Blue Pelican

Alg II - PreCal Enrichment Topics

Student Version 1.01

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Enrichment Topics

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Topic C: Point-slope and intercept forms of a line

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Enrichment Topic A

(Analysis of absolute value inequalities)

In the following two examples, we show how to **rewrite the problem without using the absolute value symbols**. From that point, the problem can be solved by techniques with which we are already familiar.

Example 1: $|4(x - 2)| < 3$ When an absolute value is **<** or **≤** a number, create two separate inequalities as follows and take the intersection (**and**) of the two answers.

Example 2: $|3x + 2| \geq 3$ When an absolute value is **>** or **≥** a number, create two separate inequalities as follows and take the union (**or**) of the two answers.

Assignment: In each problem, solve for the variable and draw the solution on a number line.

1. $|2t - 3| \leq 4$

2. $|2(p + 2) - 3| > 5$

3. $|3[2(x - 1) - x]| < 5$

4. $|12x - (x - 1)| \geq 11$

5. $8 > |x(2 - 7) + 2|$

6. $|4(1 + m)| \geq 22$

Enrichment Topic B

(An application of linear inequality systems in two variables …Linear Programming)

Systems of linear equations in two variables can often represent real life situations, especially in the business world. These inequalities are known as "restrictions" and when graphed represent an appropriate area for operation.

After a "shaded region" is produced and the corner points identified, a function (often a profit function) of the two variables can be minimized or maximized by evaluating the function at the corner points, and then by inspection, demonstrating which is greatest or smallest. **This process is known as linear programming.**

An example from the business world is shown on the next page. Linear programming is used to solve it.

Example:
A custom auto maker has the facilities to produce a total of 22 vehicles a month (cars and trucks). Parts for the trucks cost $15,000 each and $10,000 for a car. Due to tight credit, they can only borrow a total of $240,000 per month for parts.

The labor cost per truck is $2,000 and the labor for a car is $1,800. Labor costs are restricted to only $75,000 per month; otherwise, it would be necessary to pay overtime which would cut excessively into profit margins.

Create three inequalities from the above data using *x* as the number of cars and *y* as the number of trucks. The assumption is made that x ≥ 0 and y ≥ 0 since it would make no sense to consider a negative number of cars or trucks. We need not graph these two inequalities; just restrict all graphs to the first quadrant.

Assuming a profit of $3200 per car and $2300 per truck, what should be the production of each per month in order to realize the greatest total profit?

```
        Plot2  Plot3
\Y1目 -X+22
\Y2目 -(2/3)X+16
\Y3目 -.9X+37.5
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=30
 Xscl=5
 Ymin=0
 Ymax=30
 Yscl=5
 Xres=1
```

Assignment:

1. A football team runs no more than 46 running and passing plays per game. The coach insists on no more than 30% passing plays.

Let *x* be the number of runs per game and *y* be the number of passing plays. Express the solution to the above system of inequalities as a shaded region on an x-y coordinate system.

2. Assume for this particular team that a run results (on the average) in .12 points on the score board while a pass results in .3 points. Examine the corner points of the shaded region from problem 1 and determine the optimum number of runs and passes so as to maximize the number of points earned by the team per game.

Enrichment Topic C

(Point-slope & Intercept forms of a line)

The equation of a line can be easily given if we have the slope, *m*, of the line and a point, *(x₁, y₁)*, on the line:

$$y - y_1 = m(x - x_1)$$ **Point-slope form of a line**

Example 1: What is the equation of a line (in point-slope form) having the same slope as $y = 3x - 5$ and passing through $(-12, 5)$?

If the x-intercept, *a*, and y-intercept, *b*, of a line are known, then the equation of the can be given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$ **Intercept form of a line**

(Note that neither *a* nor *b* can be 0)

Example 2: What is the equation of a line in intercept form that passes through the y-axis at $(0, -5)$ and passing through the x-axis at $(-4,0)$?

Example 3: Express y = 4x –2 in intercept form.

Example 4: Express y – 5 = 2(x + 6) in intercept form.

Assignment: In problems 1-4, find the equation (in point-slope form) of the line with the given characteristics.

1. Passing through (3, –4) and parallel to the line with equation x – y = 4.	2. Having root –2 and slope 5/4.
3. Perpendicular to y = (1/2)x and passing through (5, 6).	4. Passing through the vertex of the parabola given by y = –2(x + 5)^2 – 6 and having slope –7.

In problems 5-8 find the equation of the line (in intercept form) with the given characteristics.

5. Having a root at 4 and intersecting the y-axis 3 units below the origin.	6. Passing through the points (5, 0) and (0, –14).

7. The line connecting the points at which $y = (x + 11)^2$ touches the x and y axes.

8. Having y-intercept 3 and x-intercept 50.

9. A line has equation $\dfrac{-x}{4} - \dfrac{y}{-7} = 1$. What are the x and y intercepts?

*10. A line has equation $\dfrac{x}{5} - \dfrac{3y}{7} = 2$. What are the x and y intercepts?

11. Express $y = -8x + 3$ in intercept form.

12. Express $y + 1 = 2(x - 9)$ in intercept form.

Enrichment Topic D

(The summation operator, Σ)

Consider the expression $\sum_{k=1}^{3}(k^2 + 1)$.

Read this expression as, "The summation from k equal one to three of k squared plus one."

Σ is the Greek letter, capital sigma.

Example 1: Expand and simplify $\sum_{k=1}^{3}(k^2 + 1)$.

Example 2: Expand and simplify $\sum_{i=3}^{6} x^i y^{6-i}$.

Assignment:

1. Expand and simplify $\sum_{m=0}^{3}(3m^2 - 2)$.

2. Expand and simplify $\sum_{p=-1}^{4}(p + 1)$.

3. Expand and simplify $\sum_{k=11}^{14}\left(\frac{k-3}{k-1}\right)$.

4. Expand and simplify $\sum_{z=3}^{5} x^{z+1}y^{6-z}$.

5. Expand and simplify $\sum_{i=2}^{5} 7e^{-i+1}$. Express the final answer with all positive exponents.

Enrichment Topic E

(An unusual look at probability)

Consider the two probability spinners shown here. We are interested in when the spinners stop on the red sections. The probability of getting a red with the four-segment spinner is **1/4** since there are four different colored segments but only one that is red. Similarly, the five-segment spinner's probability of coming up red is **1/5**.

The problem we are to solve here is to find the probability of getting **at least one red** when considering **both spinners simultaneously**.

Experimental determination:
With two students operating the spinners, have them spin both devices while the third records a tally on the board of **both** the total number of trials **and** the number of successes. Emphasize that a **success could either be just one red or both red**.

Do at least 20 trials and more if time permits. Make sure the students understand that the greater the number of trials done, the closer will be the agreement of this experimental probability (**successes/trials**) with the theoretical probability.

Trials	Successes
ЖЖ ЖЖ ЖЖ ЖЖ	ЖЖ IIII

Theoretical determination:
Begin with a discussion and emphasis on the fact that the sum of the probabilities of all possibilities is one. This may be a new concept to students, so make sure they understand since it is at the very heart of this lab.

As an example, students might relate to a weather prediction of 75 % rain. Of course, this means a 25% chance of no rain. Point out that these two probabilities add to 100% which is equivalent to a probability of one.

With regard to getting a red with the two spinners, all possible probabilities are:

P_0 = probability of no reds
P_1 = probability of exactly one red
P_2 = probability of two reds

Expressing the sum of these probabilities, we get the fundamental equation with which we will solve our problem:

P_0 + P_1 + P_2 = 1

At this point it is important for the students to realize that the probability we want (the probability of getting at least one red) is the **probability of getting exactly one red plus the probability of getting two reds**. This can be expressed as:

P_1 + P_2

Rewrite the "sum" equation, isolate these two terms in parenthesis, and then subtract P_0 from both sides:

P_0 + (P_1 + P_2) = 1
(P_1 + P_2) = **1 − P_0**

Instead of finding two probabilities (P_1 & P_2) and adding them, we have the easier task of finding P_0 and subtracting it from 1:

P_0 = (prob of no red on spinner$_5$) X (prob of no red on spinner$_4$)

$$P_0 = \frac{\# \, of \, non \, reds}{5 \, spaces} \, X \, \frac{\# \, of \, non \, reds}{4 \, spaces}$$

$$P_0 = \quad \frac{4}{5} \quad X \quad \frac{3}{4} \quad = \frac{12}{20}$$

Finally, the solution is:

$$P1 + P2 = 1 − P_0 = 1 − \frac{12}{20} = \frac{8}{20} = \frac{2}{5} = .4$$

Assignment:

Suppose in addition to the 4 and 5 color spinners, we additionally have a 6 color spinner that also has just one red section. Spinning all three simultaneously, what would be the probability of at least one red?

Enrichment Topic F

(Rotations)

Example 1: Consider the point (0, 5) being rotated $90°$ ccw about the origin. Plot the original point and its new rotated position. What are the new coordinates?

Example 2: Consider the triangle with vertices (0, 5), (5, 0), and (-5, 0) rotated $90°$ cw about the origin. Draw the original triangle and the new rotated version.

A point given by (x, y) can be rotated about the origin by an angle θ to a new location (x', y') given by the **rotation formulas** (read x' as "x prime"):

$x' = x\cos(θ) - y\sin(θ)$ where θ is the rotation angle (positive, ccw)
$y' = x\sin(θ) + y\cos(θ)$

Example 3: Produce the answer to Example 1 using the rotation formulas.

Assignment:

1. Plot the point (−4, 0) and plot again after manually rotating 270° cw. What are the coordinates of the new point?

2. Produce the answer to problem 1 using the rotation formulas.

3. Draw a square with vertices (2,2), (−2,2), (−2, −2), (2, −2). Then draw this same square after it has been rotated 45° ccw about the origin.

4. Draw a square centered at the origin and with diagonals (of length 2) on the two axes. Then draw this same square after being rotated 45° cw about the origin.

5. From problem 4, determine the new location of the original bottom vertex using the rotation formulas.

6. Draw a square with vertices (−2, 2), (−4, 2), (−4, 0), (−2, 0). Then draw the same square after manually being rotated about the origin 180° ccw.

Enrichment Topic G

(Absolute value parent function)

The parent function y = |x| graphs as follows: (Verify by substituting in x = 0, 1, −1, 2, −2, 3, −3, etc.)

The following examples involve graphing absolute value on a graphing calculator. See **Calculator Appendix D** and a related video for details on graphing absolute value functions.

Example 1: Simultaneously graph both y = |x| and y = −|x| on a graphing calculator. Make a sketch of the display, labeling each function. What is the general effect of multiplying the parent function by –1?

Example 2: Simultaneously graph both y = |x| and y = 2|x| on a graphing calculator. Make a sketch of the display, labeling each function. What is the general effect of multiplying the parent function by 2?

Example 3: Simultaneously graph both y = |x| and y = |x + 4| on a graphing calculator. Make a sketch of the display, labeling each function. What is the general effect of replacing *x* in the original parent function with ?

Example 4: Simultaneously graph both y = |x| and y = |x| + 3 on a graphing calculator. Make a sketch of the display, labeling each function. What is the general effect of adding 3 to the original parent function?

Example 5: One at a time and starting with the original parent function, y = |x|, perform the transformations to produce a sketch of the final transformed function, y = −2|x + 4|+ 3. (Do in the sequence stretch, reflect, translate.)

Example 6: Describe the transformations performed on the original parent function that produces $y = f(x) = -7|x + 5| - 6$.

Assignment: For each of problems 1-6, sketch and label the transformed function along with the original parent function:

1. $y = |x| - 2$

2. $f(x) = |x| + 4$

3. $y = -|x|$

4. $y = .4|x|$

5. $g(x) = |x + 4|$

6. $y = |x - 5|$

For each of problems 7-9, sketch the original parent function. Then graph one transformation at a time (in the sequence: stretch, reflect, translate) until the final transformed function is sketched. Label each sketch.

7. $y = 3|x - 4| + 6$

8. $f(x) = -|x + 6| + 2$

9. $m(x) = -8|x| - 2$

In problems 10-13, describe the transformations on the original parent function that would produce the given function:

10. $g(x) = -10|x + 7|$	11. $y = .4|x - 11| + 2$
12. $y = -|x| + 6$	13. $g(x) = |x + 4| - 16$

14. Give the function that matches the graph of this absolute value function.
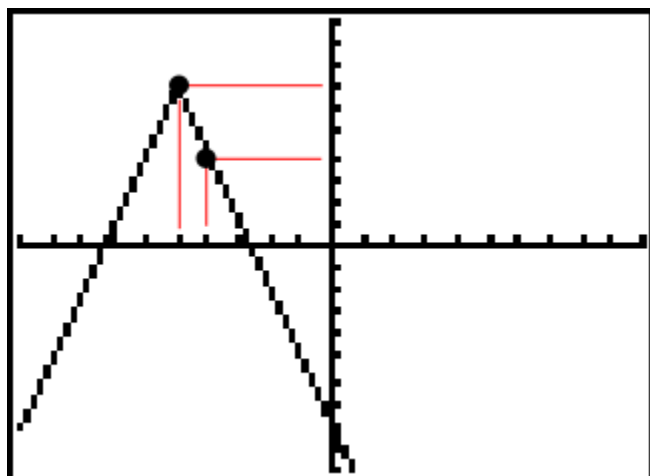
Enrichment Topic H

(Dimension changes affecting perimeter, area, and volume)

Draw a three dimensional rectangular solid and label its dimensions with *L*, *W*, & *H*. Write the formula for area of the front face, the perimeter of the base, and its volume.

Draw another three dimensional rectangular solid and label its dimensions as **twice** that of the previous rectangular solid. Write the formula for the area of the front face, the perimeter of the base, and its volume.

From the examples above we conclude that multiplying the dimensions by **2** …

- multiplies the perimeter by **2**
- multiplies the area by **2**^{2} = 4
- Multiplies the volume by **2**^{3} = 8

Generally, if the dimensions are multiplied by **n**:

- the perimeter is multiplied by **n**
- the area is multiplied by **n**^{2}
- the volume is multiplied by **n**^{3}

Example 1: A pyramid's volume is 36. What would be its new volume if all its dimensions are four times as large?	**Example 2:** A cylindrical tank is 20 m high and 12m in diameter. A new design enlarges all the dimensions by a factor of 2.7. What would be the area of the circular top of the new design?
Example 3: What would be the new volume of the tank described in Example 2?	**Example 4:** If the original pyramid of Example 1 had a base whose perimeter was 13, what would be its new perimeter?

Example 5: A rectangular solid has a volume of 200cm^3. What would be its new volume if the dimensions were all divided by a factor of 5?

Assignment:

1. A rectangle has dimensions L X W. What would be its new perimeter if both dimensions are quadrupled?

2. A rectangle has dimensions L X W. What would be its new area if both dimensions are now twice as long?

3. A sphere has a diameter given by *d*. If the sphere is enlarged so that its new diameter is *4.2d*, how much larger is the new surface area as compared to the old surface area?

4. A sphere has a diameter given by *d*. If the sphere is enlarged so that its new diameter is *4.2d*, how much larger is the new volume as compared to the old volume?

5. A sphere has a diameter given by *d*. If the sphere is shrunk so that its new diameter is *.5d*, how much smaller is the new surface area as compared to the old surface area?

6. A sphere has a diameter given by *d*. If the sphere is shrunk so that its new diameter is *.5d*, how much smaller is the new volume as compared to the old volume?

7. The perimeter of a city is p. If a city with a similar shape has twice the perimeter, how much larger is the area of the larger city as compared to the smaller city?

8. If the price of a rectangular parcel of land is $50,000, what should be the price of parcel of land in the same neighborhood if its dimensions are half as much as the larger one?

*9. A cube has dimensions x. By what factor is the volume of a new rectangular solid larger if the original cube retains all of its dimensions except its height, which is multiplied by a factor of 3?

Enrichment Topic I

(**Algebraic solution to three equations in three variables**)

Consider the three dimensional equation $3x - 4y + 6z = 12$. What would this look like if graphed in three dimensions? Most would guess a line. Actually, it's a plane.

Find the three axis intercepts by setting the other two variables to 0.

Now what if we have a system of such equations as follows? What is the meaning of the solution to this system? It's the **intersection point of the three planes**.

$$x - 3y - 2z = 9$$
$$3x + 2y + 6z = 20$$
$$4x - y + 3z = 25$$

This system can be solved by using Cramer's rule (determinants) or with an inverse matrix. In the example below, we will demonstrate an algebraic technique.

Example 1:

Example 1 (continued):

Assignment: Solve the following systems for the three variables.

1. x − y + z = 12
    x + y + z = −2
    x + 2y + 3z = −3

2. $2x + 3y + z = 1$
$-x + 4y + 2z = 15$
$x - 2y - z = -10$

Enrichment Topic J

(Algebraic solution to quadratic systems of equations)

Consider the solution to the following system:

$$x^2 - y^2 = 4 \quad \ldots\ldots\ldots \text{ a hyperbola}$$
$$x^2 - 4x + y^2 = 0 \ldots. \text{ a circle}$$

In general, a hyperbola and circle could intersect in five ways (0, 1, 2, 3, or 4 pts)... the 3 pt picture is not shown. Can you draw it?

Example 1: Solve the system above.

Assignment: Solve for the intersection points.

1. $x - 3y = -5$
    $x^2 + y^2 = 25$

2. $x^2 + 4y^2 = 16$
   $x^2 + y^2 = 9$

3. $x^2 \qquad + y^2 = 4$
    $x^2 - 2y + y^2 = 0$

Enrichment Topic K

(Derivation of the Sine Law)

Consider the following two triangles in which a perpendicular has been dropped from vertex B to side AC. The reason we consider two triangles is so that we cover both cases: when the perpendicular falls **inside** the triangle and when it falls **outside**.
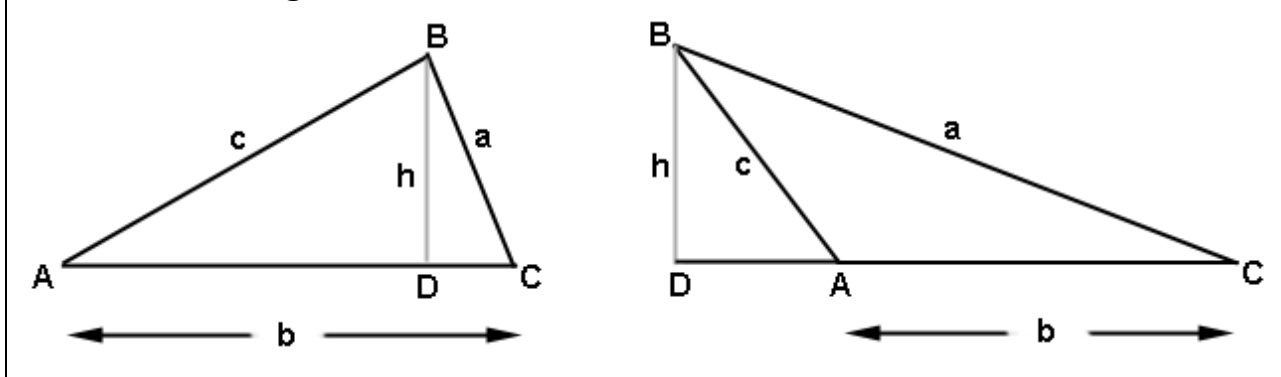
From either figure above, solve for *h* from triangle BCD in terms of *a* and *C*:

Using either triangle ABD from the left figure or triangle ABD from the right figure, solve for *h* in terms of *c* and *A* (Notice in the right picture, angle DAB and angle CAB have the same sine.):

Set the two equations for *h* equal to each other:

Similarly, it can be proved that $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ and $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

Enrichment Topic L

(Derivation of the Cosine Law)

Consider triangle ABC in which a perpendicular is dropped from B to the line described by opposite side. The left drawing below is when this perpendicular falls inside the triangle and the right picture shows it falling outside the triangle. Both scenarios will be considered in this derivation.
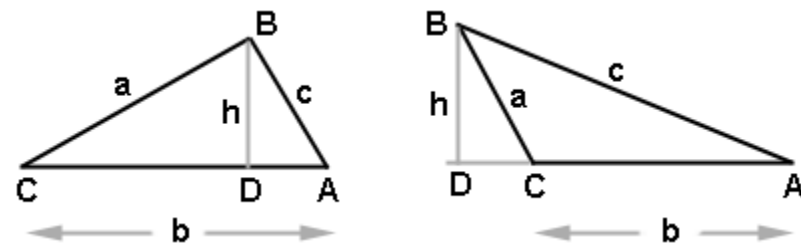
Applying the Pythagorean theorem to either right triangle, we get:

(1)

Solve for *h* from either right triangle and get:

(2)

Solve for DA in terms of a, b, and C

(3)

Now substitute equations (2) and (3) into equation (1) and get:

Enrichment Topic M

(Tangent composite function derivations)

Derivation 1: Begin with $\tan(A + B) = \dfrac{\sin(A+B)}{\cos(A+B)}$ and derive:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Derivation 2: Begin with $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \, \tan B}$ and derive:

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

Derivation 3: Begin with $\tan(A/2) = \dfrac{\sin(A/2)}{\cos(A/2)}$ and derive:

$$\tan(A/2) = \frac{\sin A}{1 + \cos A}$$

Enrichment Topic N

**Locating the vertex
of a standard-form parabola**

A quadratic function (parabola)
in standard-form is:

Suppose the parabola looks like
this where it crosses the x-axis in
two places:

Notice that the axis of symmetry is halfway between the two roots and
that the vertex lies on this line.

From the quadratic formula, the two roots are given by:

$$r_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad r_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

To find the x-value halfway between these two roots, simply average
them.

$$x_{avg} = \frac{r_1+r_2}{2}$$

Therefore, the *x* coordinate of the vertex is **x** $= \dfrac{-b}{2a}$

To find the *y* coordinate of the vertex, simply evaluate the quadratic function at –b/(2a):

The coordinates of the vertex are: $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a} \right) \right)$

The y-intercept of the parabola in standard-form is easily found by setting x = 0 and solving for *y*.

Enrichment Topic O

(Algebraic manipulation of inverse trig functions)

It must be remembered that all inverse trig functions are **angles**. For example, when we see something like $\text{Cos}^{-1}(x)$, it's a good idea to **set it equal to an angle**:

$$\text{Cos}^{-1}(x) = \theta$$

Example 1: Find the value of $\sin(\text{Cos}^{-1}(x))$

Example 2: Find the value of $\sin(\text{Sin}^{-1}(x))$

Example 3: Find the value of $\text{Sin}^{-1}(\sin(\phi))$

Example 4: $\tan(2\sin^{-1}(u))$

Example 5: Express $\sin^{-1}(p) - \cos^{-1}(p)$ as an inverse tangent.

Example 6: Prove Arcsin(x) = π/2 –Arccos(x)

Assignment: Evaluate the given expression. Both *u* and *v* are positive numbers.

1. sin(Sin^{-1}(3/19))

2. Tan^{-1}(tan(32°))

3. csc(Arccos(–3/5))

4. cos(Arccos(u) + Arccos(v))

5. cos(.5(Arccos(–u)))

6. cot(Arccot(v))

In the following problems, assume that θ is a positive acute angle. Find the inverse function value.

7. Arccos(sin(–θ))	8. Arcsin(–sin(–θ))

In the following two problems, express each sum or difference as a single angle.

9. Arcsin(3/5) + Arccos(3/5)	10. Arcsin(5/13) – Arcsin(3/5)

Enrichment Topic P

(Logarithm theorem derivations)

The following three theorems are derived in this topic:

- $\log_b(PQ) = \log_b P + \log_b Q$
- $\log_b(P/Q) = \log_b P - \log_b Q$
- $\log_b P^n = n \log_b P$

All three derivation begin with the premise that both P and Q can be written as powers of the base b:

$$(1) \qquad P = b^x \qquad \text{and} \qquad Q = b^y$$

Each of these two equations can equivalently be written in log form:

$$(2) \qquad x = \log_b(P) \qquad \text{and} \qquad y = \log_b(Q)$$

Derivation of $\log_b(PQ) = \log_b P + \log_b Q$:

Begin by writing the $\log_b$ of the product PQ. Then replace P and Q with the corresponding expressions in (1) above.

Derivation of $\log_b(P/Q) = \log_b P - \log_b Q$:

      Begin by writing the $\log_b$ of the quotient P/Q. Then replace P and Q with the corresponding expressions in (1) above.

Derivation of $\log_b(P^n) = n \log_b P$:

      Begin by writing the $\log_b$ of the quotient P^n. Then replace P with the corresponding expression in (1) above.

Enrichment Topic Q

(Arithmetic and geometric sum formulas)

Arithmetic sum, $S_n = (n/2)(a_1 + a_n)$:

Consider the arithmetic sum $3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$.

Finite geometric sum, $S_n = a_1(1 - r^n)/(1 - r)$,   where $r \neq 1$

Begin by expanding a geometric series, call it S_n. Produce a new series (from $a_n = ar^{n-1}$) by multiplying the original by $-r$, call it $-rS_n$. Add the two equations and solve for S_n as follows:

Infinite geometric sum, $S_\infty = a_1/(1-r)$, where $|r| < 1$:

Begin with $S_n = a_1(1 - r^n)/(1 - r)$ and find the limit as $n \to \infty$.

Enrichment Topic R

**Converting general form
conic sections to standard form
(Completing the square)**

In the following examples, convert the given general form conic section into its standard form, identify the conic section, make a sketch of its graph, and label completely.

Example 1: $x^2 + 10x + 8y - 7 = 0$

Example 2: $3x^2 + 3y^2 - 6x + 24y + 24 = 0$

Example 3: $3x^2 + 2y^2 - 8y + 24x + 32 = 0$

Example 4: $4y^2 - x^2 + 2x + 24y + 19 = 0$

Assignment: In the following problems, convert the given general form conic section into its standard form, identify the conic section, make a sketch of its graph, and label completely.

1. $9x^2 + 25y^2 + 36x - 150y + 36 = 0$

2. $y^2 - 4x - 8y + 28 = 0$

3. $x^2 + y^2 - 2x + 2y - 23 = 0$

4. $x^2 + 4y^2 - 10x - 40y + 121 = 0$

5. $4y^2 - 16y - 9x^2 - 36x - 164 = 0$

6. $x^2 + 4x - 8y + 12 = 0$

7. $25x^2 - 4y^2 + 16y - 116 = 0$

8. $x^2 + y^2 + 14x + 45 = 0$

Enrichment Topic S

(Conic Section Applications)

Applications of the four conic sections are beautifully illustrated at:
http://britton.disted.camosun.bc.ca/jbconics.htm

Applications of the **circle**:

No applications are given for this obvious category since they abound in everyday life.
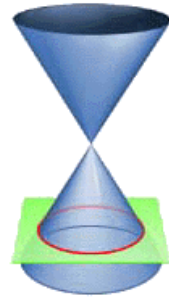
Applications of the **parabola**:

- Some comets pass around the sun in a parabolic path.
- Projectiles follow a parabolic path (a tossed ball).
- The mirrors in flashlights and reflecting telescopes are parabolic in shape.
- The cables of a suspension bridge hang in the shape of a parabola
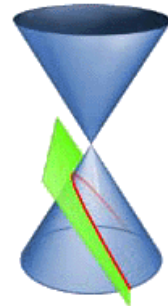
Applications of the **ellipse**:

- Planets orbit about the sun in an elliptical path with the sun at one of the foci.
- Light passing through one focal point will reflect off the parabola and pass through the other focal point. The same thing happens with sound as evidenced in "whispering galleries".
- A right circular cylinder sliced at an angle produces an ellipse
- Elliptical gears provide variable rotational speed about an axis passing through a focal point.
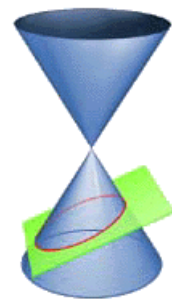
Applications of the **hyperbola**:

- Some comets pass around the sun in a hyperbolic path.
- Charged particles fired at heavy nuclei of atoms are deflected into a hyperbolic path.
- Hyperbolic tracking (a navigational technique called Loran) uses intersecting hyperbolas to locate the positions of airplanes and ships.
- The shock wave of a sonic boom intersects the ground in the shape of a hyperbola.
- Hyperbolic as well as parabolic mirrors and lenses are used in systems of telescopes.
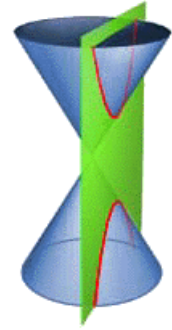
See www.youtube.com/watch?v=XDLyiEWcj_Y for a humorous video about the applications of conic sections.

Enrichment Topic T

A Close Look at Composite Functions
(Restrictions on the Domain)

Typical syntax for a composite function is *f (g(x))* or, equivalently, $(f \circ g)(x)$.

Example 1: If f(x) = 3x^2 + 2x - 1 and g(x) = 12x − 3, find f(g(x)).

Composite functions are not as exotic or contrived as one might think.

For example, consider the amount of fun *f* a person might have at a birthday party as a function of the number of games played *g.* The functional relationship is expressed as *f(g)*. The number of games *g* is, in turn, a function of the planning *x* spent on the party.

This means that the amount of fun *f* is ultimately a function of the amount of planning *x*. This is expressed as the **composite function**, ***f(g(x))***.

Notice that the range of *g(x)* becomes the domain of *f(x)*. Think of the **output** of g(x) as being the **input** for f(x).

General rules for determining the domain of a composite function, f(g(x)) = (f ∘ g)(x):

The domain of *f* must be a subset of the domain of *g*, and the domain of the composite function must include only those *x* values for which *g* maps into the domain of *f.* This leads to the following methodical method of finding the domain of the composite function f(g(x)) = (f ∘ g)(x):

- Find the domain of the "inner" function *g(x)*.
- Find the domain of the composition.
- Find the intersection of these two domains.

In determining domain, it is sometimes easier to think of what must be excluded from the domain. The following should always be excluded:

- Values of *x* that cause a denominator to be zero
- Values of *x* that cause an even root to be taken of a negative number

There are, of course, other cases such as values that cause trig functions to be undefined, etc.

Example 2: Find the composite function f(g(x)) = (f ∘ g)(x) where $f(x) = \sqrt{x}$ and g(x) = x − 2 and then specify its domain.

Example 3: Find the composite function f(g(x)) = (f ∘ g)(x) where f(x) = $\sqrt{\dfrac{1}{x+2}}$

and g(x) = $\dfrac{2x}{x-3}$ and then specify its domain.

Example 4: : Find the composite function f(g(x)) = (f ∘ g)(x) where f(x) = $\sqrt{x-1}$

and g(x) = $\dfrac{1}{x-1}$ and then specify its domain.

Example 5: Determine two functions f(x) and g(x) which, when composed, generate f(g(x)) = $(5x + 2)^3 - 3(5x + 2) - 11$.

Example 6: If the function f(x) is known to have a root at x = 19, to what *x* value will that root be moved with the composition f(g(x)) where g(x) = 6x − 9?

Assignment:

1. If $f(x) = 3x^2 + 2x - 1$ and $g(x) = 12x - 3$, find $g(\,f(x)\,)$.

2. Find the composite function $g(\,f(x)\,) = (g \circ f)(x)$ where $f(x) = \sqrt{x}$ and $g(x) = x - 2$ and then specify its domain.

3. Find the composite function g(f(x)) = (g ∘ f)(x) where $f(x) = \sqrt{\dfrac{1}{x+2}}$ and

g(x) = $\dfrac{2x}{x-3}$ and then specify its domain.

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4. Find the composite function f(g(x)) = (f ∘ g)(x) where $g(x) = \sqrt{x-4}$ and f(x)
    = $\dfrac{16}{x-2}$ and then specify its domain.

5. Determine two functions f(x) and g(x) which, when composed, generate
f(g(x)) = $4 (\sqrt{7x-2})^2 - 8(\sqrt{7x-2}) - 22$.

6. If the function h(x) is known to have a root at x = -1 , to what *x* value will that root be moved with the composition h(p(x)) where p(x) = x + 6?

7. Find f ∘ g where f(x) = $11x^2$ and g(x) = $\sqrt{x}$. What is the domain of this composite function?

8. Find $g \circ f$ where $f(x) = 11x^2$ and $g(x) = \sqrt{x}$. What is the domain of this composite function?

9. Find $f \circ f$ where $f(x) = \sqrt{x + 1}$. What is the domain of this composite function?

Enrichment Topic U

"Box" Method of Trinomial Factoring

Example 1: Use the box method to find the factors of $x^2 + 2x - 48$. Begin by placing the x^2 and -48 terms in the box and producing a product.

Next, find two terms whose product is that given above ($-48x^2$) and whose sum is 2x, and then fill in the other two positions of the box with these two terms.
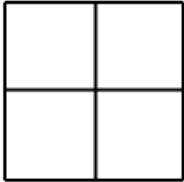
Now place the GCF of each row to the left of the row. Place the GCF of each column above the column. Finally, use these GCFs to produce the factors as shown here:
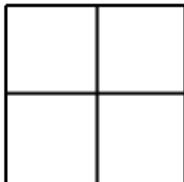
Example 2: Use the box method to find the factors of $6x^2 - 17x + 5$. Specify the product and sum that were used in arriving at the answer.
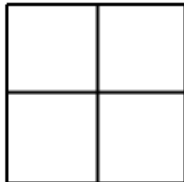
Product:

Sum:

Example 3: Use the box method to find the factors of $3x^2 + 13x - 10$. Specify the product and sum that were used in arriving at the answer.
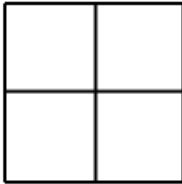
Product:

Sum:

Important sign rule!

If the item in either cell indicated here (or both) is negative, then the corresponding GCF adjacent to it will be **negative**; otherwise the GCF is positive.
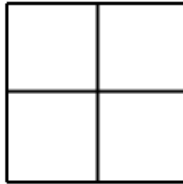
Assignment: Use the box method to find the factors of the given trinomial.
Specify the product and sum that were used in arriving at the answer.

1. $3x^2 - 2x - 5$
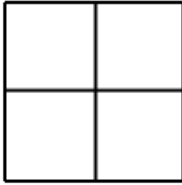
Product:

Sum:

2. $y^2 + 5y - 24$
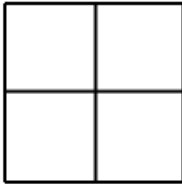
Product:

Sum:

3. $5m^2 - 13m - 6$
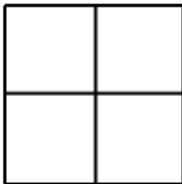
Product:

Sum:

4. $x^2 + 7x + 12$
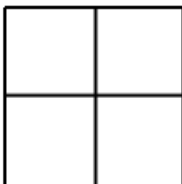
Product:

Sum:

5. $2z^2 + 5z - 12$

Product:

Sum:

6. $x^2 + 6x + 5.$

Product:

Sum:

7. $p^2 - p - 12$
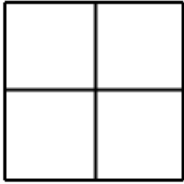

Product:
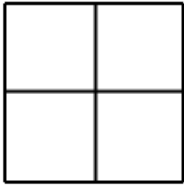
Sum:

8. $x^2 + 3x - 18$

Product:

Sum:

9. $k^2 + 3k + 2$

Product:

Sum:

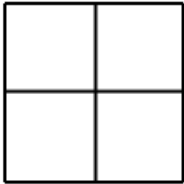10. $10x^2 - 7x + 1$

Product:

Sum:
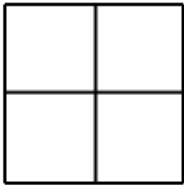
11. $4x^2 + 19x + 21$

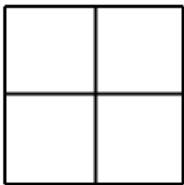Product:

Sum:

12. $t^2 - 3t - 70$

Product:

Sum:

13. $p^2 + 12p + 32$

Product:

Sum:

14. $2x^2 - 7x + 6$

Product:

Sum: