverse of $f(x) = 9x^2 - 8$.

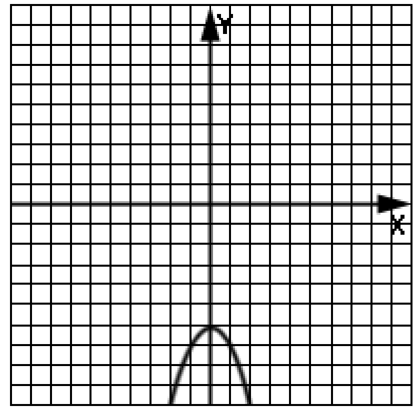
15. Write the equation for the inverse of $y = 2x + 4$. Then graph the original function, its inverse, and the line $y = x$.

Is the original relation a function?

Is the inverse a function?



16. Write the equation for the inverse of $y = x^2 - 6$. Then graph the original function, its inverse, and the line y = x.



Is the original relation a function?

Is the inverse a function?

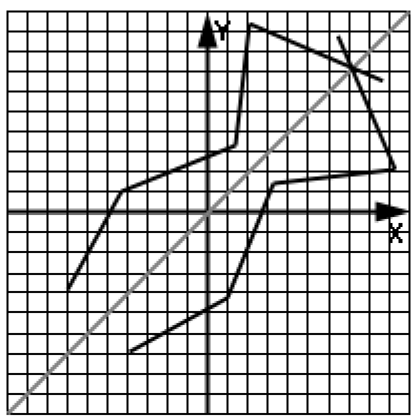
Unit 9: Lesson 04 ***Determining if two relations are inverses of each other**

If all we have is the graphs of two relations, draw a 45^0 line (y = x) and observe if they are reflections of each other across the line. If they are, they are inverses.

Determine if the two relations in each of the following examples are inverses of each other.

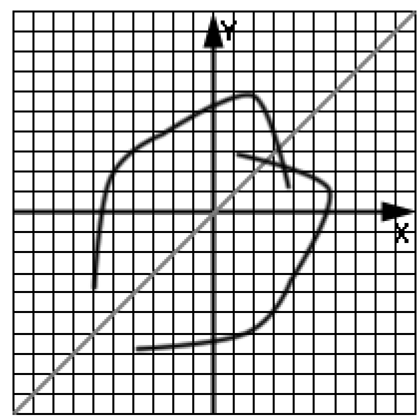
Example 1:

yes



Example 2:

No



Note in Example 2 above they are not inverses of each other because they **do not cross the 45° line at the same place**.

If we have two functions (f(x) and g(x)), passing the following tests will algebraically prove they are inverses of each other:

f(g(x)) = x

and

g(f(x)) = x

Both tests must be passed. It is possible to pass one test and fail the other. In that case the functions are not inverses.

See **Calculator Appendix H** for a unique look at inverse functions.

Example 3: Determine if $f(x) = 2x - 3$ and $g(x) = (1/2)x + 3/2$ are inverses.

$f(g(x)) = 2\left(\frac{1}{2}x + \frac{3}{2}\right) - 3$

$= x + 3 - 3 = x$ ✓

$g(f(x)) = \frac{1}{2}(2x-3) + \frac{3}{2}$

$= x - \frac{3}{2} + \frac{3}{2} = x$ ✓

Yes, they are inverses.

Example 4: Determine if $f(x) = -x + 1$ and $g(x) = -x + 2$ are inverses.

$f(g(x)) = -(-x+2) + 1$

$= x - 2 + 1$

$= x - 1$

Not inverses

If we have two sets of points that represent two different relations, it can be determined if they are inverses of each other by noting if the coordinates are reversed between the two sets.

In the following two examples, determine if the two sets of points represent inverse relations.

Example 5: f = { (3, 4), (5, 6), (–1, 5) } and g = { (4, 3), (5, –1), (6, 5) }

Yes, the pairs don't have to be in order

Example 6:

x	**y = f(x)**
11	6
–2	–13
5	4

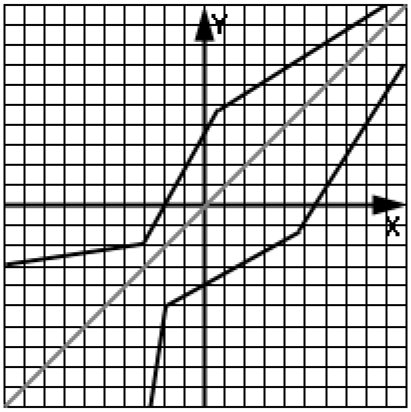
x	**y = g(x)**
6	11
–13	–2
5	4

No (5,4) is not reversed in the second table.

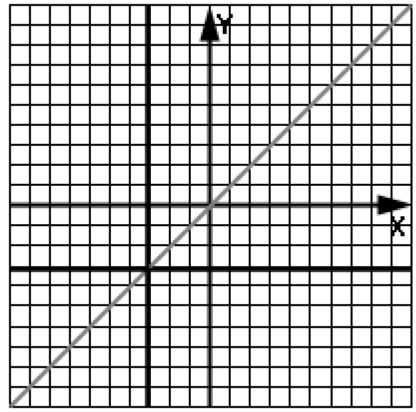
Assignment:

Determine if the two relations in each of the following examples are inverses of each other.

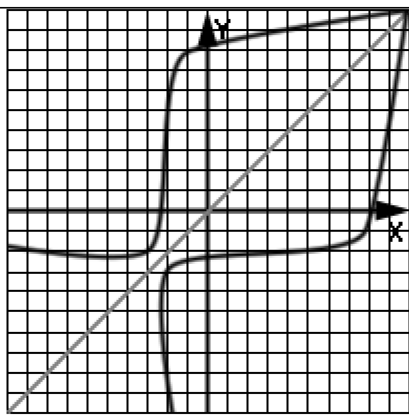
1.



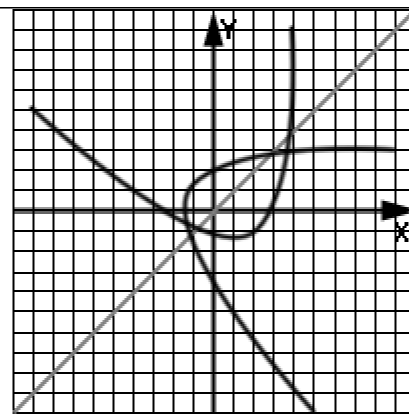
2.



3.



4.



In the following problems, two functions (f(x) and g(x)) are given. Determine algebraically if they are inverses of each other.

5. $f(x) = 2x - 2$ and $g(x) = (x + 2)/2$

6. $f(x) = x$ and $g(x) = x$

7. $f(x) = x^2 - 3x$ and $g(x) = x^3 - 2x$

8. $f(x) = x + 6$ $g(x) = x - 6$

9. $f(x) = 4x - 17$ and $g(x) = x/4 + 17/4$

10. $f(x) = x + 3$ and $g(x) = 2x^2 + 3$

In the following problems, determine if the relations represented by the sets of points are inverses of each other.

11. { (1, 5), (11, –23), (0, 1), (4, 9) } and { (–23, 11), (5, 1), (9, 4), (1, 0) }

12. { (0, 0), (1, 1), (2, 3) } and { (0, 0), (1, 1), (3, 2) }

13.

x	y
0	5
4	–6
11	22
–8	101

x	y
-6	4
5	0
22	-11
101	-8

14.

x	y
–3	2
5	2
9	–2
18	0

x	y
0	18
–2	9
2	5
2	–3

15. Consider two functions of z, w(z) and v(z). What is the test to determine if these two functions are inverses of each other?

*16. If a function is one-to-one, is there a possibility that its inverse might **not** be a function?

17. In which of problems 1 – 4, are **both** relations also functions?

18. In which of problems 1 – 4, are **both** relations not functions?

Unit 9: Cumulative Review

1. Solve $(4/5)x - x + 2 = 0$

2. Write the equation (in standard form) of the line passing through $(-1, 3)$ and the root of $4x - y = 16$.

3. Sketch the solution set of $y > -x$ and $y \le 2x + 9$.

4. Determine algebraically if the point $(-4, 2)$ is part of the solution of $y \le .5x - 1$.

5. Factor $9x^2g - 4y^2g$

6. Factor $m^2 + 4m - 45$

7. Solve $12x^2 - 21x = 6$ by factoring.

8. Solve $x^2 - 10x + 21 = 0$ using the quadratic formula.

9. Multiply $2x(x - 5)(x + 11)$

10. Simplify and leave in radical form:
$\sqrt[3]{x^7 y^{20} 16}$

11. Solve $x^2 + 6x - 11 = 0$ by completing the square.

12. The sum of three even integers is 36. What are the integers?

13. Solve $(x+1)^{1/4} = 2$

14. Solve $\sqrt[3]{3x-6} = 2$

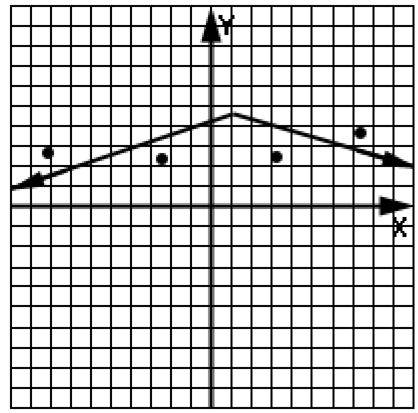
15. For the quadratic function y = f(x) = $x^2 - 4x - 1$, find the following information and graph & label the parabola:

Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

16. State the domain and range of this relation. Is it a function?

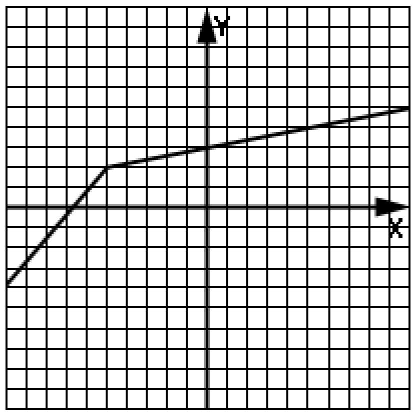
x	y
–5	–5
0	4
10	5
16	4
16	–5

17. State the domain and range of this relation. Is it a function?

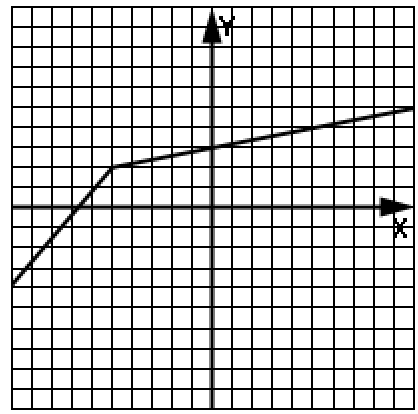


Unit 9: Review

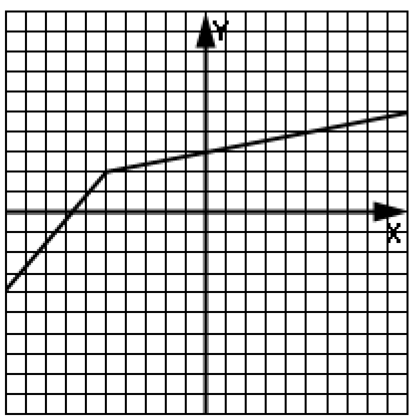
1. Sketch the reflection of the function across the x-axis:



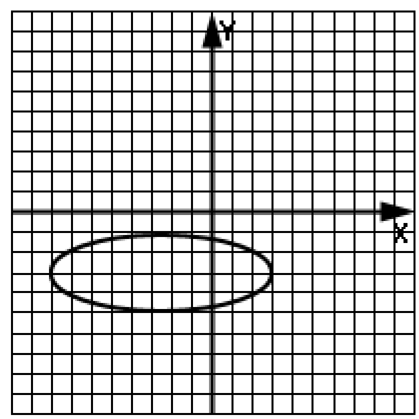
2. Sketch the reflection of the function across the y-axis:



3. Sketch the inverse of this relation.



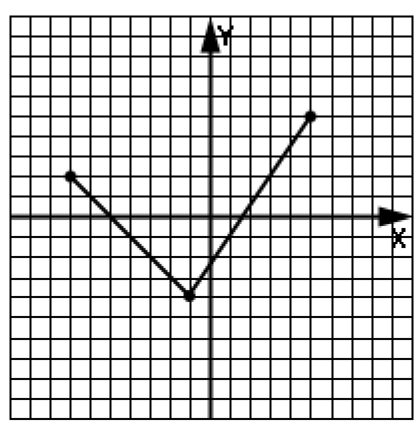
4. Translate 5 right and 6 up.



5. Fill in the second table so as to produce a reflection across the y axis. The original function is provided in the drawing. Draw the reflection.

x	y
–7	2
–1	–4
5	5

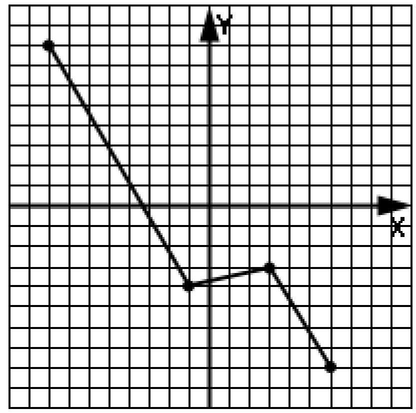
x	y



6. Fill in the second table so as to produce a reflection across the x axis. The original function is provided in the drawing. Draw the reflection.

x	y
–8	8
–1	–4
3	–3
6	–8

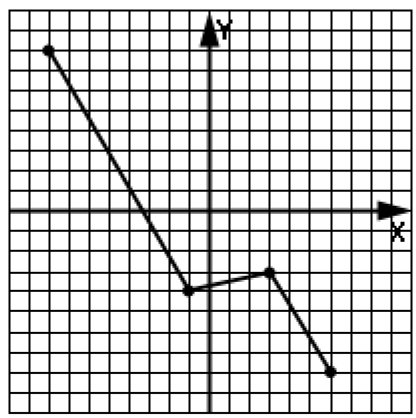
x	y



7. Fill in the second table so as to produce a translation 3 units to the right and 2 units up. The original function is provided in the drawing. Draw the translated function.

x	y
–8	8
–1	–4
3	–3
6	–8

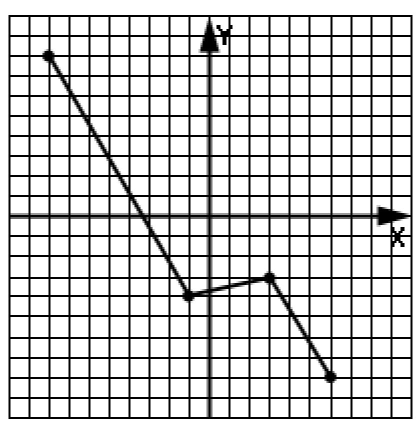
x	y



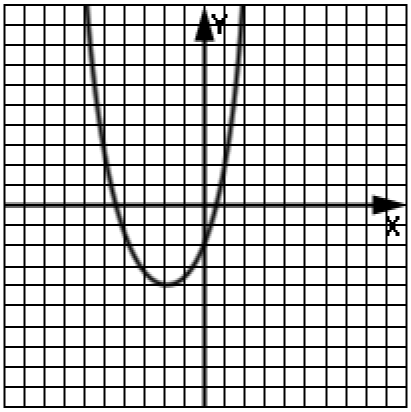
8. Fill in the second table so as to produce the inverse of the relation represented by the nodes in the left table. The original function is provided in the drawing. Draw the inverse relation.

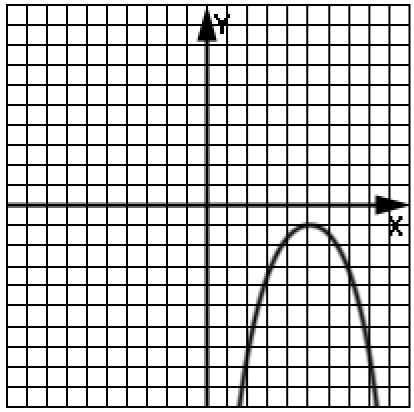
x	y
–8	8
–1	–4
3	–3
6	–8

x	y



*9. For the function g(x) shown to the left below, what would be the algebraic transformation on g(x) that would yield the function shown to the right?





10. Rewrite $y = 4(x-5)^2 + 11$ so that it will be reflected across the x–axis and translated 4 units down and 6 units to the right.

11. Find the function g(x) that is the inverse of $f(x) = -22x - 19$.

12. Find the inverse of $y = x^2 - 9$.

13. Prove that $f(x) = 7x - 9$ is the inverse of $g(x) = (x + 9)/7$

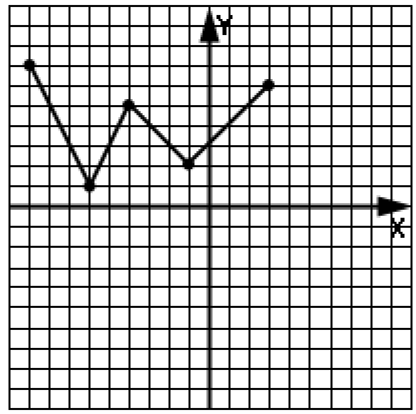
14. Fill in the second table by interchanging the x & y coordinates values of the first table. Plot and connect the points of the first table with line segments. Similarly, plot and connect the points of the second table with line segments. Draw the line described by y = x.

x	y
–9	7
–6	1
–4	5
–1	2
3	6

Is it a function?

x	y

Is it a function?



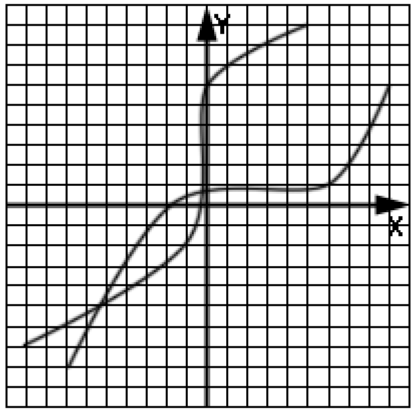
15. In problem 14 above, the second function is the _______ of the first.

16. What is the general algebraic technique of translating a function f(x) *h* units to the left/right and *k* units up/down?

17. What test can be applied to a relation that will predict if its inverse will be a function?

18. What statement concerning symmetry can be made about the two relations in problem 14?

19. Are these two functions inverses of each other?



20. Are these two functions inverses of each other?

m(y) = 8y – 3 and n(y) = (1/8)y – 1/3

21. Are the following two relations inverses of each other? Are they functions?

f(x) = { (97, –2), (5, –.2), (–.2, 5), (6, –2)}
g(x) = { (–2, 97), (–.2, 5), (5, –.2), (–2, 6)}

22. Are the relations given by these two tables inverses of each other? Are they functions?

x	y
1	–2
2	–2
–3	5
–4	6

x	y
–2	1
–2	2
6	–4
5	3

Semester 1: Review

1. Solve and graph the solution to $-4(1 + x) \leq 2x$ on a number line.

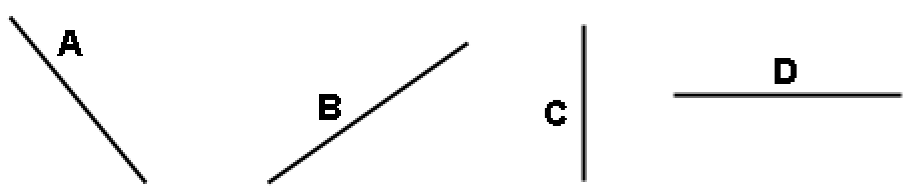
2. Find the equation of the line having the same slope as $2x - 3y = 17$ and passing through $(-1, 2)$.

3. Solve and graph the solution to $-15 \leq 5x < 45$ on a number line.

4. Four times the supplement of an angle is still only half of the angle. What is the angle?

*5. Solve $|3x + 1| \geq 17$ and graph the solution on a number line.

6. Consider lines A, B, C, and D below. Which has a positive slope? ____
Which has a negative slope? _____ Which has a slope of 0? ____
Which has an undefined slope? ____



7. Find the equation of the line passing through (4, 9) and (–2, –3).

8. Solve the following system of equations by elimination.

$2p - 3q = 1$

$5p + 2q = 0$

9. Solve the following system of equations by substitution.
$x + y – 4 = 0$ and $x – 9 + 2y = 0$

10. Ten times the number of nickels is one more than the number of dimes. The total value of the coins is $7.25. How many of each is there?

11. Use a graphing calculator to produce the solution to this system of linear equations. Sketch the display of the calculator.
$y = –5x + 111$ and $y = x + 53$

12. Determine algebraically if (–7, 1) is part of the solution of $2x – y \geq 5$.

13. Factor $4j^2 – 9k^2y^2$

14. Sketch the solution to $2x - y > 5$ and $x + y + 1 \geq 0$.

15. Factor $x^2 + 9x - 22$

16. Factor $6x^2 - 13xy + 5y^2$

17. Multiply $(4x - 1)(3x + 2)$

18. Solve by factoring $3x^2 - 27 = 0$

19. Simplify $\frac{10y}{x^{-5}}\left(\frac{5x^2}{y^{-3}}\right)^{-2}$

20. Simplify $\sqrt[3]{\frac{27}{16}}$

21. Simplify $\sqrt[4]{x^9y}\ \sqrt[4]{y^3x}$ and leave in simplest radical form.

22. Express $13^{3/5}$ in simplest radical form.

23. Express $5x\sqrt[3]{xz^6}$ in simplest exponential form.

24. Solve $(x - 3)^{1/3} = 2$

25. Solve $(4x - 2)^{3/5} = 0$

26. Solve $\sqrt{2x+3} + 4 = 0$

27. Solve $7x^2 - 14 = 0$ by taking the square root.

28. How many solutions are there to $x^5 + 6x^7 - 2x + 1 = 0$?

29. Derive the quadratic formula.

30. Solve $3x^2 - 6x + 2 = 0$ by completing the square.

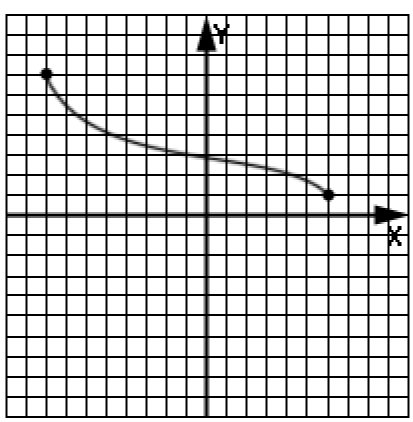
31. Solve $3x^2 - 6x + 2 = 0$ using the quadratic formula.

Unit 7 problems start here:

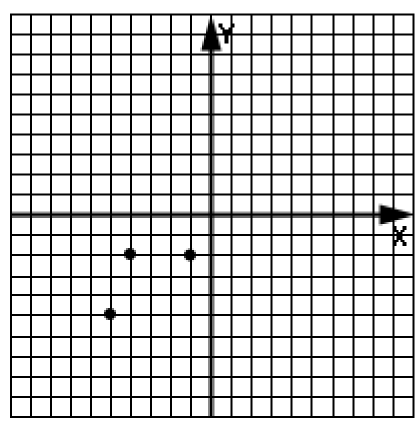
32. Is { (1, 2), (–1, –2), (2, 1), (1, –2) } a function?

33. Find f(–2) where $f(x) = x^2 - 4x + 5$.

34. Find the domain and range of this relation. Is it a function?



35. Complete this function so that it will be an even function.



36. Divide the following terms into two separate groups, one associated with the x-axis and the other with the y-axis: { independent, dependent, vertical-axis, horizontal-axis, range, domain, input, output, h, k, ordinate, abcissa, cos, sin}

37. In the function $g(a) = 3a^2 - 2a + 9$ which is the dependent and which is the independent variable?	38. What requirement concerning symmetry is necessary in order that a function be even?

Unit 8 problems start here:

39. For the quadratic function $y = f(x) = (x - 12)(x + 2)$ find the following information and graph & label the parabola:

Vertex:	Axis of Sym:	Min or Max:	Domain:
Range:	Y-intercept:	Roots:	

40. Find the equation of a parabola with vertex (3, 8) and passing through (1,-2).

41. Use a graphing calculator to find the roots of $y = 4x^2 - 3x - 5$.

Unit 9 problems start here:

42. Fill in the right table so that it represents the inverse function of the left table.

x	f(x)
–8	4
2	–1
6	–1
15	–3
–9	3

x	$f^{-1}(x)$

43. Fill in the right table so that it represents the left table translated 4 units up and 2 units to the left.

x	f(x)
–2	0
3	–2
5	–6
11	–8
12	6

x	f(x)

44. Find $f^{-1}(x)$ when $f(x)=(1/7)x + 2$.

45. Determine if $f(x) = 2x - 5$ and $g(x) = (1/2)x + 3/2$ are inverses of each other.

Miscellaneous problems start here:

46. Without finding the roots, determine the nature of the roots of
$2x^2 - 6x + 5 = 0$.

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47. What must be the value(s) of *k* so that $x^2 + 8kx + 1$ will have two different real zeros?

48. Perform a binomial expansion on $(2p - 5q^2)^4$.