Blue Pelican Geometry

First Semester

Student Version 1.01

Authors: Charles Cook, Matthew Lorber, and Jordan Brooks

Geometry Syllabus (First Semester)

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Geometry, Unit 1

Algebra Review

Unit 1:
Lesson 01

Solving linear equations and inequalities

Example 1: Solve $x + 2 = 19$

Example 2: Solve $2x = 46$

Example 3: Solve $4x – 8 = 40$

Example 4: $4x – x + 12 = 0$

Example 5: Solve $11h – 7 = 2h + 1$

Example 6: $4(x – 3) = 8$

Example 7: Solve $-3y – 4(6y + 2) = y – 9$

Example 8: Solve (1/6)g + 1/3 = (1/2)g – 1

Inequalities are solved exactly like equations with this exception:

> If both sides of the inequality are either multiplied or divided by a **negative** quantity, the **inequality symbol must be reversed**.

Example 9: Solve and graph 5p – 8 ≥ 2.

Example 10: Solve and graph
4k + 11 < 6k + 19.

Fundamental to all of algebra is knowledge and immediate recall of all sign rules. See **In-Depth Topic A** for practice with the sign rules.

Assignment: Solve for the variable in each equation or inequality. Graph the inequalities.

1. $13h - 4 = 22$	2. $4x - 9 = -3$
3. $5(3 - 2e) + e = 11$	4. $(1/2)x + 1 = 12$
5. $114 = (x + 2)15 - 3x$	6. $p - 12 = 4p + 21$

7. $-3f + 2 = 3(f - 2) - 8f$

8. $\frac{4x+1}{3} - 1 = 12$

9. $8x < x - 14$

10. $-x + 1 \geq 6x + 22$

11. $-2(x - .5) < x - 1$

12. $4(u - 3) \leq 5(u - 6)$

13. $.3(x - 5) > -60$

14. $2b + 3b - 1 = 8b - 13$

Unit 1: Lesson 02 Solving systems of two linear equations

A system of linear equations that we will consider here consists of two linear equations whose graphs (lines) generally intersect.

The (x, y) **point of intersection** is considered the **solution** of the system.

We will consider two techniques for solving such a system:

- The **elimination method** (sometimes called the addition method)
- The **substitution method**

Example 1: Solve this system using the elimination method.

$-2x + 3y = 11; \quad 2x + y = 1$

Example 2: Solve this system using the elimination method.

$2x - 3y = 4 \; ; \; x + 4y = -9$

Example 3: Solve this system using the substitution method.

$y = 3x + 4$
$x – y = 2$

Example 4: Solve this system using the substitution method.

$x – 3y = 4$
$2x + 7y = -5$

Assignment: Solve the following linear systems using the substitution method.

1. $x + y = 8$; $y = 3x$

---

2. $y = 3x - 8$; $x + y = 4$

---

3. $3x - 5y = 11$; $x = 3y + 1$

---

4. $x + 4y = 1$; $2x + y = 9$

5. $2a + 7b = 3$; $a = 1 - 4b$

6. $p - 5q = 2$; $2p + q = 4$

7. $-4a + 5b = 17$; $5a - b = 5$

Solve the following linear systems using the elimination method.

8. $4x - 3y = -2$; $2x + 3y = 26$

---

9. $a - b = 4$; $a + b = 8$

---

10. $2x - 5y = -6$; $2x - 7y = -14$

11. $3x + y = 4$; $5x - y = 12$

12. $5p + 2q = 6$; $9p + 2q = 22$

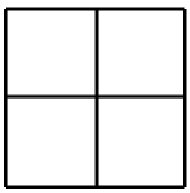
13. $5x + 12y = -1$; $8x + 12y = 20$

14. $3x - 4y = 8$; $4x + 3y = 19$

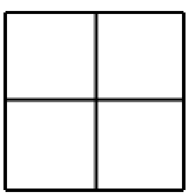
Unit 1: Lesson 03 Trinomial factoring

Example 1: Use the box method to find the factors of $x^2 + 2x - 48$. Begin by placing the x^2 and -48 terms in the box and producing a product.

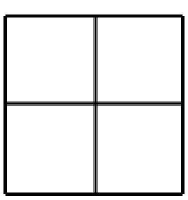
Next, find two terms whose product is that given above ($-48x^2$) and whose sum is 2x, and then fill in the other two positions of the box with these two terms.



Now place the GCF of each row to the left of the row. Place the GCF of each column above the column. Finally, use these GCFs to produce the factors as shown here:



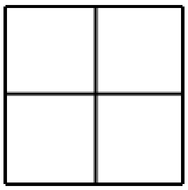
Example 2: Use the box method to find the factors of $6x^2 - 17x + 5$. Specify the product and sum that were used in arriving at the answer.



Product:

Sum:

Example 3: Use the box method to find the factors of $3x^2 + 13x - 10$. Specify the product and sum that were used in arriving at the answer.

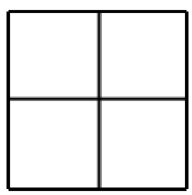


Product:

Sum:

Important sign rule!

If the item in either cell indicated here (or both) is negative, then the corresponding GCF adjacent to it will be **negative**; otherwise the GCF is positive.



Assignment: Use the box method to find the factors of the given trinomial. Specify the product and sum that were used in arriving at the answer.

1. $3x^2 - 2x - 5$

Product:

Sum:

2. $y^2 + 5y - 24$

Product:

Sum:

3. $5m^2 - 13m - 6$

Product:

Sum:

4. $x^2 + 7x + 12$

Product:

Sum:

5. $2z^2 + 5z - 12$

Product:

Sum:

6. $x^2 + 6x + 5$.

Product:

Sum:

7. $p^2 - p - 12$

Product:

Sum:

8. $x^2 + 3x - 18$

Product:

Sum:

9. $k^2 + 3k + 2$

Product:

Sum:

10. $10x^2 - 7x + 1$

Product:

Sum:

11. $4x^2 + 19x + 21$

Product:

Sum:

12. $t^2 - 3t - 70$

Product:

Sum:

13. $p^2 + 12p + 32$

Product:

Sum:

14. $2x^2 - 7x + 6$

Product:

Sum:

Unit 1: Lesson 04

Special factoring formulas
$a^2 - b^2$, $a^2 \pm 2ab + b^2$

Recall from Algebra 1 the shortcut for factoring $a^2 \pm 2ab + b^2$:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Example 1: Multiply $(x - 5)^2$

Example 2: Multiply $(3y + 2b)^2$

Example 3: Factor $x^2 - 8x + 16$

Example 4: Factor $m^2 + 18m + 81$

Recall from Algebra 1 the shortcut for factoring $a^2 - b^2$ (difference of squares):

$$a^2 - b^2 = (a - b)(a + b)$$

Example 5: Multiply $(p - 7y)(p + 7y)$

Example 6: Factor $k^2 - 100$

Assignment:

1. Multiply $(x - 8)^2$

2. Multiply $(5 + b)^2$

3. Multiply $(v - 8)^2$

4. Multiply $(2f + g)^2$

5. Factor $r^2 - 64$

6. Factor $t^2 - 22t + 121$

7. Factor $4v^2 - 12v + 9$

8. Factor $r^2 + 24r + 144$

9. Factor $4b^2 - 49$

10. Factor $4 - 4p + p^2$

11. Factor $81z^2 - 121$

*12. Factor $h^4 - 25$

13. Factor $16x^2 - 8x + 1$

14. Factor $p^2 + 22p + 121$

15. Factor $j^2x^2 - 12jx + 36$

16. Factor $16 + 8x + x^2$

Unit 1: Lesson 05 Solving quadratic equations

A quadratic equation is an equation that can be put in this form:

where *x* is a variable and *a*, *b*, and *c* are constants.

In this lesson we will review the following techniques for solving a quadratic equation.

- Factoring
- Quadratic formula
- Graphing calculator

Example 1: Solve $x^2 + 4x - 12 = 0$ by factoring.

Example 2: Solve $8x^2 - 2x - 3 = 0$ by factoring.

For quadratics that can't be factored, use the quadratic formula:

See **In-Depth Topic B** for a derivation of the quadratic formula.

Example 3: Solve $x^2 + 10x + 24 = 0$ using the quadratic formula.

Example 4: Solve $2x^2 + 3x – 2 = 0$ using the quadratic formula

Quadratic equations are easily solved using a graphing calculator by first graphing the quadratic function and then finding the zeros (where it crosses the x-axis).

Example 5: Use a graphing calculator to find the roots (zeros) of $y = -x^2 - 8x - 10$. Make a sketch of the calculator display and label the roots.

Special cases:

- **Double root:** The parabola is tangent to the x-axis (only touches in one place).
- **No solution:** The parabola never crosses the x-axis. There are no real roots; however, there **are** two roots (imaginary).

Assignment:

1. Solve $x^2 + 7x – 8 = 0$ by factoring.

2. Solve $x^2 – 29x + 180 = 0$ by factoring.

3. Solve $x^2 + x – 90 = 0$ by factoring.

4. Solve $x^2 + 14x + 33 = 0$ by factoring.

5. Solve $3x^2 + x – 14 = 0$ by factoring.

6. Solve $6p^2 – 17p + 12 = 0$ by factoring.

7. Solve $x^2 + 8x - 9 = 0$ using the quadratic formula.

8. Solve $2x^2 - 3x + 1 = 0$ using the quadratic formula.

9. Solve $10x^2 - 9x - 7 = 0$ using the quadratic formula.

10. Solve $2x^2 + 20x + 48 = 0$ using a graphing calculator. Make a sketch of the calculator display.

11. Solve $-x^2 + 12x - 34 = 0$ using a graphing calculator. Make a sketch of the calculator display.

12. Solve $-x^2 - 4x - 5 = 0$ using a graphing calculator. Make a sketch of the calculator display.

13. Solve $x^2 + 14x + 49 = 0$ using a graphing calculator. Make a sketch of the calculator display.

Unit 1: Review

1. Solve $11p - 7 = 2p + 1$

2. Solve $-3y - 4(6y + 2) = y - 9$

3. Solve and graph $5x - 8 \geq 2$.

4. Solve and graph $4k + 11 \leq 6k + 19$.

5. Solve this system using the substitution method.

$$-2x + 3y = 11; \quad 2x + y = 1$$

6. Solve this system using the elimination method.

$y = 3x + 4$
$x – y = 2$

7. Factor $2x^2 + 7x – 30$

8. Factor $x^2 – x – 12$

9. Factor $10v^2 – 7v + 1$

10. Multiply $(h - 3)^2$

11. Multiply $(2x + 5)^2$

12. Multiply $(3x - 7)(3x + 7)$

13. Factor $x^2 - 18x + 81$

14. Factor $y^2 - 25$

15. Factor $c^2 + 4c + 4$

16. Solve $x^2 + 4x - 12 = 0$ by factoring.

17. Solve $2x^2 - 7x + 1 = 0$ using the quadratic formula.

Geometry, Unit 2

Basic Definitions & Concepts
Points, Lines, and Planes

Unit 2:
Lesson 01

Basic definitions (points, lines, & planes)

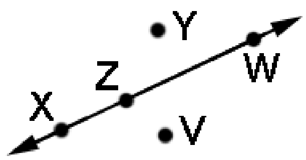
For the study of geometry, we must first establish a vocabulary. This means word **definitions** along with their corresponding **symbols**.

Term / Symbol	Definition
Point	An exact location in space. A point has no size and is infinitely small. Denote points with a capital letter. (A, B, X, Y) Multiple points can be classified as collinear, coplanar. See those definitions below.
Line	An object with no thickness that extends to infinity in two opposite directions. There are infinitely many points on the line. Denote lines with a bar that arrows on both ends. ($\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$) Lines have only one dimension, length.
Line Segment	A portion of a line having two endpoints (and all points in between). ($\overline{PQ}$, $\overline{QP}$)
Ray	A piece of a line having only one end point and extending infinitely far in one direction. ($\overrightarrow{AB}$, $\overrightarrow{AC}$) The arrow indicates the direction in which the ray extends.

Opposite rays	Two rays that share the same end point but go in opposite directions.
Plane	A flat surface that extends infinitely far in all directions within that flat surface. A plane contains an infinite number of points. The plane shown here is denoted with a capital letter, X.
Collinear points	Points that lie on the same line. Any two points are always collinear because they lie on the line joining the two points.
Coplanar points	Points that lie on the same plane. Any three points are always coplanar.

Assignment:

1. Give three points that are all collinear.



2. Which of the following set(s) of points from the drawing in problem 1 are **not** all collinear?

A. Y, X, and Z
B. X, W, and Z
C. Y, W, and V
D. X and V

3. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the front wall?

4. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the ceiling?

5. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the floor?

6. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the left wall?

7. Where would be the **end point** of a **ray** of sunshine?

8. Would a goal line on a football field be described as a point, a ray, a line, a line segment, or a plane?

9. Would the surface of a playing field be described as a point, a ray, a line, a line segment, or a plane?

10. Would the place where the 50 yard-line and the side-line meet on a football field be described as a point, a ray, a line, a line segment, or a plane?

11. Would a knot in a rope most likely be described as a point, a ray, a line, a line segment, or a plane?

12. Would the top of your kitchen table most likely be described as a point, a ray, a line, a line segment, or a plane?

13. Would a wall of your bedroom most likely be described as a point, a ray, a line, a line segment, or a plane?

14. Would the colored dots (pixels) on a computer screen most likely be described as a point, a ray, a line, a line segment, or a plane?

15. Would a star in the nighttime sky most likely be described as a point, a ray, a line, a line segment, or a plane?

16. Would a flashlight beam most likely be described as a point, a ray, a line, a line segment, or a plane?

17. Would a chocolate chip cookie most likely be described as a point, a ray, a line, a line segment, or a plane?

18. Would the speck of chocolate in a chocolate chip cookie most likely be described as a point, a ray, a line, a line segment, or a plane?

19. Would a crease in a folded piece of paper most likely be described as a point, a ray, a line, a line segment, or a plane?

20. Would the path of a bullet most likely be described as a point, a ray, a line, a line segment, or a plane?

21. Would a trampoline most likely be described as a point, a ray, a line, a line segment, or a plane?

22. Suppose you live on the same street as the school. Would the path from your house to the school likely be described as a point, a ray, a line, a line segment, or a plane?

23. Would a fly caught in a spider web most likely be described as a point, a ray, a line, a line segment, or a plane?

24. Suppose a rubber band is stretched forever in both directions (assume it never breaks). Would this most likely be described as a point, a ray, a line, a line segment, or a plane?

25. Draw a line segment and label the endpoints as A and B.

26. Draw and label $\overleftrightarrow{XY}$.

27. Draw and label $\overrightarrow{XY}$.

28. Draw and label $\overline{YZ}$.

Unit 2: Lesson 02

Postulates concerning points, lines, and planes
Practice with points, lines, and planes

A **postulate** is a statement that is **assumed to be true without requiring proof**. Following are some postulates related to points, lines, and planes.

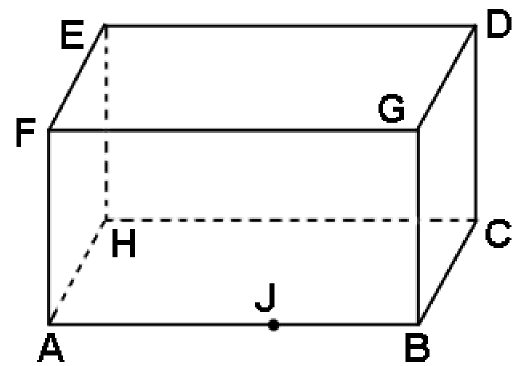
- A line contains at least two points.
- Through any two points there is exactly one line.
- If two lines intersect, then they intersect in exactly one point.
- A plane contains at least three non-collinear points.
- Planes through three points:
  - Through any three points there is **at least** one plane. If the points are collinear there are an infinite number of planes.
  - Through any three non-collinear points there is **exactly** one plane.
- If two points are in a plane, then the line that contains the points is also in the plane.
- If two different planes intersect, then their intersection is a line.

A **theorem** is a statement that must be proved.

Examples of theorems that we will encounter later:

- Vertical angles formed as the result of two intersecting lines are equal.
- The sum of the interior angles of a triangle is 180^{o}.
- The diagonals of a parallelogram bisect each other.
- The diagonals of a rhombus are perpendicular.
- and many more.

Use this drawing to answer the questions in the following examples. When possible, give one of the postulates on the preceding page to support your answer.



Example 1: Considering all the surfaces of the rectangular box, how many planes are shown?	**Example 2:** Name the intersection of planes EFD and DCG.
Example 3: Are points E, J, and C coplanar?	**Example 4:** Do points A and J determine a line?
Example 5: Name the intersection of plane FGB and $\overleftrightarrow{AH}$.	**Example 6:** How many lines are there passing through points A and D?
Example 7: How many planes are there passing through points A, J, and B?	**Example 8:** Name the intersection of $\overleftrightarrow{AJ}$ and $\overleftrightarrow{BC}$.

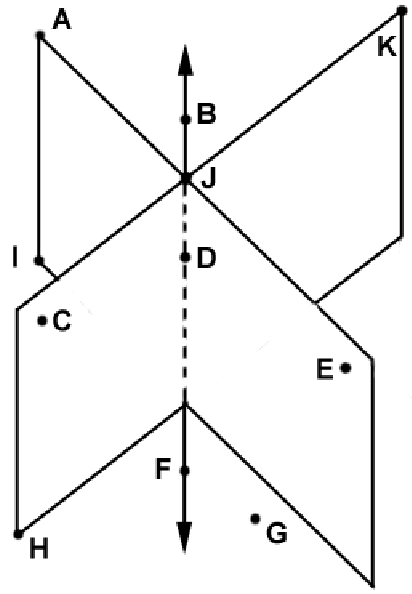
Example 9: Is $\overleftrightarrow{AE}$ in plane AHF?

Example 10: Which plane(s) contain both $\overleftrightarrow{HC}$ and $\overleftrightarrow{CB}$?

Example 11: Does $\overrightarrow{JA}$ point toward the left, right, up, down, front, or back?

Assignment:

Use this drawing to answer the questions in problems 1 - 8.

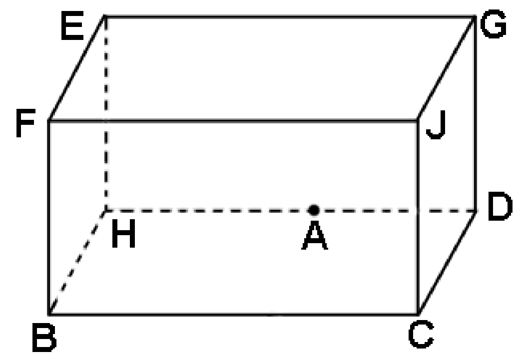


1. What are at least four possible names of the plane that slants from upper left to lower right?	2. Name the line that is the intersection of the two planes.
3. Name all the points that lie in the plane that slants from lower left to upper right.	4. Name a set of at least three collinear points that lie in the plane that slants from upper right to lower left.
5. Name all of the points in the plane that slants from upper left to lower right that are coplanar.	6. In which plane does the line $\overleftrightarrow{KH}$ lie?

7. Are points K, D, and G coplanar?

8. Are points C and E collinear?

Use this drawing to answer the questions in problems 9 - 18.



9. Name three points that are collinear.

10. Name the intersection of planes HBF and GJF.

11. How many planes make up the sides of the box? (Don't count the top or bottom.)

12. Are points H, A, and C coplanar? If possible, describe the plane in terms of a surface of the box.

13. Are points B, D, and E coplanar? If possible, describe the plane in terms of a surface of the box.

14. Does $\overrightarrow{JC}$ point toward the left, right, up, down, front, or back?

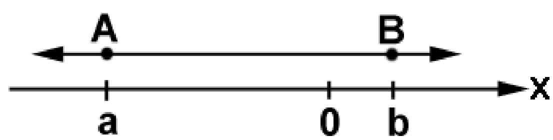
15. Do points A and F form a line? If so, is it an edge of the box?	16. Name the intersection of plane GFJ and $\overleftrightarrow{EH}$.
17. How many planes pass through points H, A, and D?	18. How many points are in plane JGD?
19. A statement that is assumed to be true without proof is called a _______.	20. A statement that must be proven is called a _________.

Unit 2: Lesson 03 — Distance on a number line (length of a line segment)

Recall that a **line segment** consists of two end points and all “in-between” points on the line connecting them.

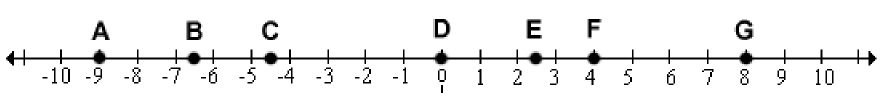
In this lesson we are concerned with the **length** of line segments.

If a line segment lies on a number line and the end points A and B are at coordinates *a* and *b* on the number line, then the **length of the line segment** is simply the **distance between the two points**.



Notice that **AB** now symbolizes the length of $\overline{\mathrm{AB}}$ while $\overline{\mathbf{AB}}$ symbolizes the line segment itself.

Use the points on this number line to find the line segment lengths in examples 1 - 6:



Example 1: AB = ?	**Example 2:** GD = ?

Example 3: FB = ?

Example 4: CE = ?

Example 5: What is the difference between XY and $\overline{XY}$?

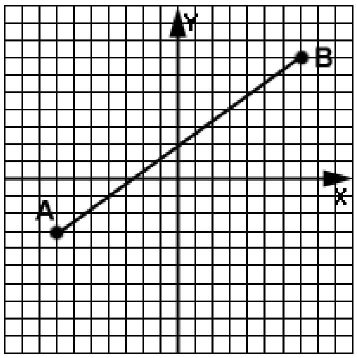
Example 6: What is the length of $\overleftrightarrow{FD}$?

For line segments that do not necessarily lie on a number line and for which the end points (x_1, y_1) & (x_2, y_2) are given, the length of the line segment is given by:

This is known as the **distance formula**.

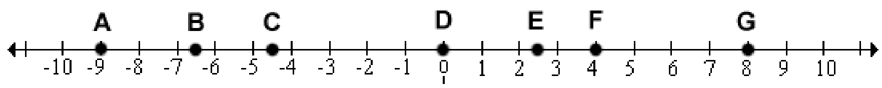
Example 7: Find the length of the line segment whose end points are (4, -2) and (-8, 6).

Example 8: Find the length of line segment $\overline{AB}$.



Assignment:

Use the points on this number line to find the line segment lengths in problems 1 - 6:



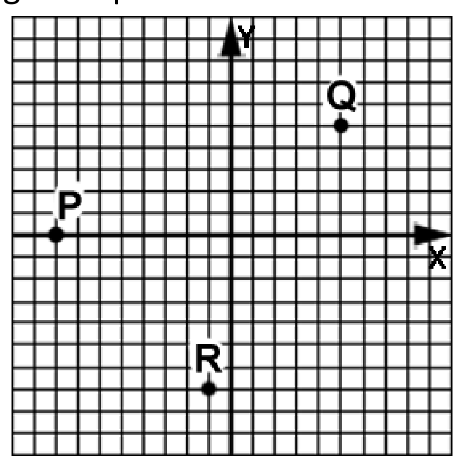
1. EB = ?	2. AE = ?
3. GA = ?	4. FC = ?
5. AD = ?	6. EG = ?

7. What is the length of line segment $\overline{WV}$ where the coordinates of W are (101, -22) and the coordinates of V are (-8, 4)?

8. How far is it between (15.2, 8.6) and (9, -10.11)?

9. Find PQ where P is located at (10,0) and Q at (-5, -12).

Use these points to find the indicated lengths in problems 10 & 11.



10. PQ = ?

11. QR = ?

Unit 2: Lesson 04

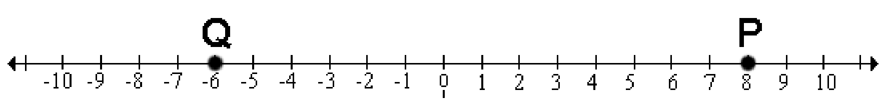
Midpoint of a line segment (midpoint formula)

The **midpoint of a line segment** is the point that is equidistant from both endpoints.

For line segment $\overline{AB}$, midpoint M is located such that AM = MB.

If the line segment $\overline{AB}$ is on a number line where the coordinate of point A is *a* and point B is *b*, then the coordinate of the midpoint *m* is the **average** of *a* and *b*.

Example 1: Find the midpoint of $\overline{PQ}$.



Example 2: Find the midpoint of $\overline{AB}$ on a number line where A is at -2 and B is at -22.

Example 3: Consider line segment $\overline{\mathrm{GH}}$ that lies on a number line. If G is located at -5 and the midpoint at 1, what is the coordinate of H?

Now consider a line segment that does not lie on a number line.

The midpoint coordinates are still found by averaging.

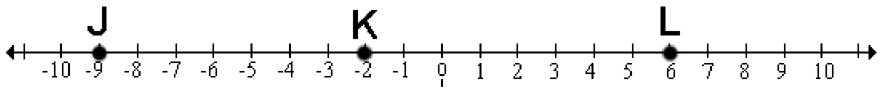
Example 4: If A has coordinates (2,-8) and B has coordinates (-2, 1), find the coordinates of the midpoint of $\overline{\mathrm{AB}}$.

Example 5: If B has coordinates (4,-100) and C has coordinates (6, -2), find the coordinates of the midpoint of $\overline{\mathrm{BC}}$.

Example 6: What are the coordinates of W if the midpoint of $\overline{\mathrm{WV}}$ is at (2, 0) and the coordinates of V are (-10, 8)?

Assignment:

Use the points shown here in problems 1 and 2.



1. Find the midpoint of $\overline{\text{JL}}$.

2. Find the midpoint of $\overline{\text{JK}}$.

3. Find the midpoint of $\overline{\text{AB}}$ when A is located at -18.2 on a number line and B is located at 9.

4. Point P is located at the origin of a number line and Q is 18 units to the left of the origin. Find the midpoint of $\overline{\text{PQ}}$.

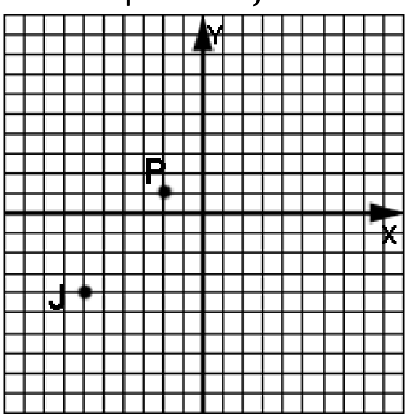
5. The midpoint of line segment $\overline{\text{PL}}$ is located at -4.2 on a number line while L is located at -.5 on the number line. What is the coordinate of P?

6. Line segment $\overline{\text{RL}}$ lies on a number line with point L located at 17. What is the coordinate of R if the midpoint of $\overline{\text{RL}}$ is at -4.6?

7. Point B has coordinates (3, -7) while point F has coordinates (11, -1). What are the coordinates of the midpoint of $\overline{BF}$?

8. Where is the midpoint of $\overline{RL}$ if R is located 3 units above the origin and L is located 8 units to the left of the origin?

9. What are the coordinates of K if P is the midpoint of $\overline{JK}$?



10. If the midpoint of $\overline{AB}$ is (4, 10) and B is located at (21, -5), what are the coordinates of A?

Unit 2: Lesson 05 Line segment bisectors

A line segment **bisector** can be a

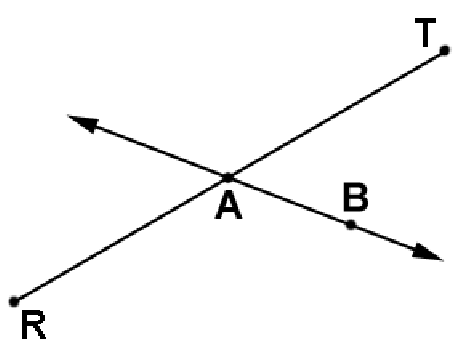
- point,
- line (or line segment), or
- plane

that intersects a line segment at its midpoint.

Example 1: Line segment $\overline{\mathrm{GH}}$ is bisected at point M. If GM = 3p + 2 and MH = 15 – 3p, find the value of *p*.

***Example 2:** P, Q, and R are collinear with Q being somewhere between P and R (but not necessarily halfway in between.) If PR = 20, PQ = j + 8, and QR = 2j + 6, determine if Q is the bisector of $\overline{\mathrm{PR}}$. (Begin by drawing $\overline{\mathrm{PR}}$ and Q.)

Example 3: Line $\overleftrightarrow{\mathrm{AB}}$ bisects $\overline{\mathrm{RT}}$ at A. If AT = 3z + 6 and RA = 11z – 18, find the value of *z*.



Now that we have a little geometry experience, it is appropriate to discuss the different types of geometry. In this course, we will study **Euclidean geometry**. See **In-Depth Topic D** for a discussion of both Euclidean and non-Euclidean geometry.

Assignment:

1. Draw an example of line segment $\overline{RT}$ being bisected by point A.

2. Draw an example of line segment $\overline{RT}$ being bisected at L by ray $\overrightarrow{KL}$.

3. Draw an example of line segment $\overline{RT}$ being bisected at Y by plane XYZ.

4. Given that $\overline{AB}$ has length 20 and C lies between A and B, draw $\overline{AB}$ and C. What is CB if AC = 9?

5. Draw line segment $\overline{UV}$ and point M as its midpoint. If UM = 4f + 2 and VM = 11 – f, what is the value of f?

6. Draw line segment $\overline{PQ}$ and point M as its midpoint. If PM = 46 and MQ = 3h – 2, what is the value of h?

*7. Consider line segment $\overline{\mathrm{HG}}$ of length 8 with point V being somewhere between H and G but not necessarily halfway in between. Draw $\overline{\mathrm{HG}}$ and V and then determine if V is the midpoint of $\overline{\mathrm{HG}}$ if HV = 8k – 12 and GV = 2k.

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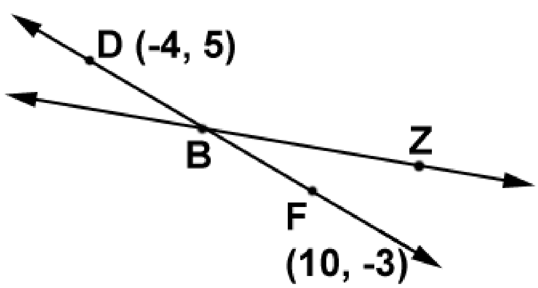
8. If $\overline{\mathrm{DF}}$ is bisected by line $\overleftrightarrow{\mathrm{BZ}}$, what are the coordinates of B?



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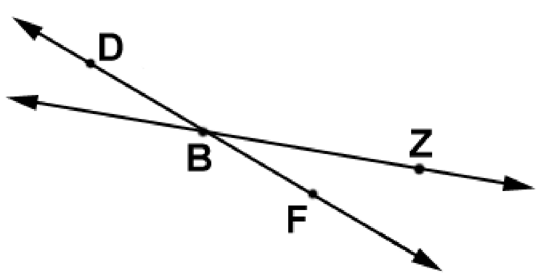
9. Determine the value of *c* if DB = 44c – 2 and BF = 2c + 82. (B is the midpoint of $\overline{\mathrm{DF}}$.)



10. Using the information in problem 9, what are the lengths of $\overline{DB}$ and $\overline{BF}$?

11. Using the results in problem 10, what is the length of $\overline{DF}$ in problem 9?

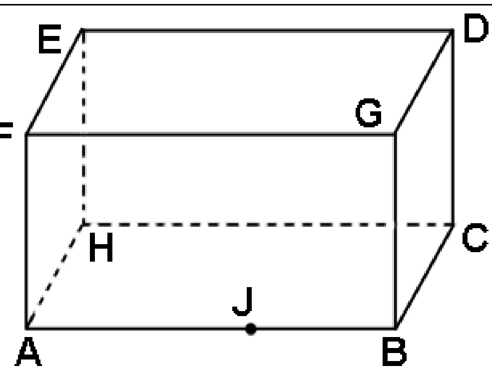
*12. Consider a slanted line coming from above that penetrates your classroom. Now consider the line segment created by the point where this line intersects the plane of the ceiling and the point where the line intersects the plane of the floor. Draw a “side view” of this description. How far above the floor would the midpoint of this line segment be located if your room has 8 ft ceilings?

Unit 02: Review

1. Would the place where the 30 yard-line and the sideline meet on a football field be considered a point, line, ray, or plane?

2. Would the surface of a football field be considered a point, line, ray, or plane?

Use this drawing to answer questions 3-6.



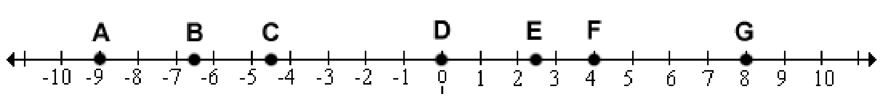
3. Name the intersection of planes EFA and AJG.

4. Are points A, G, and D coplanar?

5. Are points A, G, and D collinear?

6. Does $\overrightarrow{GF}$ point left, right, up, down, front or back?

Use this drawing for problems 7 & 8.

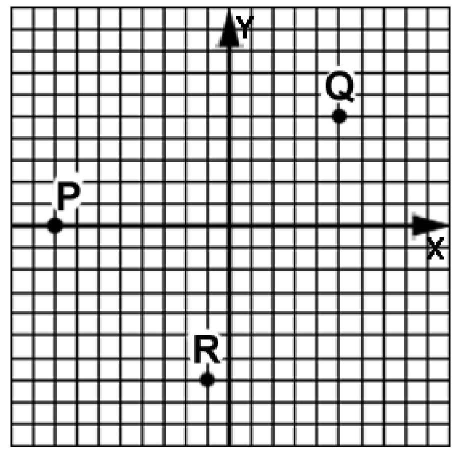


7. BD = ?

8. FB = ?

9. Find AB when A is located at (11, -19) and B is at the origin.

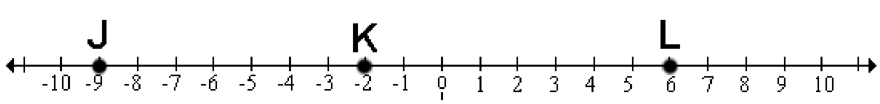
Use these points to work problems 10 and 11.



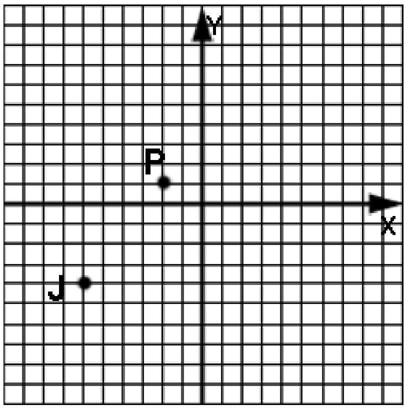
10. What is the length of $\overline{\mathrm{PR}}$?

11. What is the length of $\overleftrightarrow{\mathrm{RQ}}$?

12. Find the midpoint of KL.



13. What are the coordinates of K if J is the midpoint of $\overline{PK}$?



14. Use the points given in problem 13 to find the midpoint of line segment $\overline{PJ}$.

15. Draw line segment $\overline{PV}$ and point M as its midpoint. If PM = 5f + 2 and VM = 12– f, what is the value of f?

16. Use the information and results of problem 15 to determine PV.

17. Consider line segment $\overline{\text{HG}}$ of length 8 with point V being somewhere between H and G but not necessarily halfway in between. Draw $\overline{\text{HG}}$ and V and then determine if V is the midpoint of $\overline{\text{HG}}$ if HV = 8k – 12 and GV = 2k.

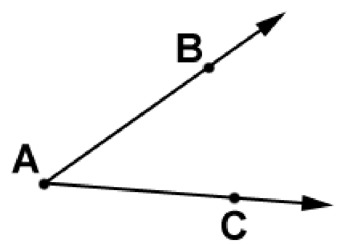
Geometry, Unit 3

Angles
Angle Relationships

Unit 3: Lesson 01 Angle fundamentals

An **angle** is an object formed by two rays with a common endpoint. The common endpoint is called the **vertex** of the angle.

Angle naming conventions:



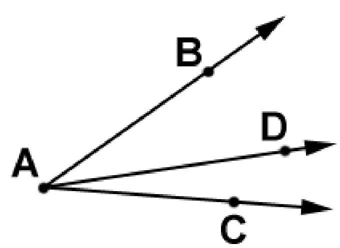
The **sides** of the angle are $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

The **vertex** is at A.

The two acceptable names of the angle using three letters are:

The "One-letter" way of naming this angle is:

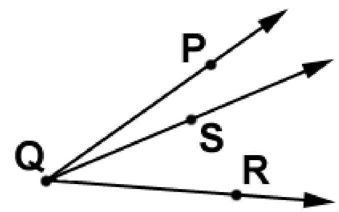
Example 1: Name all the angles shown here using the "three-letter" convention.



Why is it not possible to use the "single-letter" convention to name these angles?

An angle lies in a plane and creates three separate parts (interior, exterior, and the points along the two rays forming the angle).

Example 2: Name a labeled point that is interior to ∠PQR.

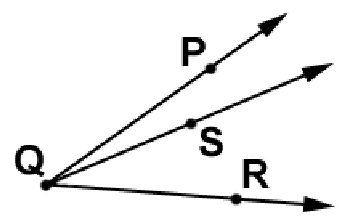


Name a labeled point that is exterior to ∠PQS.

Name a labeled point that is exterior to ∠RQS.

The measure of angle ∠A in degrees is given symbolically by **m∠A.**

Angle addition postulate: If point S is in the interior of ∠PQR as shown here, then "the two small angles add up to the big one."



Example 3: In the drawing just above, find m∠PQR if m∠PQS = 17^o and m∠SQR = 32^o.

Angles (for example ∠A) are classified **according to size**:

If m∠A = 90^o, then it's a **right angle**.

If m∠A = 180^o, then it's a **straight angle**.

If m∠A < 90^o, then it's an **acute angle**.

If m∠A > 90^o, then it's an **obtuse angle**.

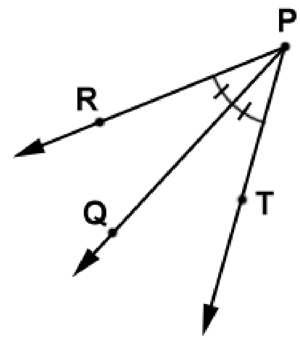
Congruent angles: Angles that have the same measure are said to be congruent.

If m∠A = m∠B then

Angle Bisectors: If $\overrightarrow{AB}$ is the bisector of ∠CAD, then ∠CAD is divided into two congruent angles.

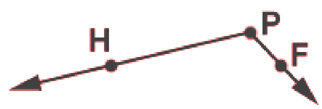
Notice the tic marks on the two angles indicating they are equal.

Example 4: If ∠RPT is bisected by $\overrightarrow{PQ}$, find *x* when m∠RPQ = 2x – 7 and m∠QPT = x + 9. Use *x* to find m∠QPT.



Assignment:

1. Give two names for this angle using the “three-letter” convention.

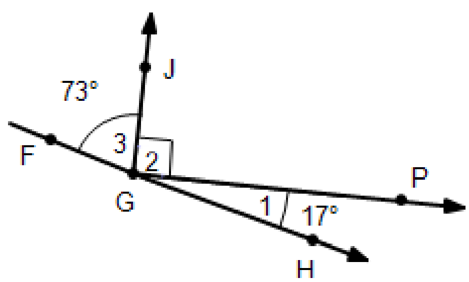


2. Use the drawing in problem 1 to name the angle using the “single-letter” convention.

3. Use the drawing in problem 1 to name the two rays that form the angle.

4. What is the vertex of the angle in problem 1?

Use this drawing for problems 5 - 15.



5. Classify ∠1.

6. Classify ∠2.

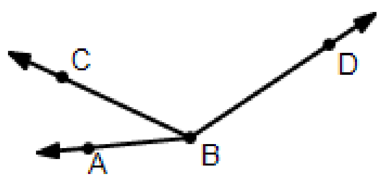
7. Classify ∠3.

8. Classify ∠FGH.

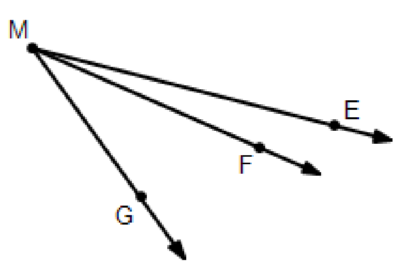
9. Classify ∠HGJ.

10. What are the two rays that make up ∠PGJ?	11. What is m∠JGF?
12. Name ∠2.	13. Name ∠1
14. m∠HGF = ?	15. m∠2 = ?

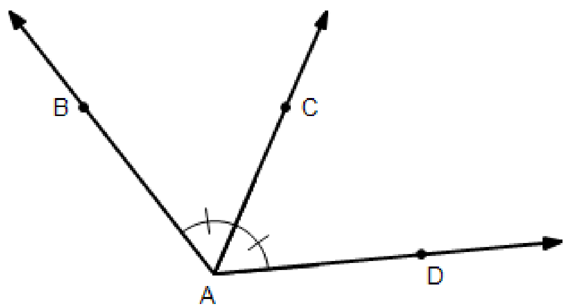
16. If m∠ABC = $(4x - 1)^{o}$, m∠CBD = $(6x + 5)^{o}$, and m∠ABD = 134^{o}, find the value of *x* and then use it to find m∠ABC.



17. m∠FME = $(8x + 5)^{o}$, m∠EMG = $(11x - 1)^{o}$, and m∠FMG = x^{o}. Find the value of *x* and then use it to find m∠FME.



18. If m∠BAC = $(8x – 3)^\circ$ and m∠BAD = $(10x + 30)^\circ$, find the value of *x* and then use it to find m∠CAD.



Unit 3: Lesson 02

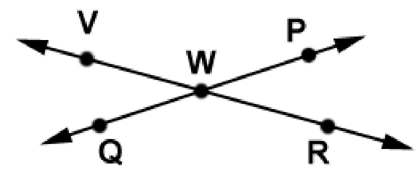
Special angle pairs, perpendicular lines
Supplementary & complementary angles

Consider special angle pairs formed by the intersection of two lines or rays.

Drawing	Name and description	Examples
	Adjacent angles: Angles that have a common vertex and a common side but no common interior points.	
	Vertical angles: Non-adjacent angles formed by an intersecting pair of lines. Vertical angles are congruent.	
	Linear pair of angles: Adjacent angles, the sum of whose measures is 180°.	

Perpendicular lines intersect to form 4 right angles. The symbol used to show that lines are perpendicular is $\perp$.

Example 1: Use this drawing to find *x* when $m\angle VWQ = 8x - 4$ and $m\angle PWR = 4x + 20$. Then use x to find $m\angle VWQ$. Assume that V, W, & R are collinear and that Q, W, and P are collinear.



Example 2: Using the drawing in Example 1, find *x* when $m\angle VWQ = 4x - 20$ and $m\angle VWP = 8x - 4$. Assume that V, W, & R are collinear and that Q, W, and P are collinear.

If the sum of the measures of two angles is 180°, the angles are said to be **supplementary**.

If the sum of the measures of two angles is 90°, the angles are said to be **complementary**.

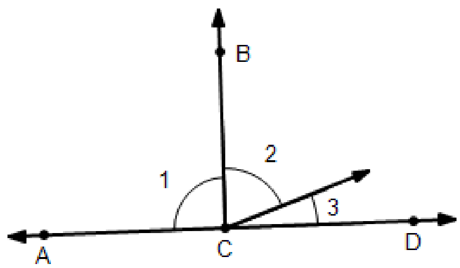
To be either supplementary or complementary, the two angles do not necessarily have to be adjacent.

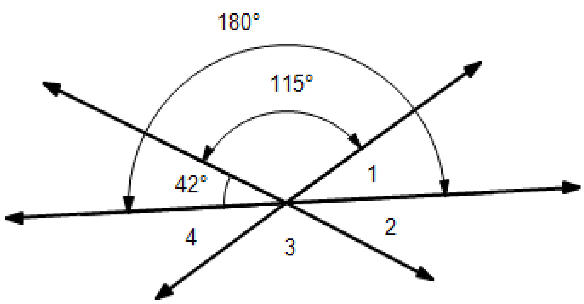
Example 3: If $\angle 3$ and $\angle 7$ are complementary with $m\angle 3 = 4z - 11$ and $m\angle 7 = z - 9$, find z and then use it to find the measure of $\angle 3$.

See **Theorem Proof A** for a proof of vertical angles being equal.

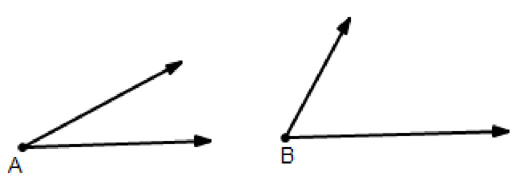
Assignment:

1. $\overleftrightarrow{AD} \perp \overleftrightarrow{CB}$, m∠3 = (6x – 1)° and m∠2 = (8x + 7)°. Find the value of *x* and use it to find m∠2.

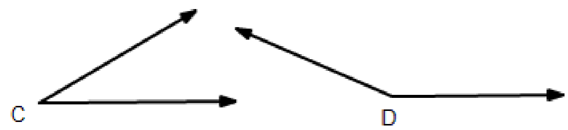


2. Find m∠1. 	3. Using the drawing in problem 2, find m∠2.
4. Using the drawing in problem 2, find m∠3.	5. Using the drawing in problem 2, find m∠4.

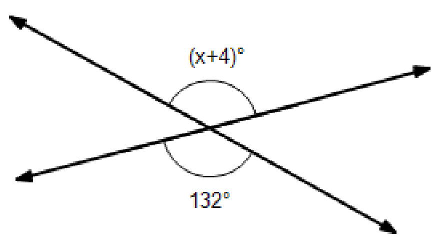
6. If ∠A and ∠B are complementary, find the measure of each. (m∠A = c and m∠B = 2c.)



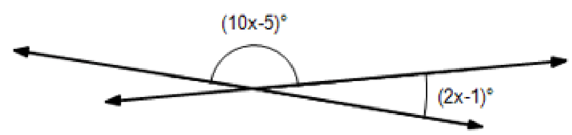
7. If ∠C and ∠D are supplementary, find the measure of each. (m∠C = 4g + 2 and m∠D = 6g – 12.)



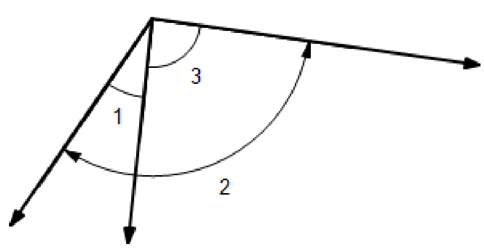
8. Find *x*.



9. Find *x*.



10. Classify ∠1 as acute, obtuse, right, or straight.



11. Using the drawing in problem 10, classify ∠2 as acute, obtuse, right, or straight.

12. Using the drawing in problem 1, classify $\angle 1$ as acute, obtuse, right, or straight.

13. Using the drawing in problem 1, classify $\angle ACD$ as acute, obtuse, right, or straight.

14. If $m\angle A = (120 + x)^{o}$ and $m\angle B = (6 + x)^{o}$, what is the measure of angle A if $\angle A$ and $\angle B$ are known to be supplementary?

15. Find the measure of two complementary angles, $\angle C$ and $\angle D$, if $m\angle C = (6x + 4)^{o}$ and $m\angle D = (4x + 6)^{o}$.

Unit 3:
Lesson 03

Angle word problems

Example 1: Find the measure of an angle if its measure is 50^{o} more than the measure of its supplement.

Example 2: Find the measure of an angle if its measure is 19^{o} less than its complement.

Example 3: Find the measure of an angle if its measure is 40^{o} less than twice its supplement.

***Example 4:** Is it possible to have an angle whose supplement is 20^o more than twice its complement?

Assignment:

1. Find the measure of an angle if its measure is 42^{o} more than the measure of its supplement.

---

2. Find the measure of an angle if its measure is 50^{o} less than its complement.

---

3. Find the measure of an angle if its measure is 10^{o} more than three times its supplement.

4. Find the measure of an angle if its measure is 20^o less than three times its complement.

*5. Is it possible to have an angle whose supplement is 20^o less than twice its complement?

6. Find the measure of an angle if its measure is half that of its complement.

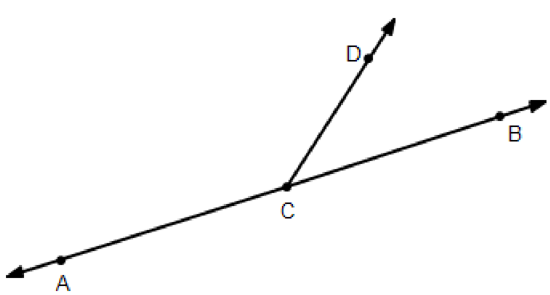
7. Find the measure of an angle if its measure is double the measure of its supplement.

---

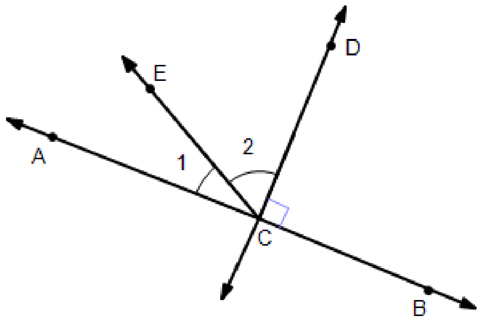
8. Is it possible for the complement of an angle to be equal to its supplement?

---

9. If points A, C, and B are collinear and m$\angle ACD$ is three times the measure of its supplement, what is the measure of $\angle BCD$?



10. $\overleftrightarrow{CD}$ is perpendicular to $\overline{AB}$. If m∠ECB = 140°, what is the sum of the measure of the supplement of $\angle ECB$ and $\angle 2$?



11. Using the drawing and information in problem 10, what is the measure of $\angle 2$?

Unit 3:
Lesson 04

Construction fundamentals
Copying segments & angles; bisecting segments & angles

In this lesson we will learn how to use a **straight edge** (ruler) and a **compass** to

- copy a line segment,
- construct a perpendicular bisector of a line segment,
- copy an angle, and
- bisect an angle.

Copying a line segment:

Begin with a given line segment $\overline{AB}$.

Place the point of the compass at point A and adjust the compass so that the pencil, is at point B.

Use a straight edge to draw a line segment and mark point P at one end.

Place the point of the compass a point P and strike an arc at point Q.

The line segment $\overline{AB}$ is now congruent to $\overline{PQ}$.

Bisecting a line segment (perpendicular bisector):

Begin with a given line segment $\overline{AB}$.

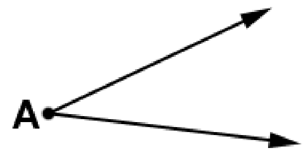
Place the point of the compass at point A, adjust its span greater than half AB, and then strike an arc above and below $\overline{AB}$.

With the same setting on the compass, place its point at B and strike arcs above and below $\overline{AB}$ that intersect the previous arcs. Call the points where the arcs meet points P and Q.

Use a straight edge to draw $\overline{PQ}$. Designate the point where $\overline{PQ}$ intersects $\overline{AB}$ as M.

Copy an angle:

Begin with angle $\angle A$:



Place the point of the compass at A and strike arcs on the two rays. Label the points where the arcs intersect the rays at P and Q.

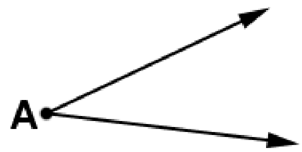
Draw a ray and label the end point as H. With the same setting on the compass as before and with the point of the compass at H, strike a large arc as shown. Label the point where the arc intersects the ray as K.

Go back to the previous drawing, place the point of the compass at P and adjust the span of the compass to reach point Q. Now place the point of the compass at K and strike an arc intersecting the previous large arc as shown. Call this point of intersection L.

Draw the ray $\overrightarrow{HL}$.

Bisecting an angle:

Begin with angle $\angle A$:



With the point of the compass at A, strike a large arc that intersects both rays. Call these points of intersection P and Q.

With a span set on the compass that is greater than the distance between P and Q, and with the point of the compass at point P, strike an arc as shown. Similarly, with the point of the compass at Q, strike another arc intersecting the first. Call the point of intersection of the arcs, R.

With a straight edge, draw ray $\overrightarrow{AR}$.

Assignment: Use construction techniques for these problems.

1. Make a copy ($\overline{PQ}$) of line segment $\overline{BD}$.

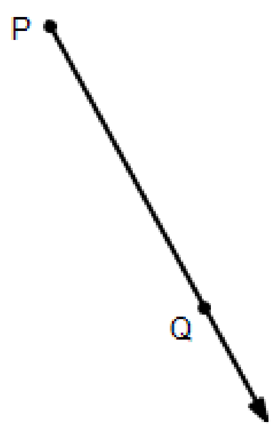
2. Make a copy ($\overline{PQ}$) of line segment $\overline{CA}$.

3. Make a copy ($\overline{PQ}$) of line segment $\overline{HA}$.

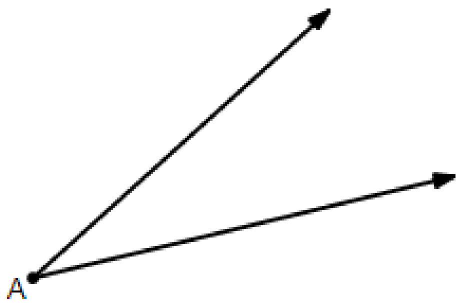
4. Construct the perpendicular bisector ($\overline{PQ}$) of $\overline{HB}$.

5. Construct the perpendicular bisector ($\overline{PQ}$) of $\overline{AB}$.

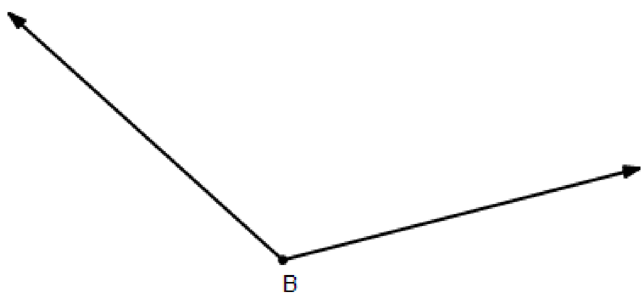
6. Construct the perpendicular bisector ($\overline{AB}$) of $\overline{PQ}$.



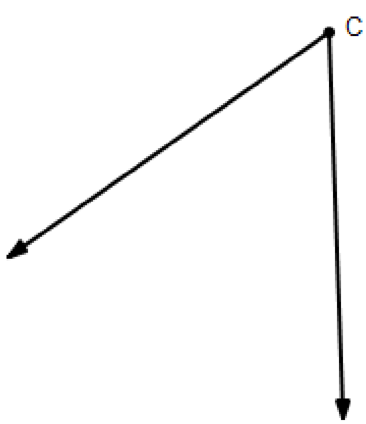
7. Make a copy of angle $\angle A$.



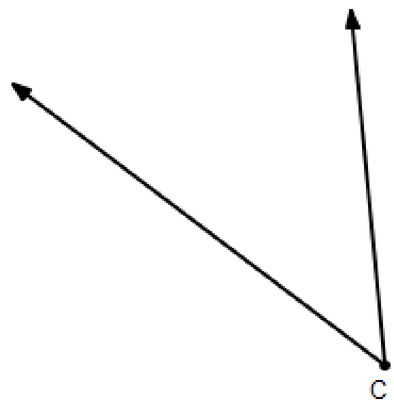
8. Make a copy of angle $\angle B$.



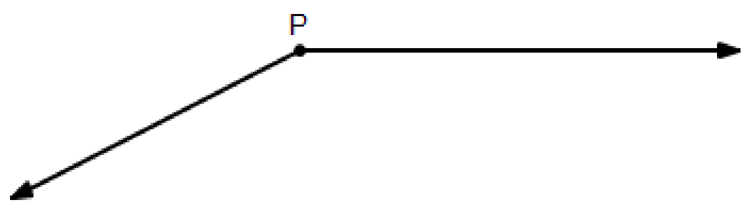
9. Make a copy of angle $\angle C$.



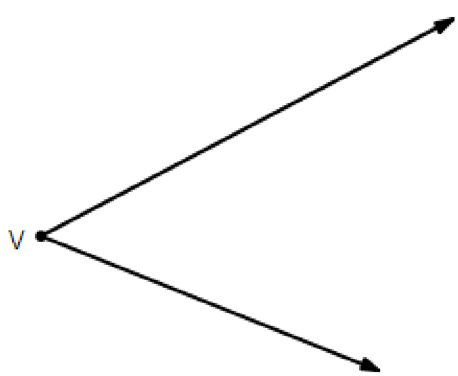
10. Create a ray ($\overrightarrow{CQ}$) that biscects $\angle C$.



11. Create a ray ($\overrightarrow{PQ}$) that bisects $\angle P$.



12. Create a ray ($\overrightarrow{VR}$) that biscects $\angle V$.



Unit 3: Cumulative Review

1. Factor $4x^2 + 19x + 21$.

2. Factor $h^2 - 16$.

3. Factor $p^2 + 14p + 49$.

4. Multiply $(2x - 8)(2x + 8)$.

5. Solve $5(x + 3) = 11x - 3$.

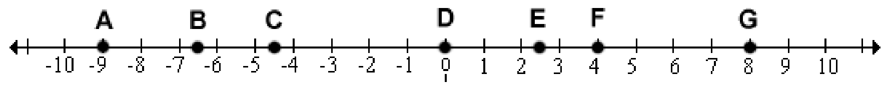
6. Solve $x^2 + x - 56 = 0$ by factoring.

7. Write the quadratic formula.

8. Solve $2x^2 - 3x + 1 = 0$ using the quadratic formula.

9. Solve $-x^2 + 12x - 34 = 0$ using a graphing calculator. Make a sketch of the calculator display.	10. Would the location on a map where two streets intersect be described as a point, line, line segment, ray, or plane?
11. Suppose one end of a rubber band is attached to a wall and the other end stretched forever (assume it doesn't break). What best describes this? (A point, line, line segment, ray, or plane)	12. A building sits beside a parking lot. Would the place where the parking lot first meets the building be described as a point, line, line segment, ray, or plane?

Use this number line for problems 13 & 14.



13. What is FB?

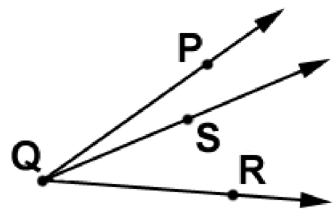
14. What is the midpoint of $\overline{\text{FB}}$?

15. What is the distance between (-8, 2) and (16, -4)?

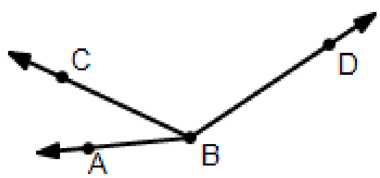
16. What are the coordinates of the point midway between (-8, 2) and (16, -4)?

Unit 3: Review

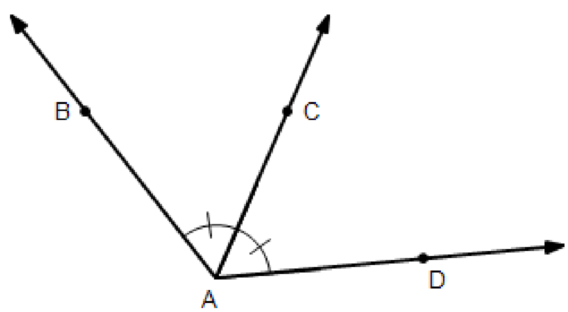
1. Find m∠PQR if m∠PQS = 17° and m∠SQR = 32°.



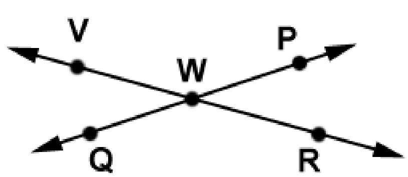
2. If m∠ABC = (2x – 1)°, m∠CBD = (6x + 3)°, and m∠ABD = 130°, find the value of *x* and then use it to find m∠DBC.



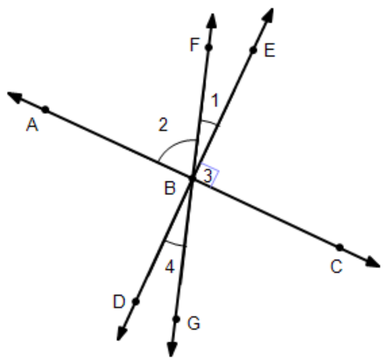
3. If m∠BAC = (4x – 2)° and m∠BAD = 126°, what is m∠CAD?



4. Name two different pairs of vertical angles. What is the relationship of vertical angles?



5. $\overleftrightarrow{AC} \perp \overleftrightarrow{DE}$. Name the angle that is the supplement to $\angle 3$.



6. Using the drawing in problem 5, name the angle that is the complement to $\angle 1$.

7. Using the drawing in problem 5, if $m\angle 2 = 70^o$, what is $m\angle 1$?

8. Using the drawing in problem 5, if $m\angle 2 = 70^o$, what is $m\angle 4$?

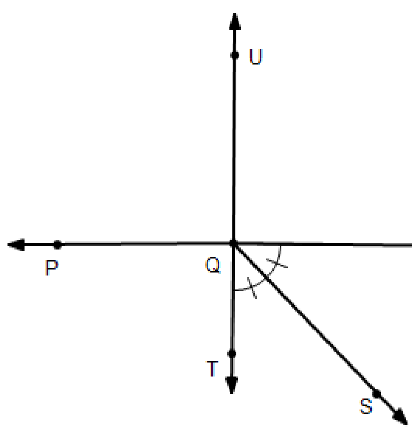
9. Using the drawing in problem 5, what is $m\angle ABF + m\angle FBE + m\angle EBC$?

10. Using the drawing in problem 5, if $\overrightarrow{BF}$ bisects $\angle ABE$, what would be the measure of $\angle 2$?

11. Find the measure of an angle if its measure is 30° less than its complement.

12. Find the measure of an angle if its measure is 20° more than one-third its supplement.

13. $\overleftrightarrow{PR} \perp \overleftrightarrow{UT}$. Name three other angles that have the same measure as $\angle TQR$.



14. Using the drawing in problem 13, what is the sum of the angles $\angle RQS$ and $\angle SQT$?

15. Using the drawing in problem 13, what is the sum of the angles $\angle PQT$ and $\angle UQR$?

16. Using the drawing in problem 13, classify $\angle RQS$ as acute, obtuse, right, or straight.

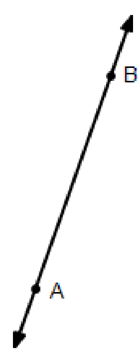
17. Using the drawing in problem 13, classify ∠PQS as acute, obtuse, right, or straight.

18. Using the drawing in problem 13, classify ∠PQR as acute, obtuse, right, or straight.

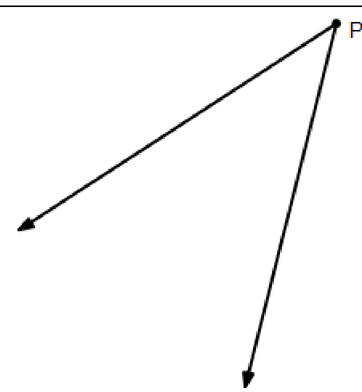
19. Using the drawing in problem 13, classify ∠RQT as acute, obtuse, right, or straight.

20. Using the drawing in problem 13, ∠UQR and what other angle form a pair of vertical angles?

21. Construct a perpendicular bisector of $\overleftrightarrow{AB}$.



22. Construct a bisector of ∠P.



Geometry, Unit 4

Parallel Lines & Planes and Transversals

Unit 4: Lesson 01 — Parallel lines & planes fundamentals
Definitions of transversal angle pairs

Definitions:

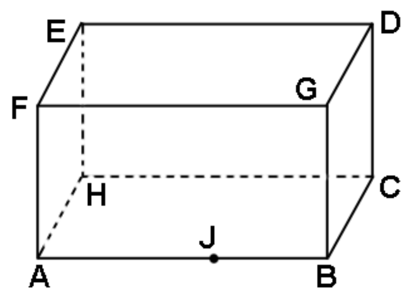
Parallel lines are lines in the same plane that never intersect.
They have the same slope and are always the same distance apart.

Symbolism: To indicate $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, write $\overleftrightarrow{AB} \,||\, \overleftrightarrow{CD}$.

Skew lines are lines that do not lie in the same plane. As a result, they never intersect.

Parallel planes are planes that do not intersect.
ABC | | DEF indicates plane ABC is parallel to plane DEF.

Use this drawing in examples 1-4:



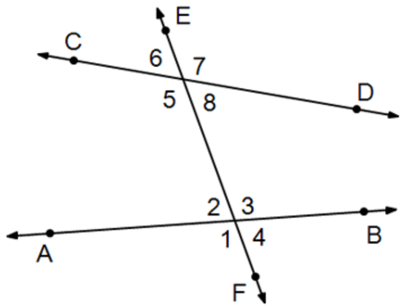
Example 1: Name all the lines parallel to $\overleftrightarrow{AB}$.

Example 2: Name a plane parallel to EFA.

Example 3: $\overleftrightarrow{AB}$ and _______ are skew lines.

Example 4: Do $\overleftrightarrow{AH}$ and $\overleftrightarrow{GD}$ lie in the same plane?

Consider two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ cut by line $\overleftrightarrow{EF}$. $\overleftrightarrow{EF}$ is called a **transversal**. Notice that 8 angles are formed.



The following defines various pairs of these angles. All definitions involve **interior** and **exterior** angles. Angles 2, 3, 5, and 8 are interior angles while angles 1, 4, 6, and 7 are exterior angles.

Definitions:

Alternate interior angles

Two non-adjacent, interior angles on opposite sides of the transversal.

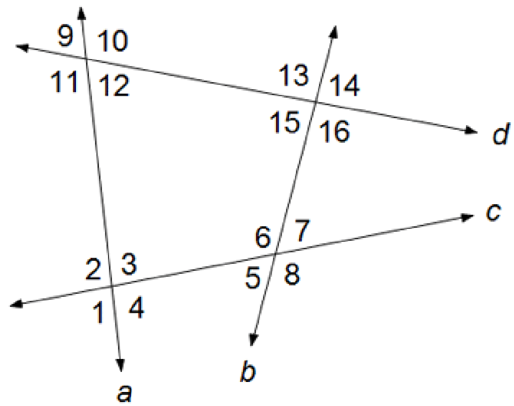
Alternate exterior angles

Two non-adjacent, exterior angles on opposite sides of the transversal.

Corresponding angles

Two angles that are in similar positions relative to the lines and transversal.

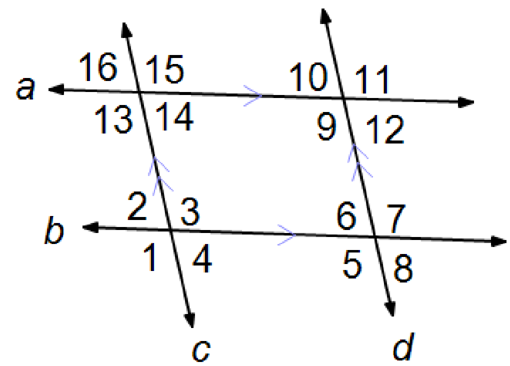
Use this drawing for examples 5-12. In each example, classify the type of each given angle pair as alternate interior, alternate exterior, corresponding, or none (no special relationship).



Example 5: $\angle 11$ & $\angle 14$	**Example 6:** $\angle 2$ & $\angle 9$
Example 7: $\angle 1$ & $\angle 14$	**Example 8:** $\angle 8$ & $\angle 4$
Example 9: $\angle 6$ & $\angle 16$	**Example 10:** $\angle 10$ & $\angle 15$
Example 11: $\angle 5$ & $\angle 12$	**Example 12:** $\angle 9$ & $\angle 16$

Assignment:

Use this drawing for problems 1-14. In each example, classify the type of each angle pair as alternate interior angles, alternate exterior angles, corresponding angles, supplementary angles, vertical angles, or none (no special relationship).

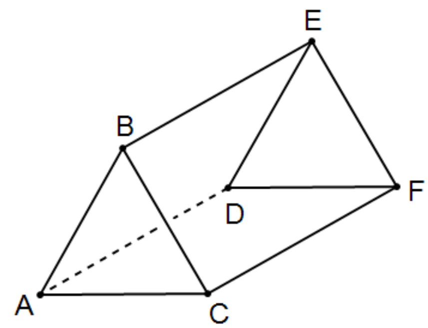


1. $\angle 2$ & $\angle 8$	2. $\angle 16$ & $\angle 8$
3. $\angle 14$ & $\angle 10$	4. $\angle 6$ & $\angle 7$
5. $\angle 1$ & $\angle 3$	6. $\angle 10$ & $\angle 8$
7. $\angle 5$ & $\angle 3$	8. $\angle 10$ & $\angle 6$
9. $\angle 16$ & $\angle 4$	10. $\angle 11$ & $\angle 3$
11. $\angle 6$ & $\angle 8$	12. $\angle 13$ & $\angle 14$

13. $\angle 2$ & $\angle 8$

14. $\angle 15$ & $\angle 9$

Use this drawing for problems 15-18.



15. Name all segments parallel to $\overline{\mathrm{AD}}$.

16. Name a plane parallel to ABC.

17. Name all segments that are skew to $\overline{\mathrm{AB}}$.

18. Name two transversals that cut $\overline{\mathrm{CF}}$ and $\overline{\mathrm{BE}}$.

19. Are the lines formed by two goal lines on a football field parallel, intersecting, or skew lines?

20. Are the lines formed by a center stripe on a road and a telephone pole beside the road parallel, intersecting, or skew lines?

21. Are the planes of two opposite walls in your bedroom parallel or do they intersect?

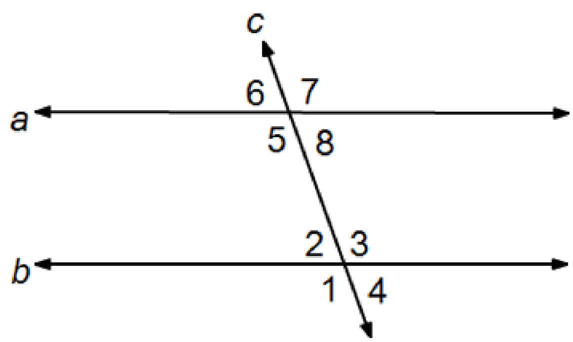
22. Are the lines on ruled paper that is used in school skew or parallel?

23. How many angles are formed when two lines in a plane are cut by a transversal?

24. If two quills on a porcupine are selected at random, are they more likely to represent skew or parallel lines?

Unit 4: Lesson 02 Parallel lines cut by a transversal

If two **parallel** lines (a & b) are cut by a transversal (c), then the following theorems apply:



- Alternate interior angles are congruent (measures are equal)

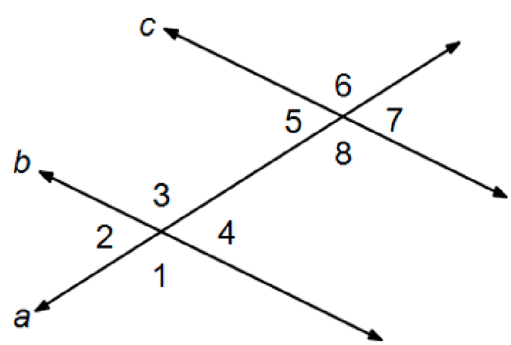
  See **Theorem Proof B** for a proof this important theorem.

- Alternate exterior angles are congruent (measures are equal)

  See **Theorem Proof B** for a proof this important theorem.

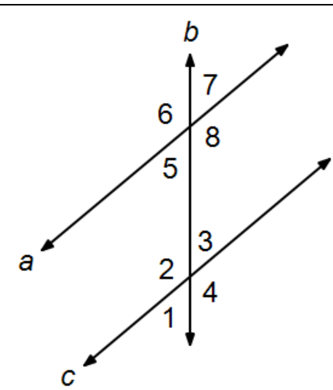
- Postulate: Corresponding angles are congruent (measures are equal)

Use this drawing for examples 1- 10. Assume c || b and specify each angle pair as congruent or supplementary. Justify your answers.



Example 1: $\angle 1$ and $\angle 3$	**Example 2:** $\angle 2$ and $\angle 7$
Example 3: $\angle 6$ and $\angle 1$	**Example 4:** $\angle 8$ and $\angle 3$
Example 5: $\angle 5$ and $\angle 7$	**Example 6:** $\angle 6$ and $\angle 7$
Example 7: $\angle 5$ and $\angle 2$	**Example 8:** $\angle 2$ and $\angle 3$
Example 9: $\angle 3$ and $\angle 8$	**Example 10:** $\angle 4$ and $\angle 7$

Use this drawing for examples 11-15.
Assume a || c.

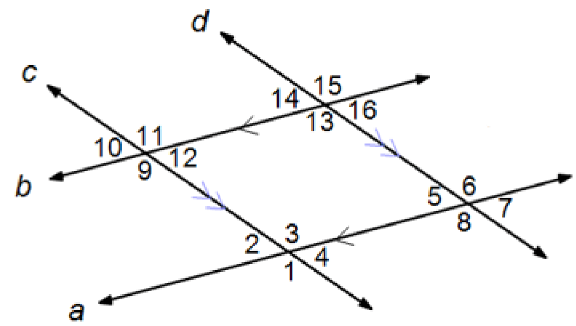


Example 11: If $m\angle 6 = 130^o$, find $m\angle 4$.	**Example 12:** If $m\angle 3 = 60^o$, find $m\angle 2$.
Example 13: If $m\angle 1 = 33^o$, find $m\angle 5$.	**Example 14:** If $m\angle 2 = 135^o$, find $m\angle 7$.

Example 15: If $m\angle 3 = 2x + 2$ and $m\angle 5 = 6x - 118$, what is the value of *x*? What is $m\angle 3$?

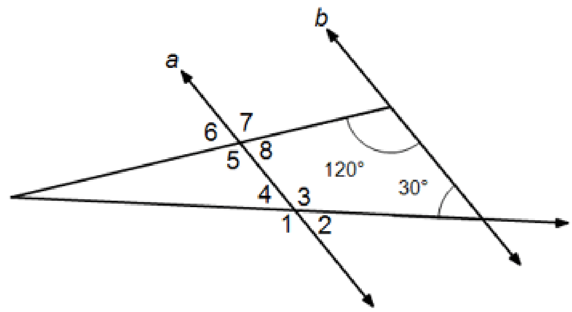
Assignment:

Use this drawing in problems 1-8 and assume c || d and a || b. Classify the given angle pair and state whether they are congruent or supplementary.



1. ∠1 and ∠6	2. ∠13 and ∠16
3. ∠10 and ∠14	4. ∠13 and ∠15
5. ∠2 and ∠12	6. ∠9 and ∠13
7. ∠10 and ∠16	8. ∠3 and ∠8

Use this drawing for problems 9-14.
Lines *a* and *b* are parallel.



9. What is the measure of ∠5?	10. What is the measure of ∠2?
11. What is the measure of ∠4?	12. To what other angle(s) is the 120^{o} angle equal?

13. What is the measure of ∠3?

14. What is the measure of ∠6?

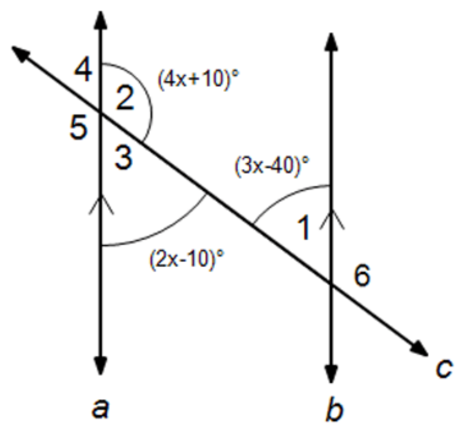
Unit 4: Lesson 03

More practice with parallel lines and transversals
Same-side angles

From problems in the previous lesson, we notice that angles on the **same side of a transversal** (cutting two parallel lines) and in the interior are **supplementary.**

Similarly, same-side exterior angles are **supplementary**.

In the following examples, use this drawing to classify the type of angle pair and use the pair to find x. Lines *a* and *b* are parallel.



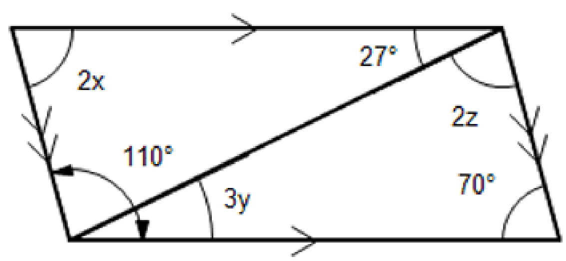
Example 1: ∠6 and ∠4 where m∠6 = 120°

Example 2: ∠1 and ∠2

Example 3: $\angle 1$ and $\angle 4$

Example 4: $\angle 3$ and $\angle 2$

Use this drawing with the following examples. One set of parallel lines are shown with the single-arrow and another set with the double-arrows.



Example 5: Find the value of *x*.

Example 6: Find the value of *y*.

Example 7: Find the value of *z***.**

Assignment:

Apply angle-pair answers to problems 1-3.

A. Corresponding angles
B. Alternate interior angles
C. Alternate exterior angles
D. Same-side interior angles
E. Same-side exterior angles
F. Vertical angles
G. Adjacent angles that form a straight line (linear pair)

1. Which angle pairs are always congruent if a transversal cuts two parallel lines?

2. Which angle pairs are always congruent if a transversal cuts two non-parallel lines?

3. Which angle pairs are always supplementary if a transversal cuts two parallel lines?

4. Draw two parallel lines & a transversal and mark a pair of corresponding angles.

5. Draw two parallel lines & a transversal and mark a pair of alternate interior angles.

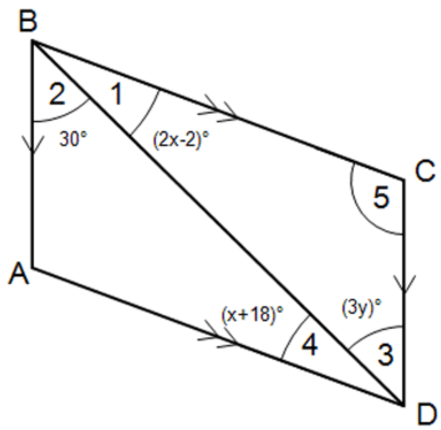
6. Draw two parallel lines & a transversal and mark a pair of alternate exterior angles.

7. Draw two parallel lines & a transversal and mark a pair of same-side interior angles.

8. Draw two parallel lines & a transversal and mark a pair of same-side exterior angles.

9. Draw two parallel lines & a transversal and mark a linear pair of angles.

Use this drawing in problems 10 - 12.



10. Find *x* and use it to find m∠4.

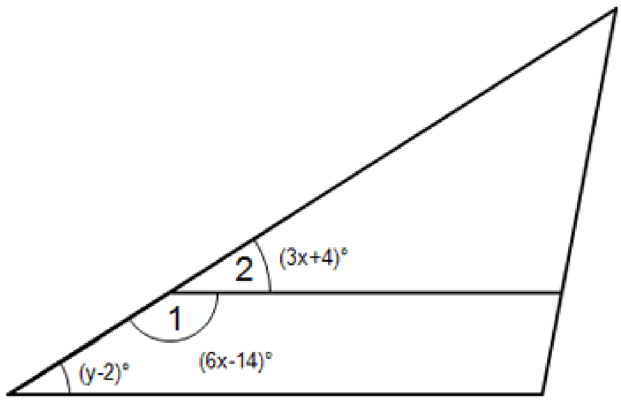
11. Find *y*.

12. What type of an angle pair is ∠ABC and ∠5?

13. Use the previous result for *x* to find m∠5.

14. Use the previous results for *x* and *y* to find m∠ADC.

Use this drawing in problems 15 & 16.



15. Find the value of x and then use it to find m∠2.

16. Use the value of x just found to find the value of y.

Unit 4: Lesson 04

Parallel line construction
Parallel lines: multiple variable problems

Given line $\overleftrightarrow{AB}$, construct $\overleftrightarrow{CD}$ that is parallel to it.

Begin with $\overleftrightarrow{AB}$ and at two different points P & Q on $\overleftrightarrow{AB}$, construct perpendiculars to $\overleftrightarrow{AB}$.

With the point of the compass at P set some convenient span on the compass, and then on the perpendicular through P, strike an arc at C.

Similarly, with the point of the compass on Q and with the same span, strike an arc on that perpendicular at D.

With a straight edge, draw a line connecting the two points C & D. This line is parallel to $\overleftrightarrow{AB}$. The distance between the two lines is the span setting on the compass when the two arcs were struck.

These constructions are demonstrated clearly in the associated video.

***Example 1:** Find *p*, *q*, and *r* if

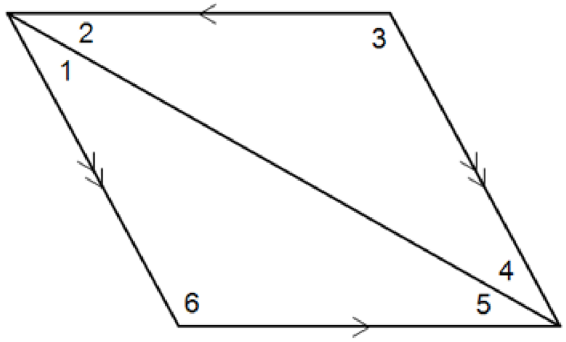
$m\angle 2 = 6q + 5,$

$m\angle 1 = 5p - 16,$

$m\angle 5 = 12r - 1,$

$m\angle 4 = 3p,$ and

$m\angle 6 = 97$

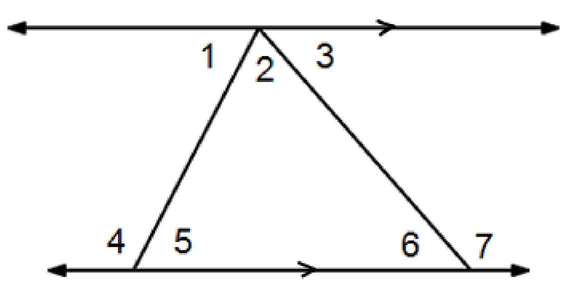


Example 2: Find *a* and *b* if

$m\angle 6 = 3 + 8a$

$m\angle 7 = 20b - 3$

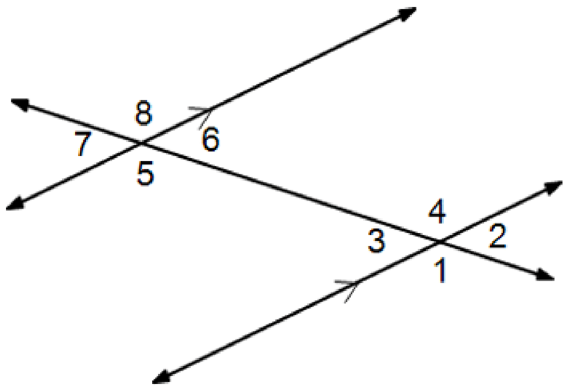
$m\angle 3 = 11a - 12$



Assignment:

1. Find *x* and *y* if

   $m\angle 6 = (6x - 4)^o$
   $m\angle 3 = (7x - 10)^o$
   $m\angle 4 = (4y - 8\ )^o$



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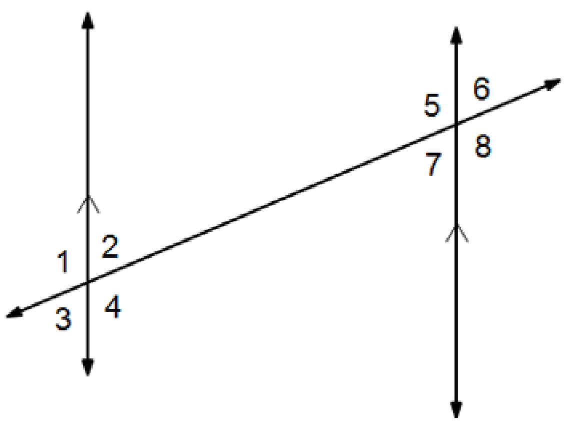
2. Find *x* and *y* if

   $m\angle 2 = (2x)^o$
   $m\angle 4 = (3y + 10)^o$
   $m\angle 7 = (y + 10\ )^o$



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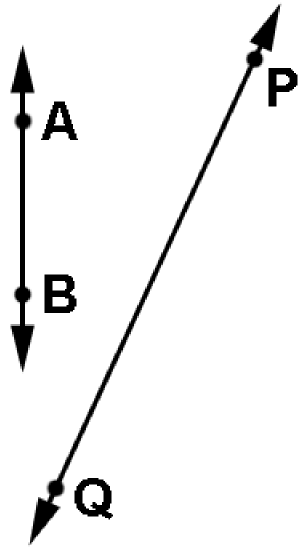
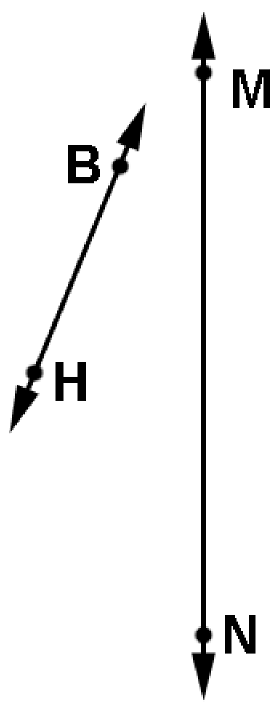
3. A linear pair have measures $(2x + 1)^o$ and $(4x - 7)^0$. What are the measures of the two angles?

4. Construct a line parallel to $\overleftrightarrow{PQ}$ and so that the two lines are AB apart.



5. Construct a line parallel to $\overleftrightarrow{MN}$ so that the two lines are HB apart.



6. $\angle A$ and $\angle B$ are a pair of vertical angles and $\angle C$ is the supplement of $\angle A$. If the measures of $\angle A$, $\angle B$, and $\angle C$ respectively are $(2x)^o$, $(40)^o$, and $(3y + 80)^o$, find *x* and *y*.

Unit 4: Cumulative Review

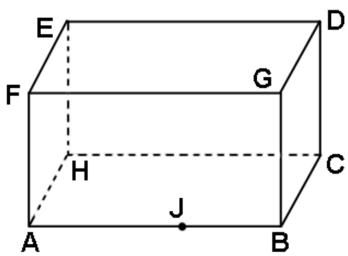
1. Solve this system using substitution:

   y = 3x + 4

   x – y = 6

2. Solve -2x – 4(6x + 2) = x – 5

Use this drawing to answer questions 3 - 5.

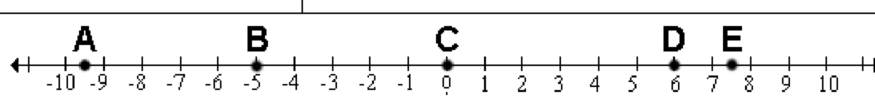


3. Name the intersection of planes HJC and BCG.

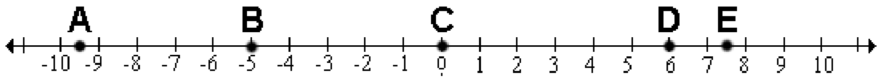
4. Does $\overrightarrow{HE}$ point left, right, up, down, front or back?

5. In what plane does the midpoint of $\overline{EG}$ lie?

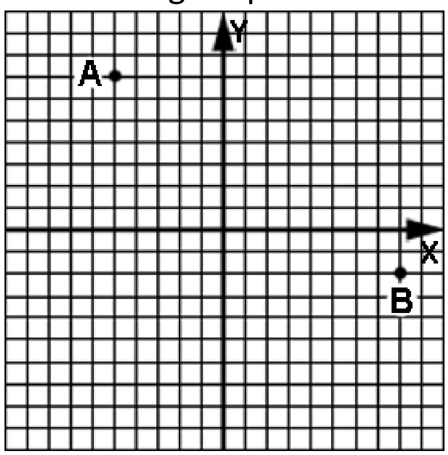
6. What is DA?



7. What is the midpoint of $\overline{AB}$?



Use this drawing for problems 8 – 10.

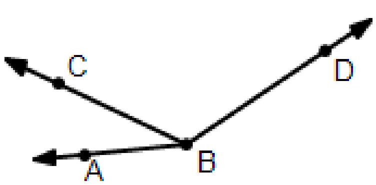


8. Find AB.

9. Find the midpoint of $\overline{AB}$.

10. If B is the midpoint of $\overline{AC}$, find the coordinates of point C.

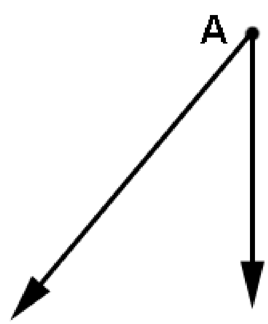
11. If m∠ABC = $(2x – 1)^{o}$, m∠CBD = $(6x + 3)^{o}$, and m∠ABD = 130^{o}, find the value of *x* and then use it to find m∠CBD.



12. Find the measure of an angle if its measure is 60° less than 2 times its complement.

---

13. Construct the bisector of angle A.



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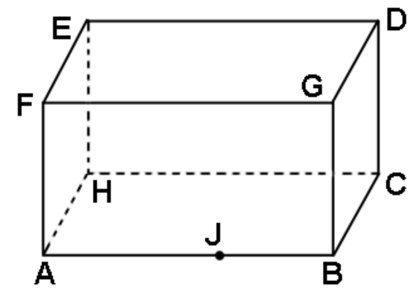
14. Construct the perpendicular bisector of $\overline{AB}$.

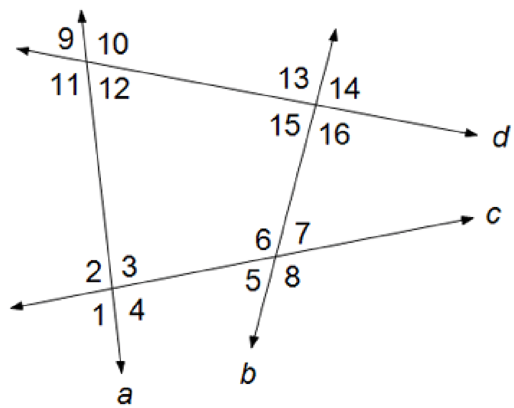
Unit 4: Review

Use this drawing in problems 1 & 2:



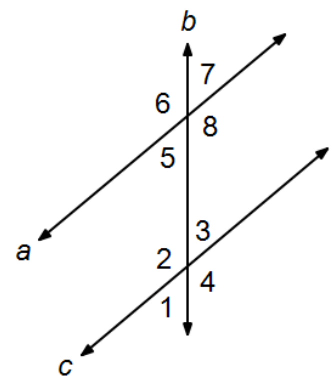
1. Name all the lines parallel to $\overleftrightarrow{BC}$.	2. Name a plane parallel to HJC.

Use this drawing in problems 3 - 8. In each problem, classify each given angle pair as alternate interior, alternate exterior, corresponding, same-side interior, or same-side exterior.



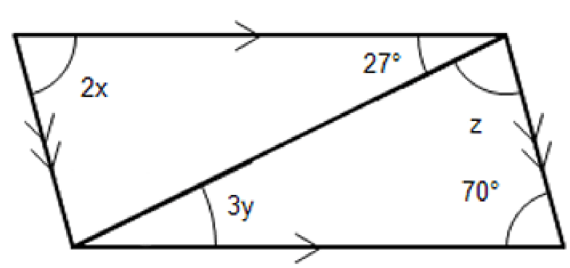
3. $\angle 6$ & $\angle 15$	4. $\angle 3$ & $\angle 5$
5. $\angle 10$ & $\angle 3$	6. $\angle 6$ & $\angle 8$
7. $\angle 9$ & $\angle 4$	8. $\angle 10$ & $\angle 4$

Use this drawing for problems 9-12.
Assume a | | c.



9. If $m\angle 4 = 145^{o}$, find $m\angle 7$.	10. If $m\angle 7 = 45^{o}$, find $m\angle 1$.
11. If m∠6 = 140°, find m∠7.	12. If m∠5 = $(5x - 10)^{o}$ and m∠3 = $(3x + 10)^{o}$, find *x*.

Use this drawing for problems 13-15:



13. Find the value of x.

---

14. Find the value of y.

---

15. Find the value of z.

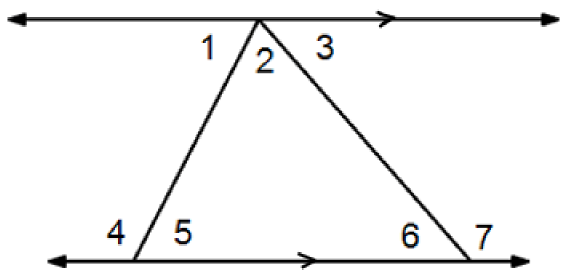
---

16. Find *a* and *b* if

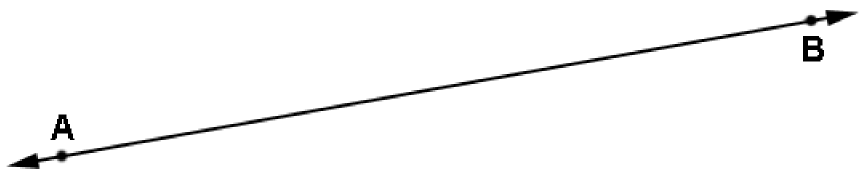
$m\angle 4 = 21b - 6$

$m\angle 5 = 7a + 4$

$m\angle 1 = 5a + 14$



17. Construct a line parallel to $\overline{\mathrm{AB}}$.



Geometry, Unit 5

Triangles & other Polygons

Unit 5: Lesson 01 Triangle fundamentals; sum of the interior angles

Triangle:

Definition: A closed figure formed by the joining of three non-collinear points with 3 line segments.

Naming convention: The triangle with vertices A, B, and C is named **ΔABC**.

Components: The components of ΔABC are

- Vertices – points A, B, and C
- Sides – line segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$
- Angles – $\angle A$, $\angle B$, and $\angle C$

Triangle classification according to **angle size**:

Acute triangle – a triangle in which the measures of each angle is less than 90^o

Obtuse triangle – a triangle in which the measure of one angle is greater than 90^o

Right triangle – a triangle in which the measure of one angle is exactly 90^o

Equiangular triangle – a triangle in which the measures of all 3 angles are equal

Triangle classification according to **lengths of sides**:

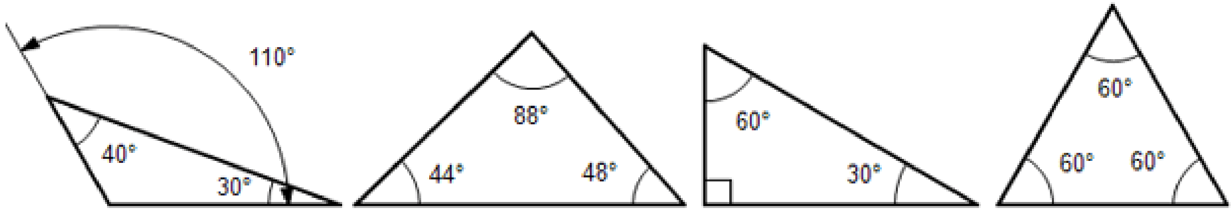
Scalene triangle – a triangle in which the lengths of all sides are different. (Notice the tick marks.)

Isosceles triangle – a triangle in which two sides have equal lengths. (Notice the tick marks.)

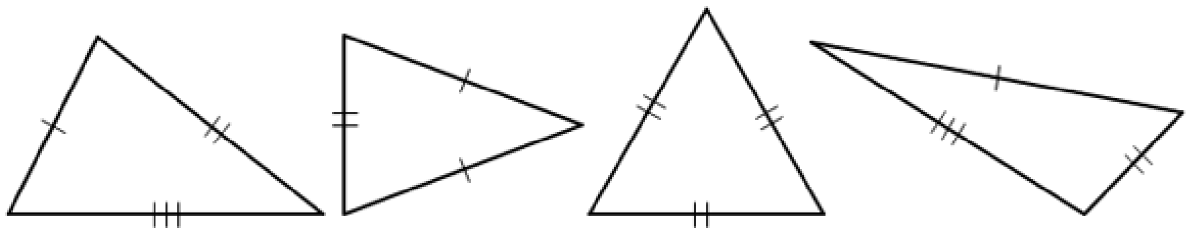
Theorem: Base angles of an isosceles triangle are equal. See **Theorem Proof D** for a two column proof of this theorem.

Equilateral triangle – a triangle in which all three sides have the same length. (Notice the tick marks.)

Example 1: Write the word acute, obtuse, right, or equiangular under each triangle so as to identify that type of triangle.



Example 2: Write the word scalene, isosceles, or equilateral under each triangle so as to identify that type of triangle.



Notice in example 1 above that in each triangle, the sum of all three angles is always 180^o.

Theorem: The sum of the interior angles of a triangle is 180^o. This is arguably the 2nd most important theorem in geometry. (The Pythagorean Theorem is considered most important.)

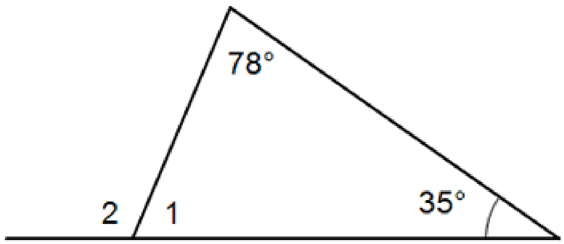
Begin with ΔABC having $\angle 1$, $\angle 2$, and $\angle 3$ as shown. Draw a line parallel to side $\overline{AB}$ thus forming $\angle 4$ and $\angle 5$.

Notice in this drawing that $\angle 1$ and $\angle 2$ are the same respectively as $\angle 4$ and $\angle 5$. Since $\angle 4$, $\angle 3$, and $\angle 5$ form a straight line, their sum equals 180^o. (See **Theorem Proof C** for a formal, 2 column proof.)

Example 3: If two angles of a triangle have measures of 28^o and 113^o, what is the measure of the third angle?

The **exterior angle** of a triangle is formed by one side of a triangle and the extension of another side. (Interior angles are **inside** the triangle.)

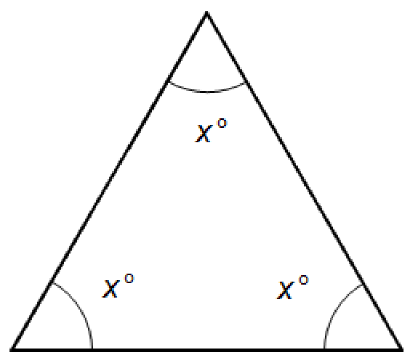
Example 4: Find the measure of ∠2.



From the results of example 4 we can conclude:

Theorem: The measure of an exterior angle of a triangle is equal to the sum of the other two remote interior angles.

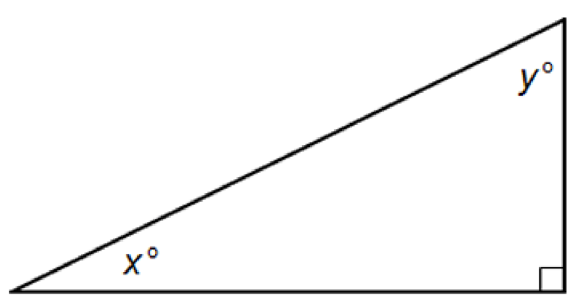
Example 5: Find the value of *x*.



From the results of example 5 we can conclude:

The measure of each angle of an equiangular triangle is 60°.

Example 6: Find the sum of x + y.



From the results of example 6 we can conclude:

The two acute angles of a right triangle are complementary.

See **In-Depth Topic E** for how to construct an equilateral triangle and a related construction, inscribing a hexagon inside a circle.

Assignment:

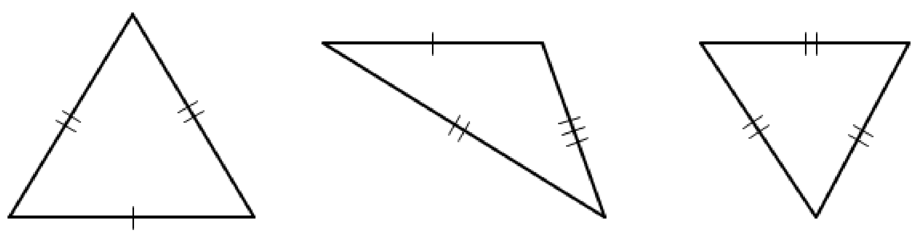
1. The sum of the measures of all three interior angles of a triangle is _______.

2. The measure of an exterior angle of a triangle is equal to_______________.

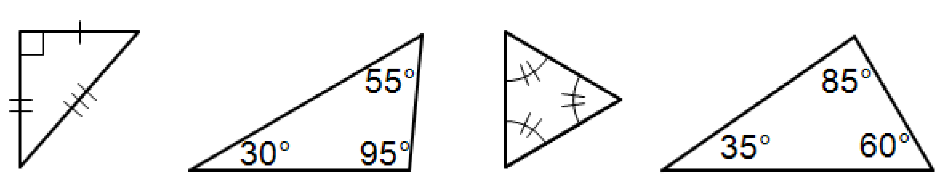
3. The two acute angles of a right triangle are______________.

4. The measure of each angle of an equilateral triangle is_______.

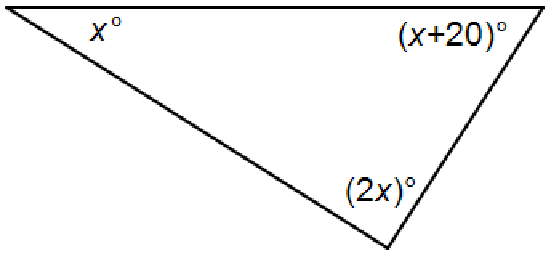
5. Write the word scalene, isosceles, or equilateral under each triangle so as to identify that type of triangle.



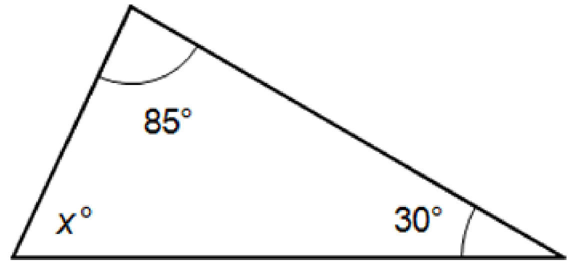
6. Write the word acute, obtuse, right, or equiangular under each triangle so as to identify that type of triangle.



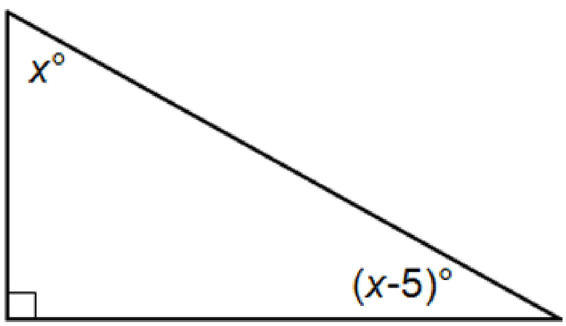
7. Find the value of x.



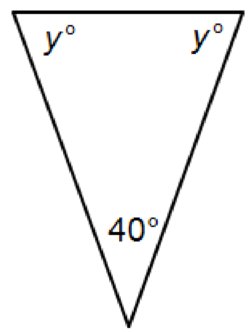
8. Find the value of x.



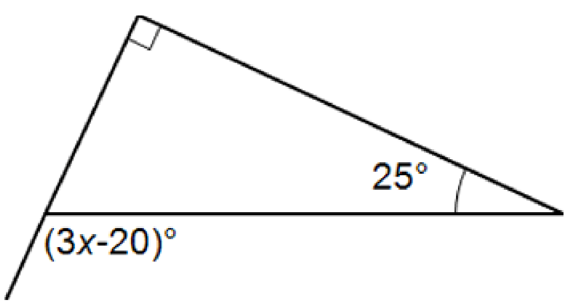
9. Find the value of x.



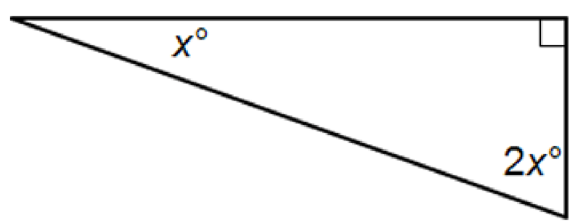
10. Find the value of y.



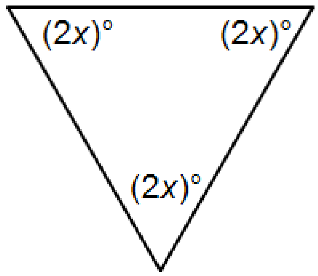
11. Find the value of x.



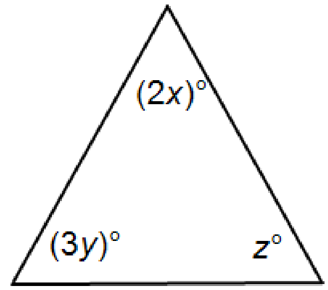
12. Find the value of x.



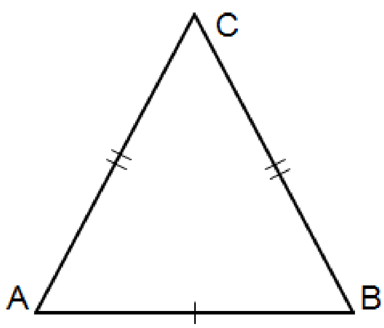
13. Find the value of *x*.



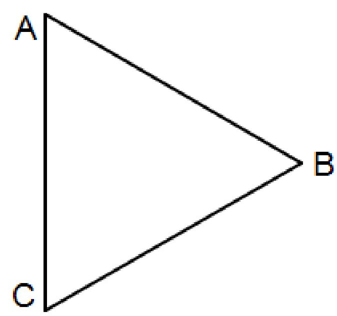
14. Find the value of *x, y* and *z* if ΔABC is equiangular.



15. If AC = 15, what is BC?



16. If the triangle shown is equilateral and the vertical side has a length of 8.2, what are the lengths of the other two sides?



17. A triangle in which the lengths of all sides are different is called a __________ triangle.

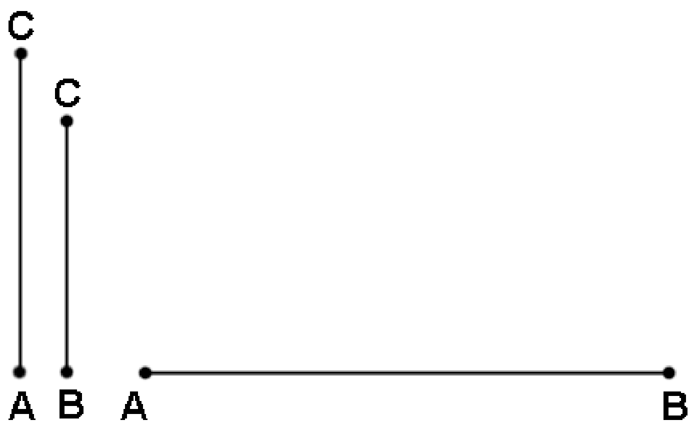
18. A triangle in which only two sides have equal lengths is called an __________ triangle.

Unit 5: Lesson 02 Triangle inequalities; Constructing a triangle

Beginning with $\overline{AB}$ as the base of a triangle, construct a triangle having $\overline{AC}$ and $\overline{BC}$ as the other two sides.

With the point of the compass at A and with a span equal to AC, strike an arc as shown. With the point of the compass at B and with a span equal to BC, strike another arc as shown.

The intersection of the two arcs is the third vertex C of the triangle. Complete the triangle by using a straight edge to connect points A & C and B & C. (This process is demonstrated in the video associated with this lesson.)



Notice in the construction above if AC and/or BC had been too short, the arcs would have never intersected.

In order for a triangle to be constructed, the **sum of the two smaller sides must be greater that the larger side.**

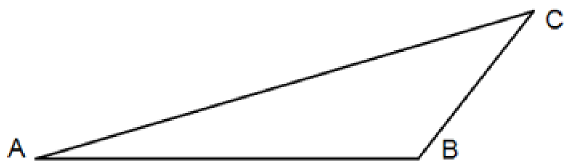
Example 1: Is it possible for a triangle to have sides of length 3, 7, and 9? If possible, classify the triangle.

Example 2: Is it possible for a triangle to have sides of length 20, 6, and 11? If possible, classify the triangle.

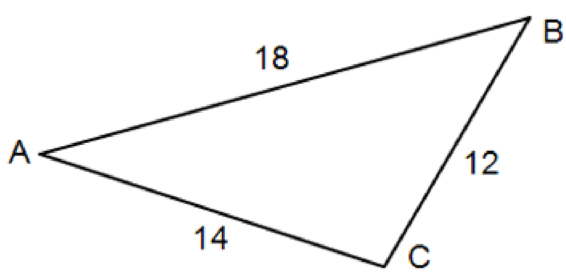
In a triangle the largest side is opposite the largest angle and the smallest side is opposite the smallest angle.

Likewise, in a triangle the largest angle is opposite the largest side and the smallest angle is opposite the smallest side.

Example 3: If $m\angle B = 130^{o}$, $m\angle A = 20^{o}$, and $m\angle C = 30^{o}$, name and list the sides from smallest to largest.



Example 4: If ΔABC has sides with lengths as shown, list the angles from largest to smallest. Classify the triangle type with regard to the lengths of the sides.



Assignment: In problems 1 - 4, decide (yes or no) if it is possible to construct a triangle having sides of the three given lengths. If possible, classify the triangle.

1. Lengths 2, 4, 7

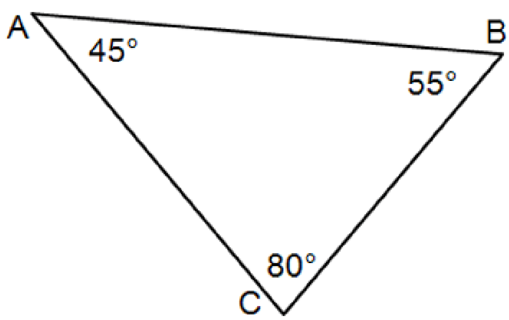
2. Lengths 8, 9, 8

3. Lengths 21, 21, 21

4. Lengths 7, 10, 12

In problems 5 - 6, name the line segment that forms the triangle's shortest side.

5.



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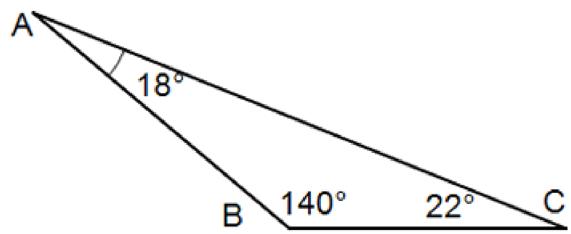
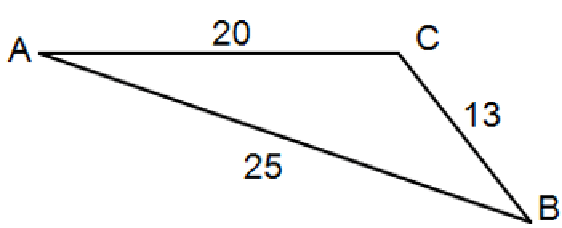


In problems 7 - 8, name the triangle's largest angle.

7.



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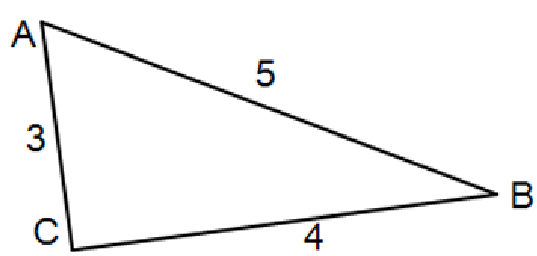
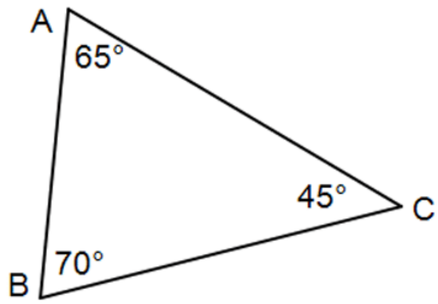


In problems 9 – 10 name and list the sides in order from largest to smallest.

9.



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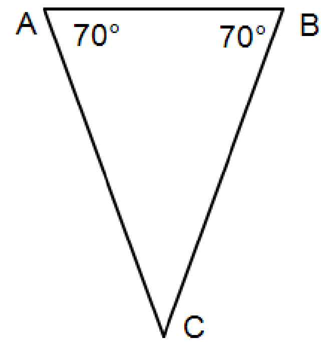
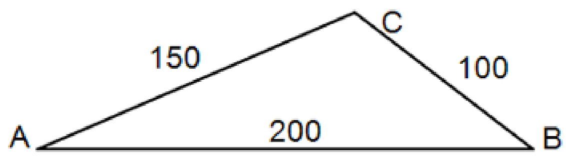


In problems 11 – 12 name and list the angles in order from smallest to largest.

11.



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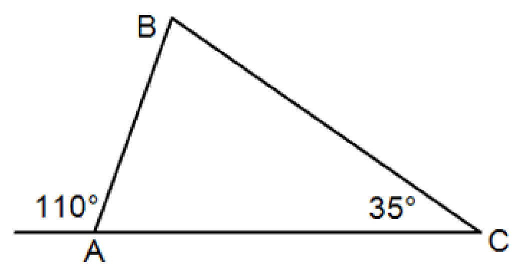
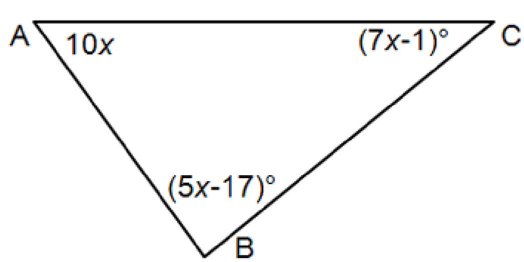
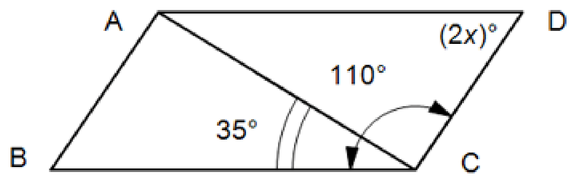


13. Find *x* and then list the angles from largest to smallest.



*14. Find *x* and then find m∠ACD.



Unit 5: Lesson 03 Polygons (interior angles)

Polygon:

A polygon is a closed figure with three or more sides. The line segments forming the sides intersect at their end points. These end points are called the **vertices** of the polygon.

A polygon has the same number of vertices as the number of sides.

Polygons fall into two broad categories: **concave** & **convex**

Polygons are named according to the number of sides:

Number of sides	Polygon Name	Number of sides	Polygon Name
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	11	Undecagon
7	Heptagon	12	Dodecagon
Generally, an n-sided polygon is called an n-gon.			

Theorem:

The **sum** of the measures of the **interior angles** of a convex polygon is $180(n-2)^{o}$ where *n* is the number of sides. See **In-Depth Topic G**, for a "proof", justification of this theorem.

Example 1: Apply the theorem above to a triangle to verify the theorem from Lesson 1, "The sum of the interior angles of a triangle is 180^{o}."

Example 2: Find the sum of the interior angles of a hexagon.

Regular polygon:

A polygon in which all angles are equal (equiangular) and all sides are equal (equilateral).

Corollary – A proposition(that needs little or no proof) that follows from something (a Theorem) already proven.

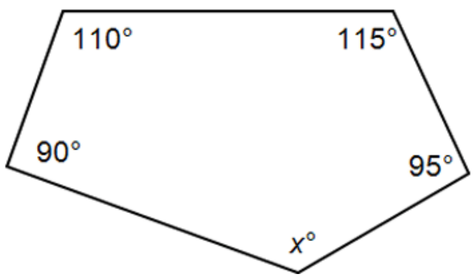
The measure of each interior angle of an n-sided regular polygon is:

$$\frac{180(n-2)}{n}$$

Example 3: What is the measure of the individual angles of a regular pentagon?

Example 4: If the measure of an interior angle of a regular polygon is 135°, how many sides does the polygon have?

Example 5: Find *x*.



Assignment: In problems 1 – 6 find the sum of the interior angles for the given polygon. Give the measure of each interior angle assuming the polygon is regular.

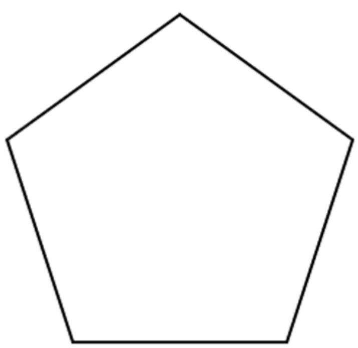
1. Quadrilateral

2. 28-gon

3. Decagon

4. Hexagon

5.



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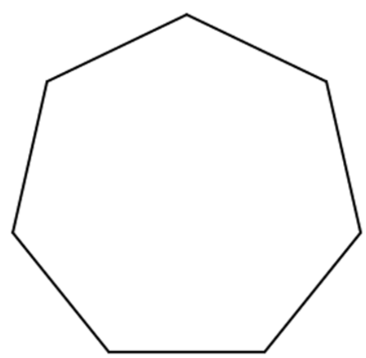
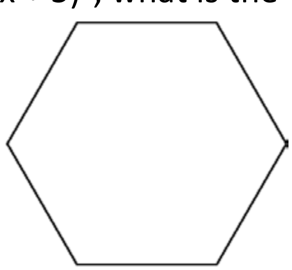
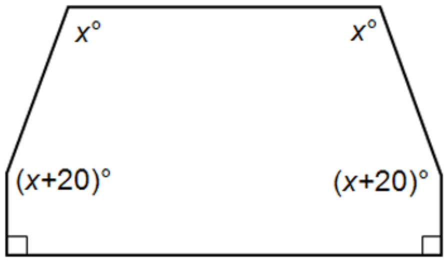


7. The polygon shown here is “regular”. If each interior angle has a measure of $(x + 5)^o$, what is the value of *x*?

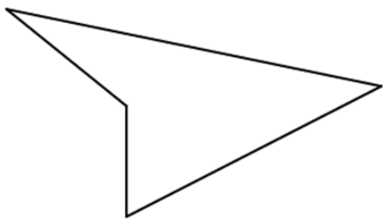


8. What is the value of x?



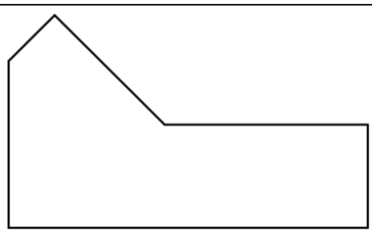
In problems 9 – 12, classify the polygon (hexagon, quadrilateral, etc.) and state whether it is convex or concave.

9.



10.

11.



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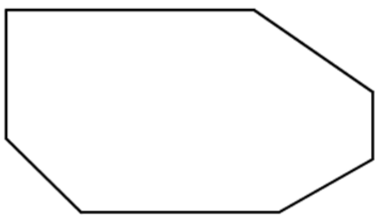
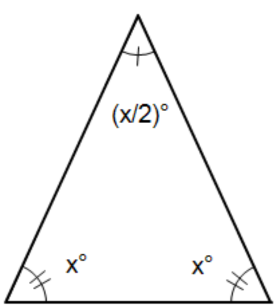


In problems 13 – 16, find x.

13.



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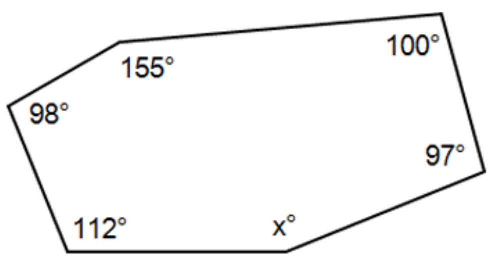


15.

16.

Unit 5: Lesson 04 **Exterior angles of a polygon**

An **Exterior angle** of a polygon is:

the angle formed by a side of a polygon and the extension of an adjacent side.

Example 1: Extend the sides of these polygons and mark all of the external angles.

Now consider the sum of the exterior angles of a convex polygon. Think of observing a person walking around this polygon and being just above and looking down.

The person turns a little at each vertex, and by the time he comes back to the starting point, his accumulated turns have amounted to one full revolution. (The video accompanying this lesson has a nice presentation of this.)

The **sum of the exterior angles** of any convex polygon is **360^o**.

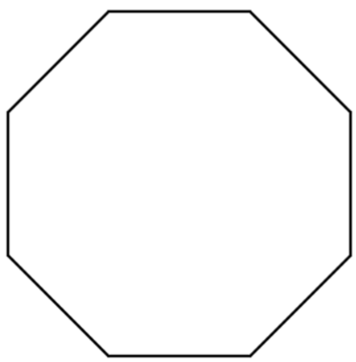
Example 2: The measure of each interior angle of a regular polygon is 150^o. What is the sum of the measures of all the exterior angles?

Regular polygons are both equiangular and equilateral.

The measure of each exterior angle of a regular n-gon is:

$$\frac{\mathbf{360}}{\mathbf{n}}$$

Example 3: What is the measure of each exterior angle of this regular polygon?



See **In-Depth Topic F** for a “proof” of the theorem concerning the exterior angles of a polygon.

Assignment: In problems 1 – 6, find the measure of each interior angles and the measure of each exterior angles of the given regular polygons.

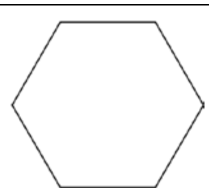
1. A triangle

2. A quadrilateral

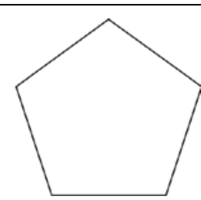
3. An octagon

4. A dodecagon

5.



6.



In problems 7 – 10 use the given information to determine the number of sides of the regular polygon.

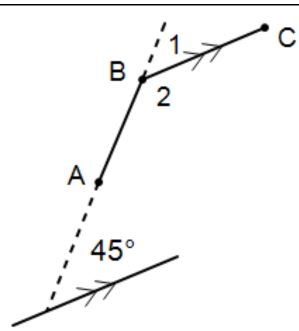
7. Measure of interior angle = 120°.

8. Measure of exterior angle = 30°.

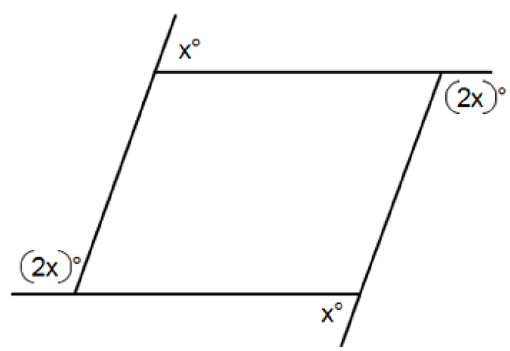
9. A polygon in which the sum of the interior angles is 900°.

10. A polygon in which the sum of the exterior angles is 360°.

11. $\angle 2$ is one of the interior angles of a regular polygon. How many sides does this polygon have?

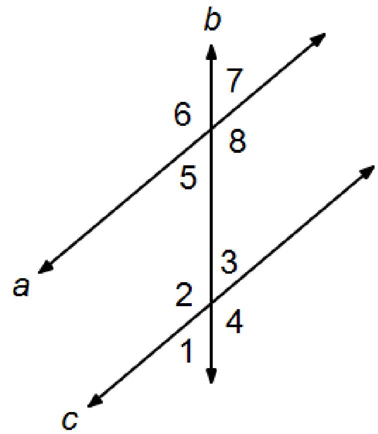


12. Find the value of *x*.



Unit 5: Cumulative Review

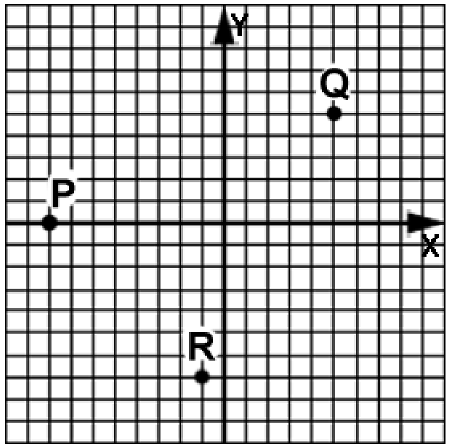
Use this drawing for problems 1- 4.
Assume a || c.



1. If $m\angle 3 = 52^{o}$, find $m\angle 8$.	2. If $m\angle 4 = 135^{o}$, find $m\angle 6$.
3. If $m\angle 3 = 58^{o}$, find $m\angle 2$.	4. If $m\angle 5 = (5x - 10)^{o}$ and $m\angle 3 = (3x + 10)^{o}$, find *x*.

5. Find the measure of an angle if its measure is 18^{o} less than twice its complement.

Use these points in problems 6 – 8.

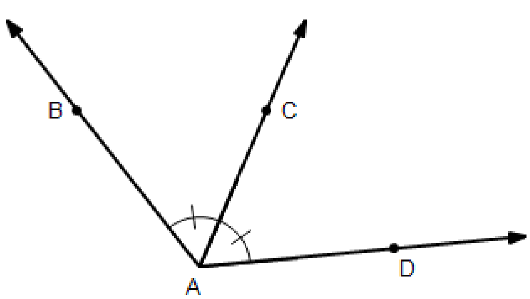


6. Find the midpoint of $\overline{QR}$.

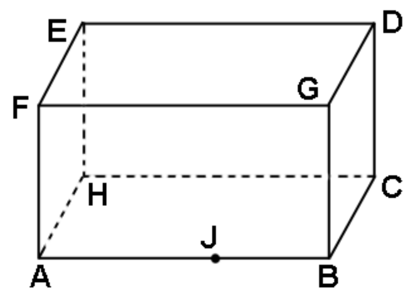
7. Find PR.

8. If R is the midpoint of $\overline{QH}$, find the coordinates of point H.

9. If m∠BAC = $(4x - 4)^{o}$ and m∠BAD = 128^{o}, what is m∠CAD?

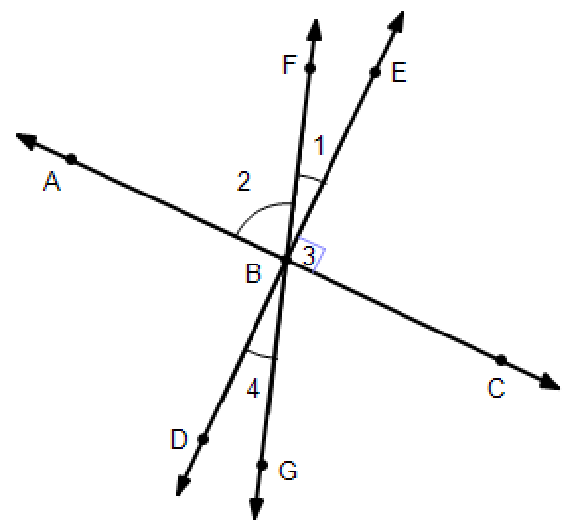


Use this drawing to answer questions 10 & 11.



10. Name the intersection of planes ACB and EDC.	11. Are points A, G, and D collinear?

Use this drawing for problems 12 - 15.
Assume $\overleftrightarrow{AC} \perp \overleftrightarrow{DE}$.



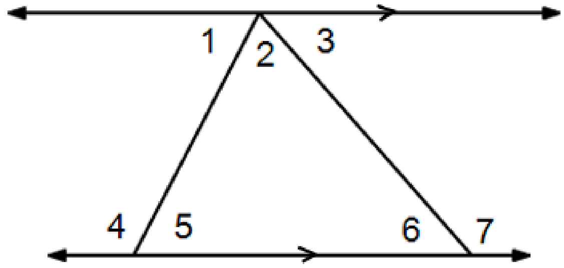
12. If m∠2 = 72°, what is m∠1?	13. If m∠2 = 72°, what is m∠4?
14. ∠4 and ∠GBC have what special relationship?	15. What is the measure of ∠ABE?

16. Find *a* and *b* if

$m\angle 4 = 21b - 7$

$m\angle 5 = 7a + 5$

$m\angle 1 = 5a + 15$



Unit 5: Review

1. In an acute triangle, each angle is less than__________.

2. In an obtuse triangle, one angle is greater than __________.

3. In a right triangle, one angle is exactly ___________.

4. In an equiangular triangle, all angles are __________ and each measure is_________.

5. In a scalene triangle, all angles and sides are ___________.

6. In an isosceles triangle, only two sides have the same __________.

7. The smallest angle of a triangle is always opposite the _______________.

8. The sum of the measures of the interior angles of a triangle is ________.

9. The sum of the measures of the exterior angles of a convex polygon is ________.

10. An exterior angle of a triangle is equal to the sum of the two ____________________.

11. The sum of the two shortest sides of a triangle must be _______________ the longest side.

12. The longest side of a triangle is always ___________ the largest angle.

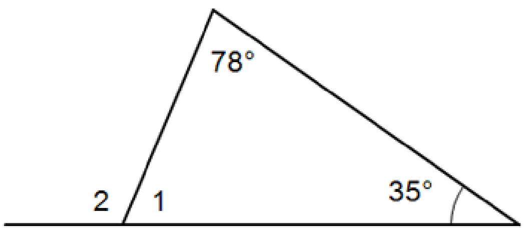
13. In an equilateral triangle, each side has the same __________.

14. The smallest angle of a triangle is always opposite the _____________.

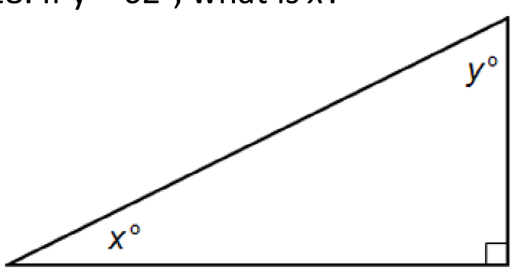
15. The sum of the measures of the interior angles of a convex polygon is ___________.

16. A ___________ pentagon is one in which all angles are equal and all sides are equal.

17. Find the measure of $\angle 1$ and $\angle 2$.



18. If y = 62°, what is *x*?

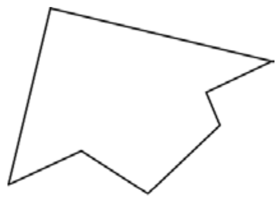


19. Is it possible for a triangle to have sides of length 12, 8, and 6?

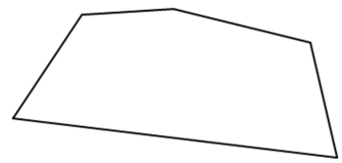
20. Is it possible for a triangle to have sides of length 4, 10, and 5?

21. If the measure of an interior angle of a regular polygon is 135°, how many sides does the polygon have?

22. Classify this polygon as either concave or convex.



23. Classify this polygon as either concave or convex.



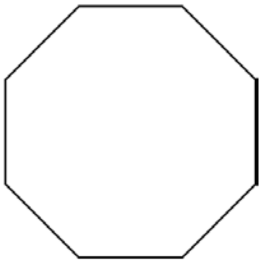
For each of the regular polygons in problems 24-27, find the sum of the interior angles, the measure of each interior angle, the sum of the exterior angles, and the measure of each exterior angle.

24. A pentagon

25. A 13-gon

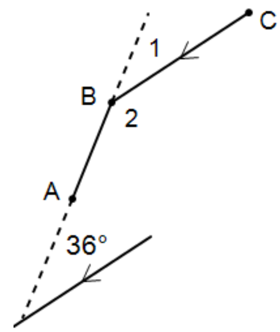
26. A decagon

27.

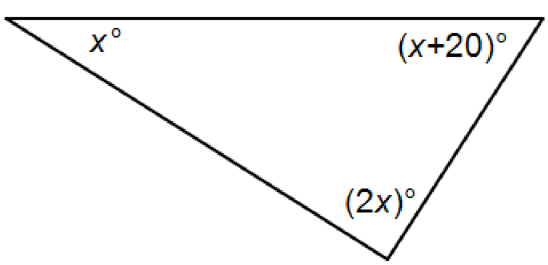


28. If the measure of an interior angle of a regular polygon is 157.5°, what is the number of sides of the polygon?

29. ∠2 is one of the interior angles of a regular polygon. How many sides does this polygon have?



30. Find the value of *x*.



Geometry, Unit 6

Quadrilaterals
Parallelograms & Trapezoids

Unit 6:
Lesson 01

Parallelogram fundamentals

Quadrilateral: a polygon having four sides.

Parallelogram: a quadrilateral in which opposite sides are parallel.

Naming convention:

Vertices:

Opposite sides:

Opposite angles:

Pairs of consecutive angles:

Diagonals: line segments connecting opposite vertices.

Theorems:

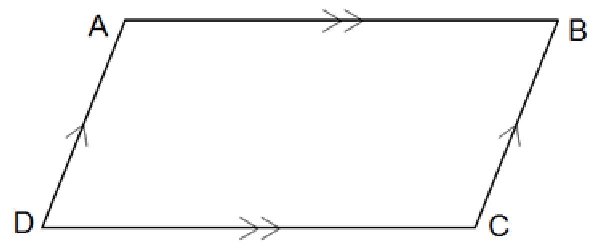
Opposite sides of a parallelogram are congruent (equal lengths). See **Theorem Proof G**.

Consecutive angle pairs of a parallelogram are supplementary.

Opposite angles are congruent. (See **Theorem Proof F**.)

Diagonals of a parallelogram bisect each other. (**Theorem Proof H**.)

Use this parallelogram in examples 1 – 8.



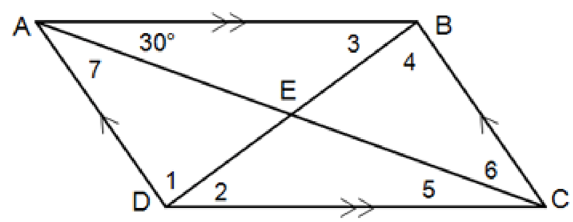
Example 1: $\overline{AD}$ is parallel to what other line segment.	**Example 2:** AB is equal to the length of what other segment?
Example 3: Name the parallelogram.	**Example 4:** Name the vertices of ▱ABCD.
Example 5: Name the angle opposite $\angle C$.	**Example 6:** Name the diagonal having a positive slope.

Example 7: Name two angles that are supplementary to $\angle C$.

Example 8: $\overline{AC}$ bisects what other line segment?

Example 9: Draw ▱GHJK and label m∠K = x^{o} and m∠J as (2x – 2)^{o}. Solve for *x*.

Use this drawing for examples 10-13.
$m\angle BAD = 65^{o}$, and $DB = 12$.



Example 10: Find $m\angle 7$, $m\angle 5$.

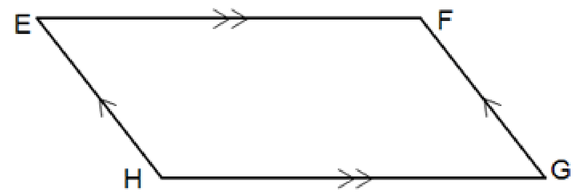
Example 11: Find $m\angle ADC$.

Example 12: Find $m\angle 6$.

Example 13: Find EB.

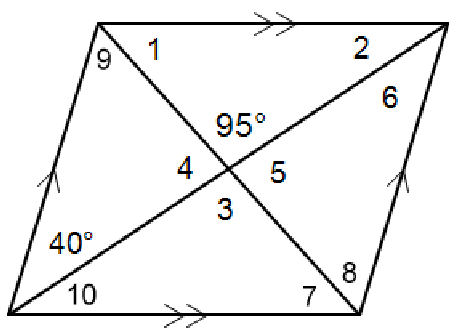
Assignment:

Use this parallelogram in examples 1 – 8.

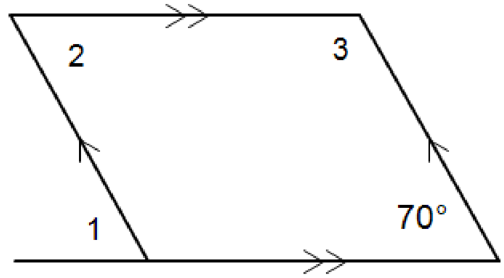


1. $\overline{\mathrm{EH}}$ is parallel to what other line segment?	2. HG is equal to the length of what other segment?
3. Name the parallelogram.	4. Name the vertices of ▱EFGH.
5. Name the angle opposite $\angle$E.	6. Name the diagonal having a negative slope.
7. Name two angles that are supplementary to $\angle$F.	8. $\overline{\mathrm{FH}}$ bisects what other line segment?

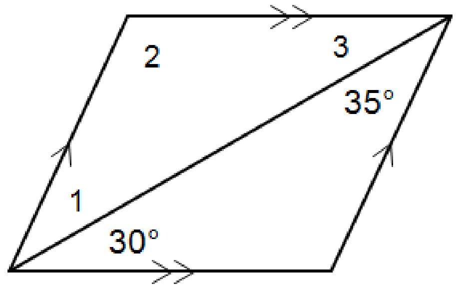
*9. If $\angle 1 = 60^{o}$, find all other numbered angles.



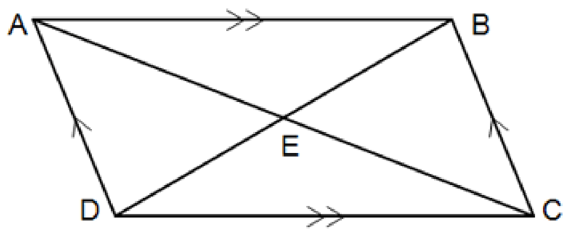
10. Find the measures of $\angle 1$, $\angle 2$, & $\angle 3$.



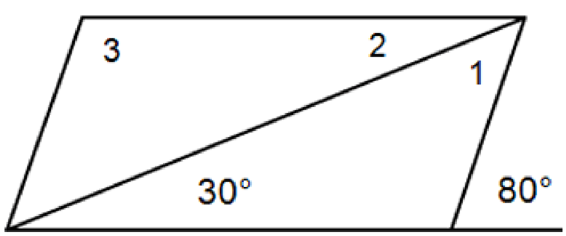
11. Find the measures of $\angle 1$, $\angle 2$, & $\angle 3$.



12. If EB = 3x + 1 and DB = 8x – 2, find *x*.



13. Find the measures of $\angle 1$, $\angle 2$, & $\angle 3$.



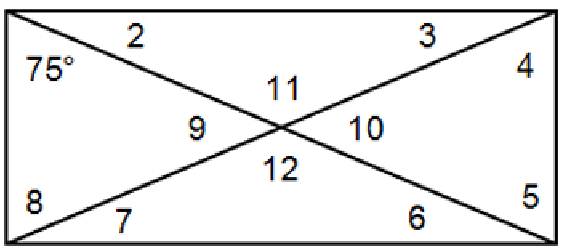
Unit 6: Lesson 02 **Rectangles**

Rectangle – a parallelogram in which all four angles are right angles.

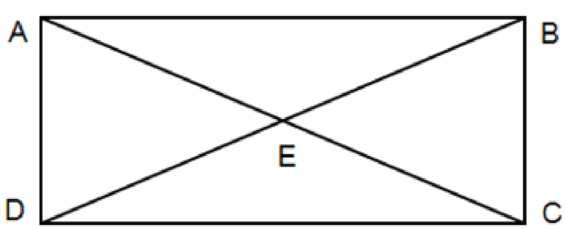
All the properties of parallelograms apply as well as this additional one:

The **diagonals** of a rectangle **are congruent** (same length).

Example 1: Find the measures of ∠2 - ∠12 in this rectangle.

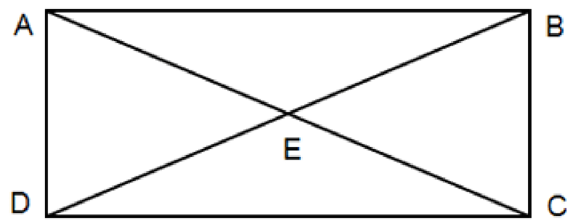


Example 2: In rectangle ABCD, DE = x – 3 while AE = 4x – 6. What is the value of *x*?



Assignment:

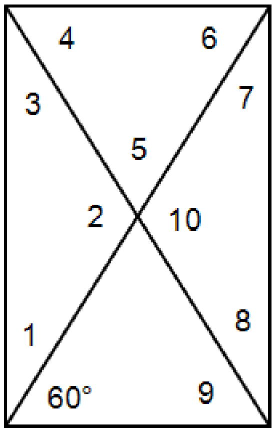
Use this drawing for problems 1 – 4.
Parallelogram ABCD is a rectangle.



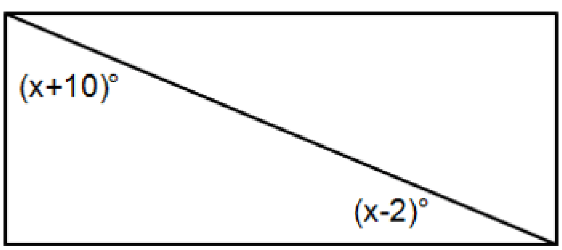
1. If EB = 16 and DB = 20z – 8, find *z*.	2. If AE = y + 4 and DB = 8y + 1, find *y*.
3. If BC = 11 what is AD?	4. If AC = 20 what is DE?

*5. If the coordinates of W are (-5, 6) while the coordinates of Y are (2, 3) of rectangle WXYZ, what is XZ?

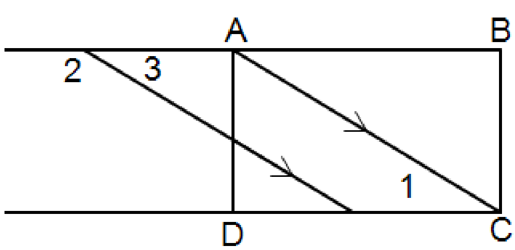
6. Find the measures of all the numbered angles in this rectangle.



7. Find the value of *x* in this rectangle.



8. If ▱ABCD is a rectangle and m ∠1 = 30^{o}, find m∠2.



9. Which one(s) of the following are true of both parallograms and rectangles?

a. The diagonals are equal in length
b. Opposite sides are equal
c. Opposite angles are equal
d. Adjacent angles are supplementary
e. All angles are equal

---

10. Which one(s) of the following are true?

a. All rectangles are also polygons
b. All polygons are also rectangles
c. The diagonals of all polygons bisect each other
d. The diagonals of a rectangle bisect each other
e. The diagonals of all polygons are congruent
f. Both diagonals of all rectangles are congruent

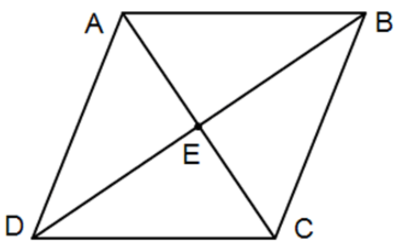
Unit 6: Lesson 03 Rhombi & squares

Rhombus (plural is rhombi) – a special parallelogram in which all four sides are congruent (same lengths).

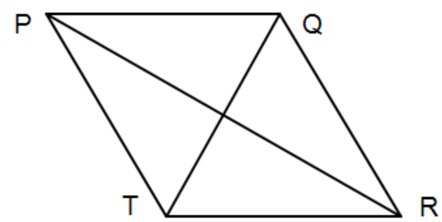
All of the properties of parallelograms still apply in addition to two more.

- The **diagonals are perpendicular**.
- The **diagonals bisect the angles** at all four vertices.

Example 1: If m∠DEC = $(4x + 10)^\circ$, find x in rhombus ABCD.



Example 2: If m∠PTR = 140°, find m∠RPQ in rhombus PQRT.

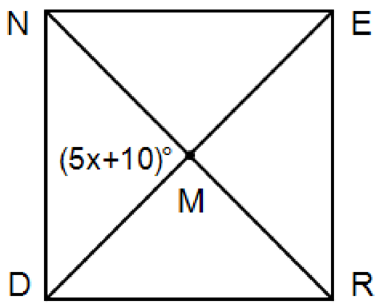


Square – a special parallelogram that is both a rhombus and a rectangle.

In addition to all of the properties of parallelograms, and rhombi, the square has this additional property:

The two diagonals are congruent (equal lengths).

Example 3: In the square shown, find *x* and m∠EDR.

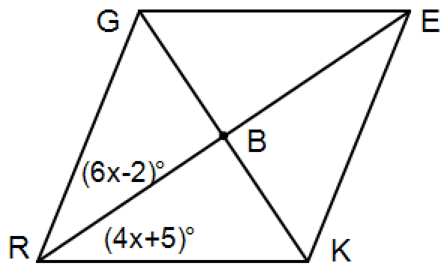


Example 4: In the square shown in example 3, find ND if RE = 24. What is the measure of m∠EMR?

See **In-Depth Topic E** for how to construct a square.

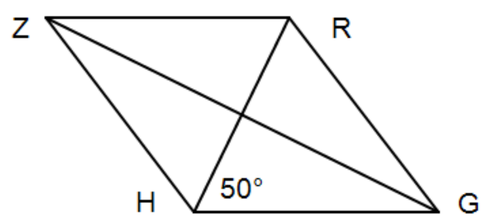
Assignment: In all problems, assume that the parallelogram shown is a rhombus.

1. Find the value of *x*.

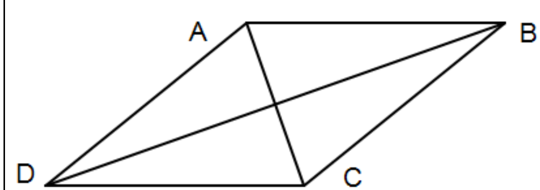


2. Use the drawing in problem 1 to find *y* if m∠EBK = (4y – 10)°.

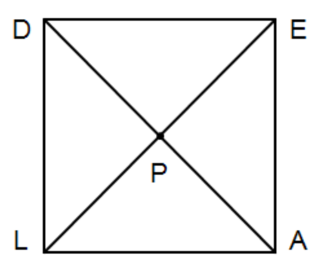
3. Find the measure of m∠GRZ.



4. If m∠BCD = 140°, what is the measure of m∠CBA?



5. If LE = 5x – 6 and PA = x + 1, what is *x* if the rhombus is also a square?



6. Using the drawing in problem 5, what is LA if EA = 6 if the rhombus is also a square?

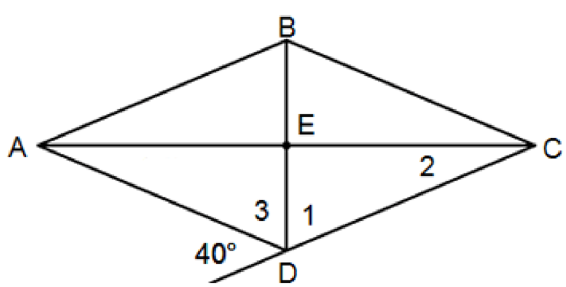
7. Consider rhombus ABCD. If AB = 19, is BC = 19?

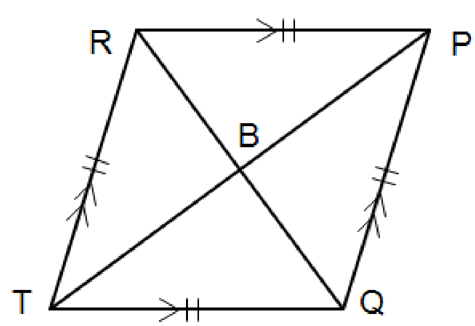
8. Consider rhombus ABCD. If AC = 13, is it necessary that BD = 13?

9. Consider rhombus ABCD. Is $\angle AMB$ congruent to $\angle CMB$ if M is the point at which the diagonals intersect?

10. Consider square PQRT. Is PR = TQ?

11. Find $m\angle 1$ and $m\angle 2$.





12. What must be true of $m\angle RTQ$ so that $RQ = TP$?

13. Using the drawing in problem 12, if $m\angle RTQ = 90^o$, what is the measure of $\angle RPB$?

14. Using the drawing in problem 12, is RQ = TP?

15. Which one(s) of the following are true of both a square and a rhombus?

a. The diagonals are equal in length
b. The diagonals bisect each other
c. The diagonals are perpendicular
d. All sides are equal in length
e. All interior angles are equal
f. All interior angles are right angles

16. Which one(s) of the following are true?

a. All rhombi are parallelograms
b. All parallelograms are rhombi
c. A rhombus is a square
d. A square is a rhombus
e. All rhombi are rectangles
f. All rectangles are squares

Unit 6: Lesson 04 Trapezoids

Trapezoid – a quadrilateral with exactly one pair of **parallel sides**.

Bases: the two parallel sides.

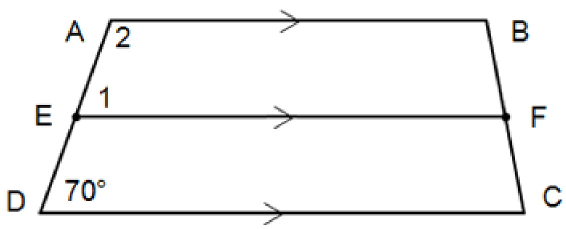
Legs: the two non-parallel sides.

Base angles: the two angles at the longer base.

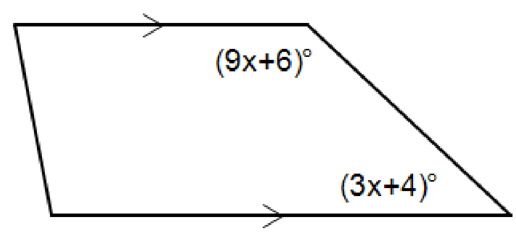
Median: the line segment joining the midpoints of the two legs.

The median is **parallel** to the two bases.

Example 1: In trapezoid ABCD, $\overline{EF}$ is the median. Find $m\angle 1$ and $m\angle 2$.



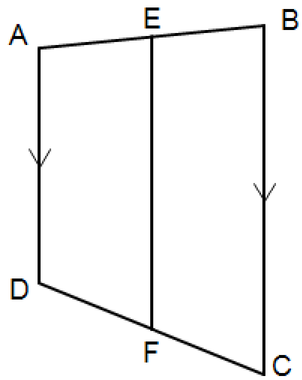
Example 2: Find *x*.



Theorem: The **length** of the median of a trapezoid is the **average of the two bases:**

$$\textbf{median} = \frac{\textbf{base1} + \textbf{base2}}{\textbf{2}}$$

Example 3: In trapezoid ABCD, $\overline{EF}$ is the median. If AD= 19 and BC = 23, what is EF?

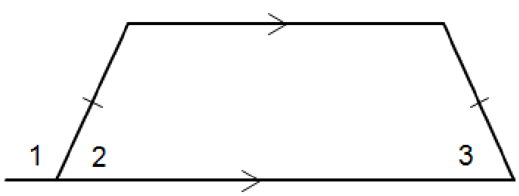


Example 4: Using the drawing in example 3, if EF = 20 and BC = 25, what is the value of AD? ($\overline{EF}$ is the median.)

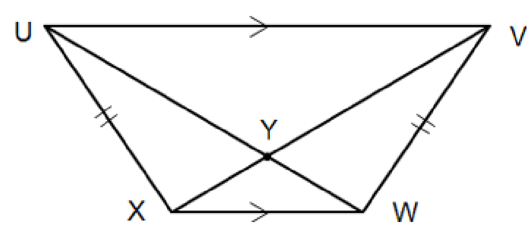
Isosceles trapezoid – a trapezoid in which the two legs are congruent.

- The diagonals are congruent (same length).
- Base angles are equal.

Example 5: If m∠1 = 104°, what is the measure of ∠2? Find m∠3.

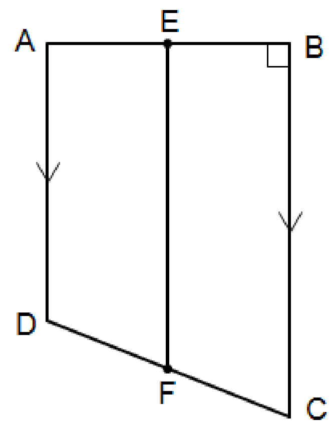


Example 6: If UW = 26 and XV = 8x – 6, what is the value of *x*?



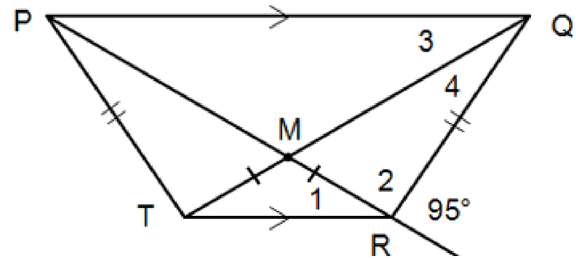
Assignment:

Use trapezoid ABCD for problems 1 – 4 where $\overline{\mathrm{EF}}$ is the median.



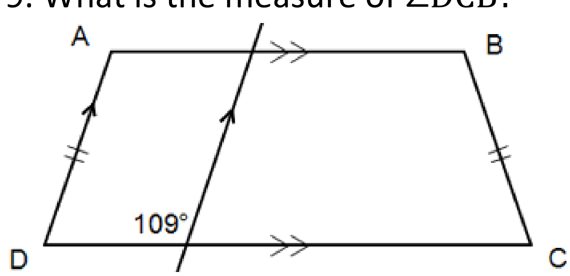
1. If the base angles measure 90^{o} and 80^{o}, what is the measure of $\angle\mathrm{CDA}$?	2. If AD is 2x + 6 and CB is 4x – 2, what is the length of the median?
3. If AD is 18 and the length of the median is 20, what is the length of the longer base?	4. If CB = 7x, EF = 5x – 3, and AD = x – 2, what is *x*?

Use this isosceles trapezoid for problems 5 – 8.

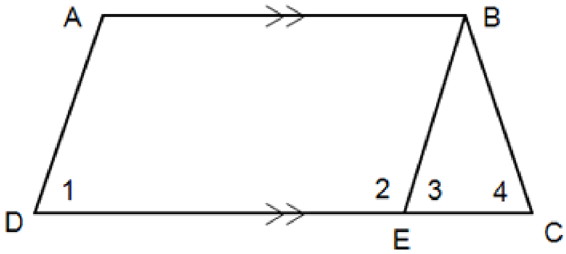


5. If m∠3 = 40^o, find m∠1 and m∠2.	6. If PM = 8 and MR = 4, what is the length of diagonal $\overline{TQ}$?
7. If m∠3 = 35^o and m∠4 = 25^o, what is the measure of ∠TPQ?	8. Find the value of *x* if PT = 11 and RQ = 6x – 8.

9. What is the measure of ∠DCB?



10. If ABCD is an isosceles trapezoid, what are the measure of ∠2, ∠3, & ∠4 when ∠1 = 75° and BC = BE?



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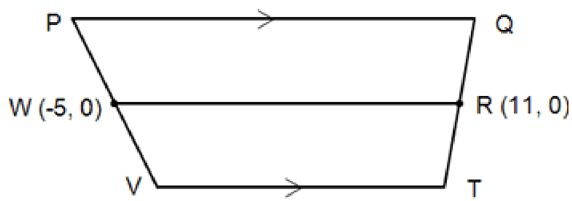
*11. If $\overline{WR}$ is a median of trapezoid PQTV, what is the sum of the lengths of the two bases?



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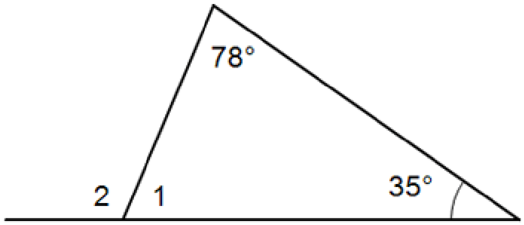
12. Which one(s) of the following are true?

a. All trapezoids are quadrilaterals
b. All quadrilaterals are trapezoids
c. It is possible for a trapezoid to have two pairs of parallel sides.
d. It is possible for a trapezoid to have two right angles
e. The diagonals of a trapezoid can never be congruent

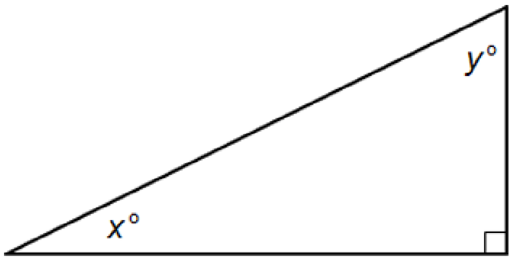
.

Unit 6: Cumulative Review

1. Find the measure of ∠1 and ∠2.



2. If x = 38°, what is *y*?



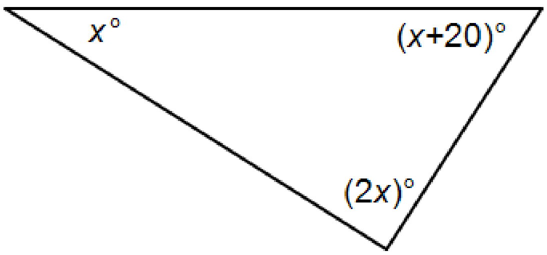
3. Is it possible for a triangle to have sides of length 40, 50, and 60?

4. Is it possible for a triangle to have sides of length 11, 20, and 8?

5. Find the sum of the interior angles, the measure of each interior angle, the sum of the exterior angles, and the measure of each exterior angle of a regular pentagon.

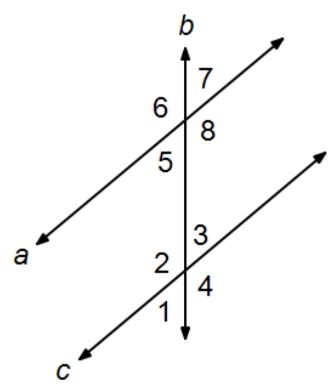
6. Find the distance between (-4, 5) and (-6, -12).

7. Find the value of *x*.



8. What are the coordinates of the midpoint of $\overline{\mathrm{AB}}$ when A is located 3 units to the right of the origin and B is loacted 18 units below the origin?

Use this drawing for problems 9 & 10.
Assume a || c.



9. If $\mathrm{m}\angle 1 = 42^{\circ}$, find $\mathrm{m}\angle 6$.

10. If m∠5 = $(5x - 10)^{\circ}$ and m∠3 = $(3x + 10)^{\circ}$, find *x*.

11. Find the intersection point of the graphs of the two lines whose equations are given here. Solve using **substitution**.

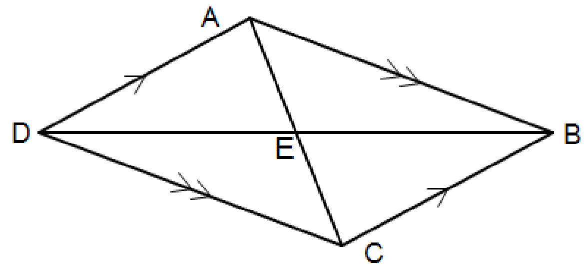
$3x - 2y = 5$
$2x - y = 11$

12. Find the intersection point of the graphs of the two lines whose equations are given here. Solve using **elimination**.

$3x - 2y = 5$
$2x - y = 11$

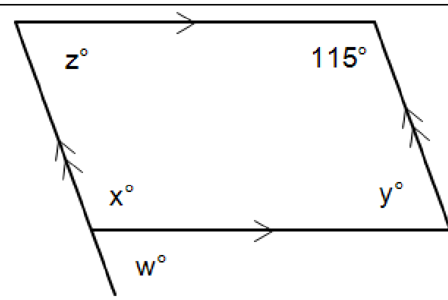
Unit 6: Review

Use this drawing in problems 1 – 4.



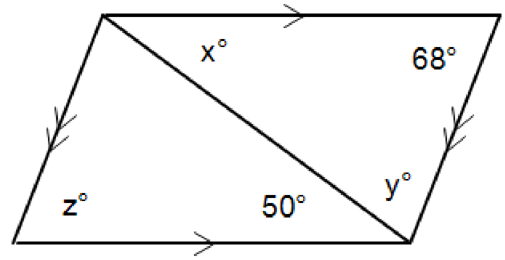
1. If AD = 37, what is the length of $\overline{BC}$?	2. If m∠ABC = 70°, what is m∠ADC?
3. If CA = 20, what is EC?	4. If m∠DAB = 112°, what is m∠ADC?

Use this drawing in problems 5 – 8.



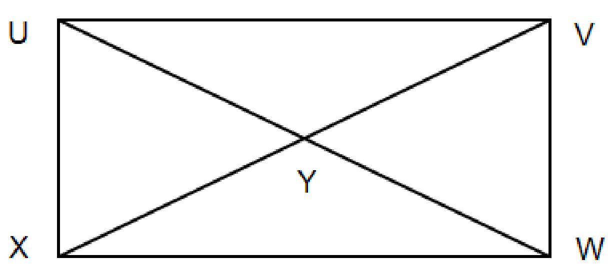
5. Find the value of *w*.	6. Find the value of *x*.
7. Find the value of *y*.	8. Find the value of *z*.

Use this drawing in problems 9 – 12.

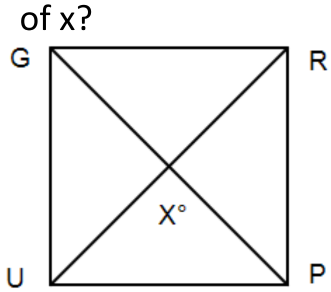


9. What is the relationship between the diagonal shown and the one not shown?	10. Find the value of *x*.
11. Find the value of *y*.	12. Find the value of *z*.

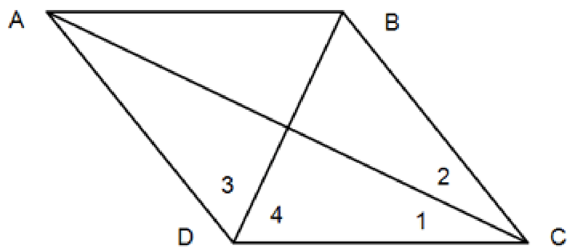
13. Parallelogram UVWX is a rectangle. If YW = 8x – 2 and XV = 22, what is *x*?



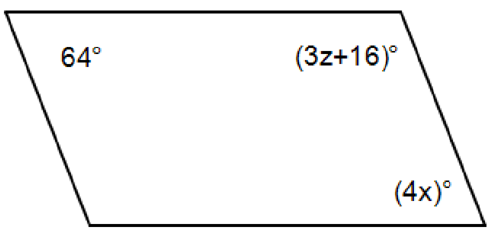
14. Find UP in square GRPU if the perimeter of the square is 48. What is the value of x?



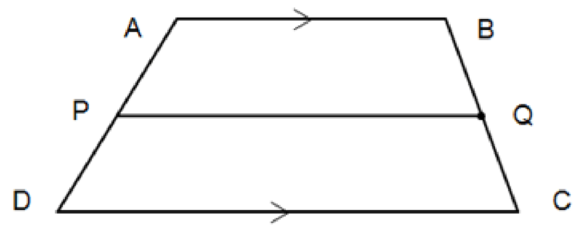
15. In rhombus ABCD m∠4 = $25x^{o}$ and m∠1 = $(10x – 5)^{o}$. Find *x*.



*16. Find the values of *x* and *z* such that the quadrilateral shown here is also a parallelogram.



Use the trapezoid shown here in problems 17 & 18.



17. If AB = 4x – 10, DC = 3x + 8, and the median has a length of 13, find *x*.

18. If AB = 10 and PQ = 13.5, what is DC?

Use the provided word bank as the best match for these descriptions.

	19. a four-sided polygon
	20. a quadrilateral in which opposite sides are parallel
	21. a segment that joins non-consecutive vertices
	22. a parallelogram with four right angles
	23. a parallelogram with four congruent sides
	24. a quadrilateral that is both a rectangle and a rhombus
	25. a quadrilateral with exactly one pair of parallel sides
	26. a line segment joining the midpoints of the legs of a trapezoid

Word bank

A	Quadrilateral
B	Diagonal
C	Median
D	Square
E	Trapezoid
F	Rhombus
G	Parallelogram
H	Rectangle

Geometry, Unit 7

Right Triangles
Trig ratios (sine, cosine, & tangent)

Unit 7: Lesson 01 The Pythagorean Theorem

Typically, the vertices and sides of a right triangle are named as shown here. Notice the small letters denoting the lengths of the sides are opposite the angles with corresponding capital letters.

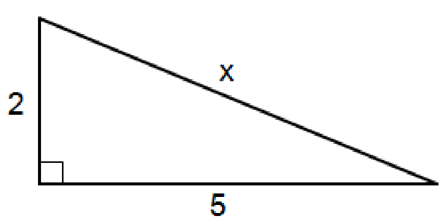
The relationship between the sides *a*, *b*, and *c* is known as the Pythagorean Theorem and is given by:

> *The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.* See the **Theorem Proof J** for a proof of this theorem.

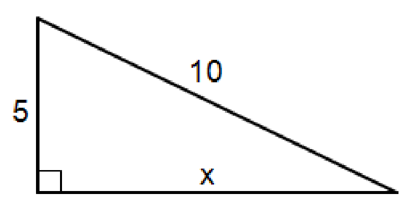
$$\mathbf{a^2 + b^2 = c^2}$$

Given the lengths of any two sides of a right triangle, the third side can be found with this equation.

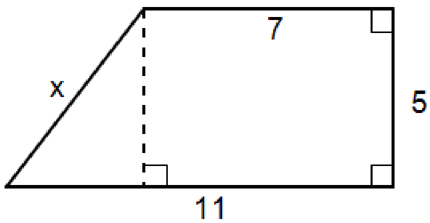
Example 1: Find *x*.



Example 2: Find *x*.



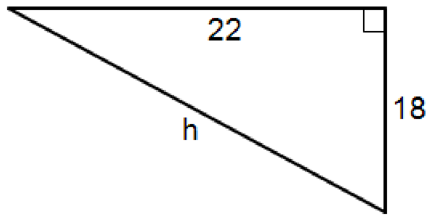
Example 3: Find *x*.



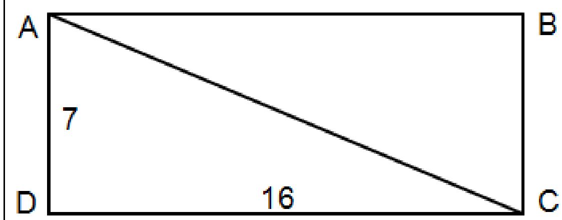
Example 4: What is the length of a diagonal of a rectangle of length 200 and width 100?

Assignment:

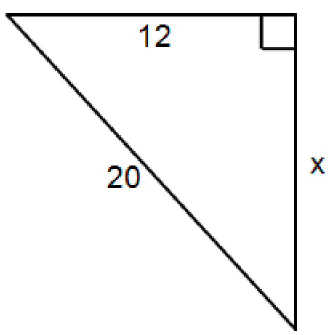
1. Find *h*.



2. Find the length of diagonal $\overline{AC}$ in rectangle ABCD.

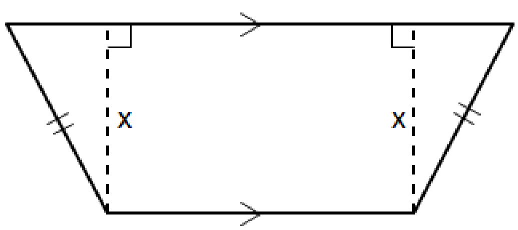


3. Find *x*.

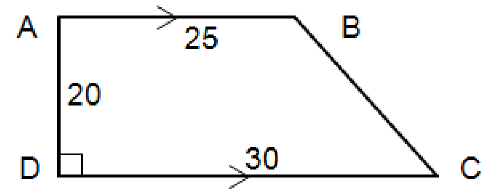


4. If the diagonal of a square has a length of 15, how long is each side of the square?

5. In this isosceles trapezoid, what is *x* if the length of the longer base is 17 and the length of the shorter base is 12? The length of the legs is 10.



Use this drawing for problems 6 and 7.



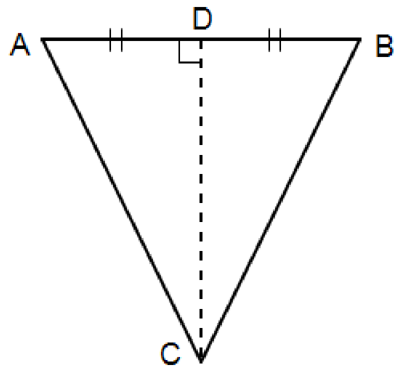
6. What is the length of diagonal $\overline{AC}$?

7. What is the length of diagonal $\overline{DB}$?

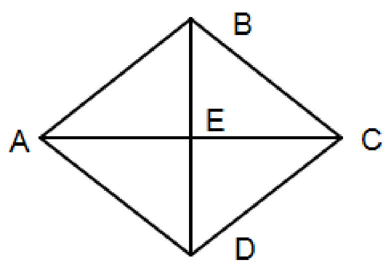
8. A rectangle has a length of 27 and a width of 19. How long is a diagonal?

9. A square has a diagonal of length 15.6. How long is a side of the square?

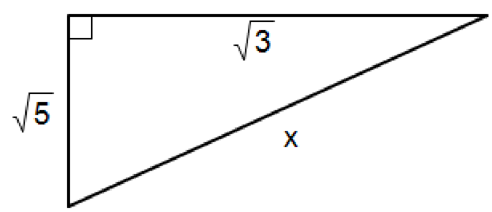
10. Isosceles triangle ABC has a base of length 10. How long are the legs if the altitude DC is 17?



11. Quadrilateral ABCD is a rhombus in which the diagonals are of equal length. If BE = 4, how long is each side of the rhombus?



12. Find *x*.



13. The shadow of a vertical flagpole is 7 ft long. If the flagpole is 15 ft high, how far is it from the end of the shadow to the top of the flagpole?

Unit 7: Lesson 02

Pythagorean triples
Converse of the Pythagorean Theorem

Pythagorean Triple:

> Any group of three positive integers (*a*, *b*, and *c*) that satisfy the Pythagorean theorem ($c^2 = a^2 + b^2$) is known as a **Pythagorean triple**. (Notice that *c* is always the largest of the three integers.)

Example 1: Determine if 3, 4, and 5 constitute a Pythagorean triple.

Example 2: Determine if 4, 5, and 7 constitute a Pythagorean triple.

Recall that the Pythagorean theorem states:

> *In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.*

The **converse** of the Pythagorean theorem states:

> *If in a triangle the square of one side is equal to the sum of the squares of the other two sides, then the triangle is a **right triangle**.*

Example 3: Determine if a triangle with sides 7, 25, and 24 is a right triangle.

Example 4: Determine if a triangle with sides 111, 114, and 15 is a right triangle.

The relationship of c^2 with the sum $a^2 + b^2$ can be used to categorize a triangle as:

- a right triangle
  (one angle is 90^o)

- an acute triangle
  (all angles are less than 90^o)

- an obtuse triangle
  (one angle is greater than 90^o)

In the following examples, the lengths of three sides of a triangle are given. First, determine if a triangle can be formed (longest sides must be less than the sum of the two smaller sides). Then determine if the triangle is right, acute, or obtuse.

Example 5: 7, 8, 6

Example 6: 11, 5, 17

Example 7: 40, 9, 41

Example 8: 7, 10, 16

Assignment:

1. Determine if 5, 13, and 12 constitute a Pythagorean triple.	2. Determine if 17, 7, and 15 constitute a Pythagorean triple.
3. Determine if a triangle with sides 7, 23, and 24 is a right triangle.	4. Determine if a triangle with sides 77, 36, and 85 is a right triangle.

In the following examples, the lengths of three sides of a triangle are given. First, determine if a triangle can be formed (longest sides must be less than the sum of the two smaller sides). Then determine if the triangle is right, acute, or obtuse.

5. Sides of length 8, 10, & 12	6. Sides of length 1, 2, & 3

7. Sides of length 34, 16, & 30

8. Sides of length 8, 20, & 13

9. Sides of length 8, 9, & 12

10. Sides of length 9, 14, & 27

11. Sides of length 8, 19, & 20

12. Sides of length $\sqrt{2}, \sqrt{3}$, & $\sqrt{5}$

13. Sides of length $\sqrt{3}$, 15, & 20

*14. Sides of length $\sqrt{13}$, 5, & $\sqrt{38}$

15. Write the Pythagorean theorem in words.

16. Write the converse of the Pythagorean theorem in words.

Unit 7: Lesson 03 A special triangle (45, 45, 90) Introduction to trig ratios

Draw square ABCD with one diagonal. Label the lengths of the sides as *x* and the diagonal as *h*.

Recall that each corner of the square is a right angle and that the diagonal bisects the interior angles, thus producing 45^{o} angles.

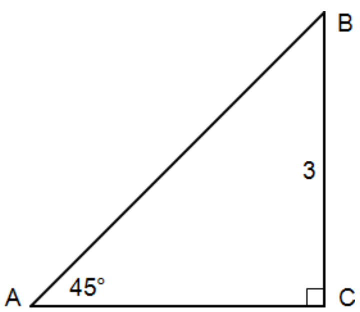
From the square above, draw triangle BCD and use the Pythagorean theorem to solve for *h*.

The following concerning a 45-45-90 triangle should be memorized.

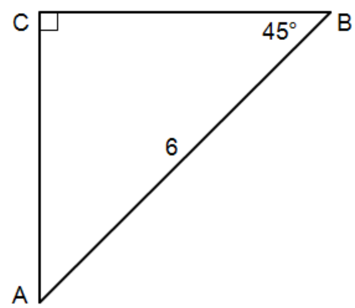
If we substitute x = 1 we have:

Memorize this!

Example 1: Use the "x-labeling" of a 45-45-90 triangle to find the lengths that are missing.



Example 2: Use the "x-labeling" of a 45-45-90 triangle to find the lengths that are missing.



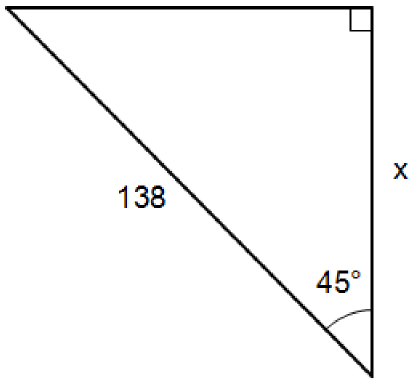
Trigonometric ratios for a 45-45-90 triangle:

The **sine** of an angle is the **ratio** of the side opposite an angle to the hypotenuse.

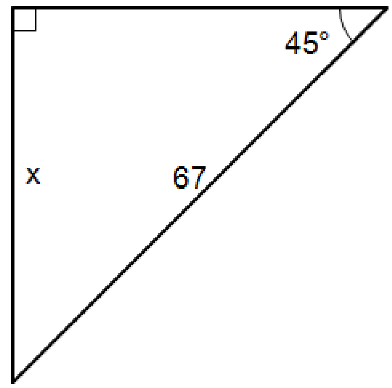
The **cosine** of an angle is the **ratio** of the side adjacent an angle to the hypotenuse.

The **tangent** of an angle is the **ratio** of the side opposite an angle to the side adjacent to the angle.

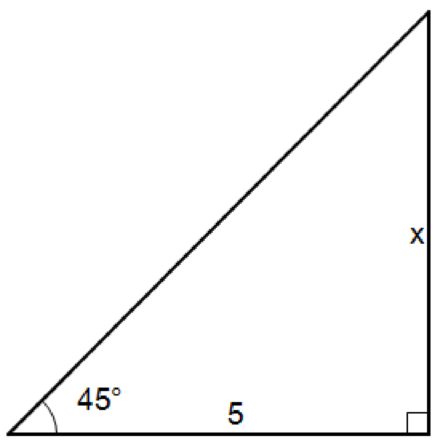
Example 3: Use a trig ratio to find *x*.

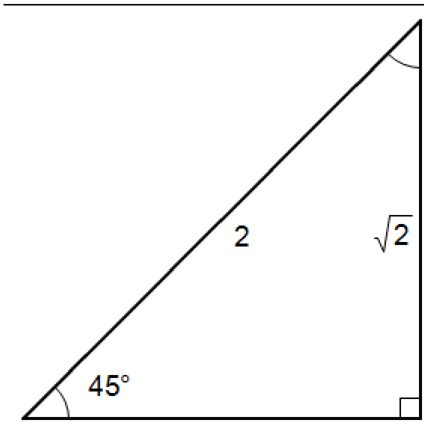


Example 4: Use a trig ratio to find *x*.



Example 5: Use a trig ratio to find *x*.





Example 6: Use a trig ratio to determine if the sides shown for this 45-45-90 triangle are possible.

Assignment:

1. What is the sine of 45^o?

2. What is the cosine of 45^o?

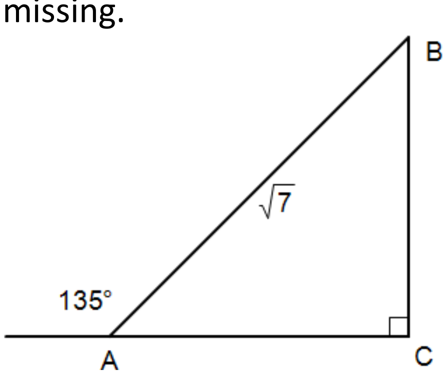
3. What is the tangent of 45^o?

4. What is the abbreviation for sine?

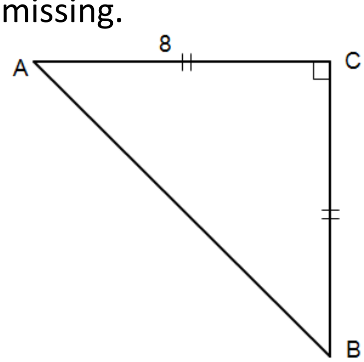
5. What is the abbreviation for cosine?

6. What is the abbreviation for tangent?

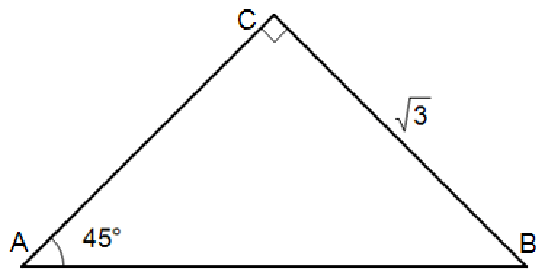
7. Use the "x-labeling" of a 45-45-90 triangle to find the lengths that are missing.



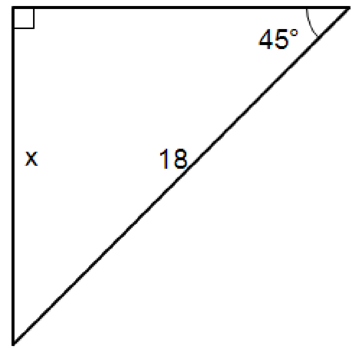
8. Use the "x-labeling" of a 45-45-90 triangle to find the lengths that are missing.



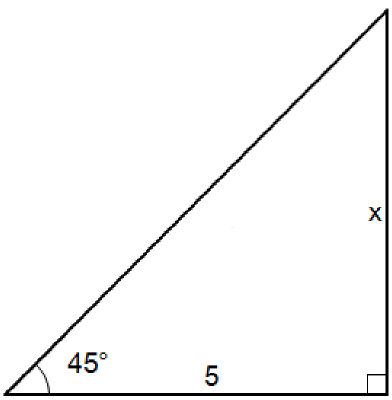
9. Use the "x-labeling" of a 45-45-90 triangle to find the lengths that are missing.



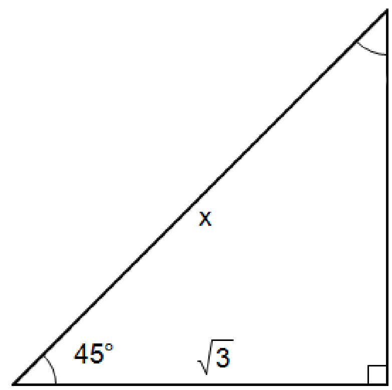
10. Use a trig ratio to find *x*.



11. Use a trig ratio to find *x*.

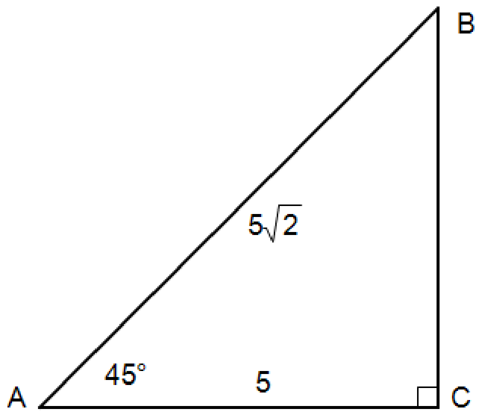


12. Use a trig ratio to find *x*.

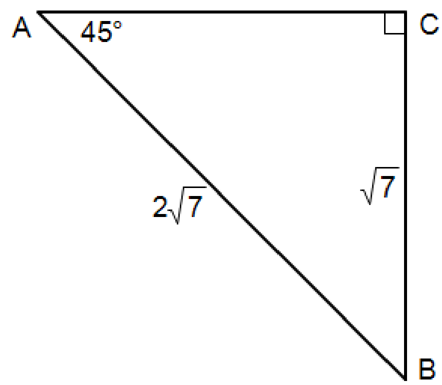


13. Comment on why the sine and cosine of 45° are the same.

14. Use a trig ratio to determine if the sides shown for this 45-45-90 triangle are possible.



15. Use a trig ratio to determine if the sides shown for this 45-45-90 triangle are possible.



16. Draw a right triangle and use standard labeling with *A, B, C, a, b,* & *c*.

Unit 7:
Lesson 04

Another special triangle (30-60-90)

Draw an equilateral triangle and drop a perpendicular (the altitude) from the top vertex A to the base.

Notice that this forms a 30-60-90 triangle. Draw this triangle labeling the hypotenuse as 2*x* and short side as *x*.

Use the Pythagorean Theorem to solve for *a* (the altitude) in terms of *x*.

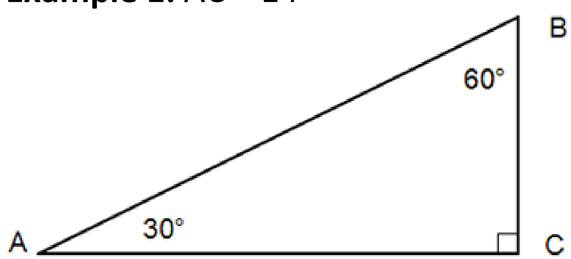
The following concerning a 30-60-90 triangle should be memorized.

If we substitute x = 1 have:

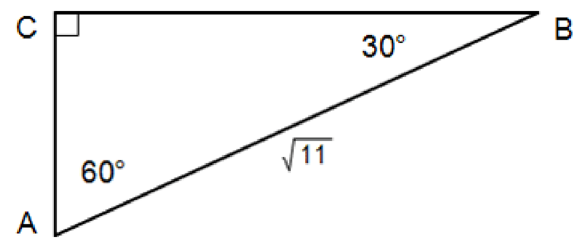
Memorize this!

In examples 1 & 2, use “x-labeling” of a 30-60-90 triangle to find the lengths that are missing.

Example 1: AC = 14



Example 2:



Trigonometric ratios for a 30-60-90 triangle:

$\sin(30^o) =$

$\cos(30^o) =$

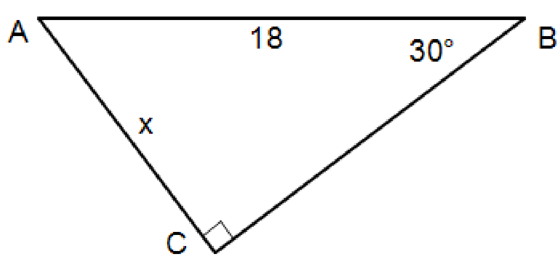
$\tan(30^o) =$

$\sin(60^o) =$

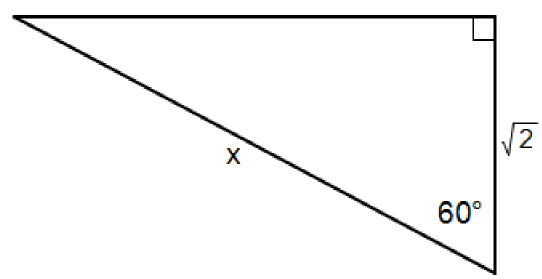
$\cos(60^o) =$

$\tan(60^o) =$

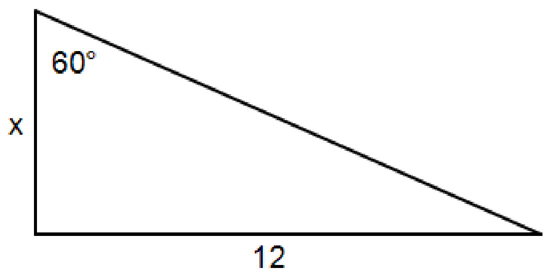
Example 3: Use a trig ratio to solve for *x*.



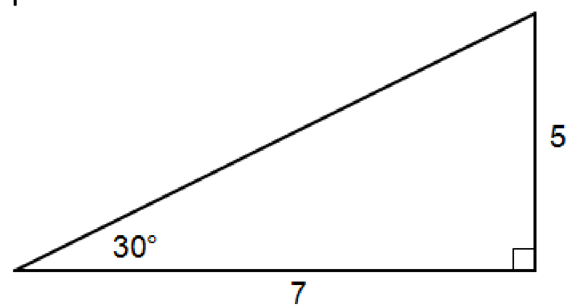
Example 4: Use a trig ratio to solve for *x*.



Example 5: Use a trig ratio to solve for *x*.



Example 6: Use a trig ratio to determine if the lengths of the sides shown for this 30-60-90 triangle are possible.

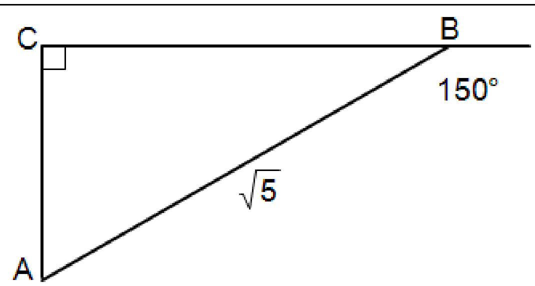


Assignment:

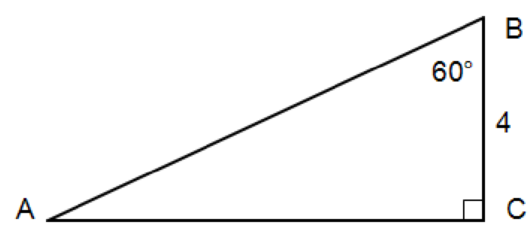
1. What is the sine of 30^o?	2. What is the cosine of 30^o?
3. What is the tangent of 30^o?	4. What is the sine of 60^o?
5. What is the cosine of 60^o?	6. What is the tangent of 60^o?

In problems 7 - 9, use the "x-labeling" of a 30-60-90 triangle to find the lengths that are missing.

7.



8.



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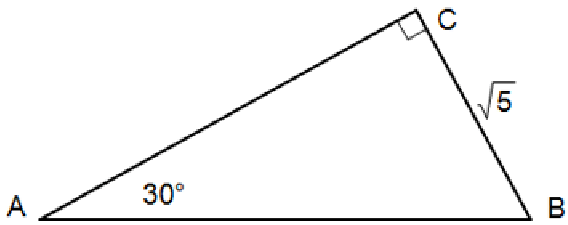
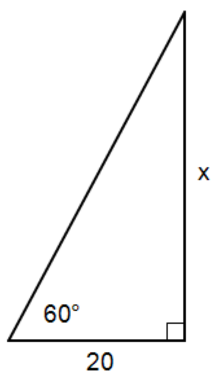
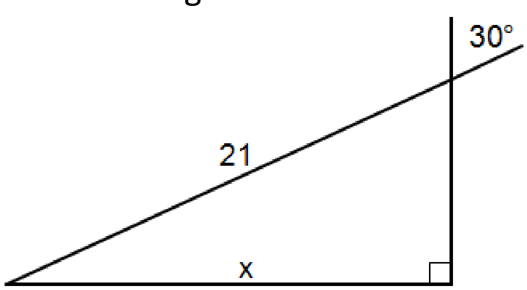


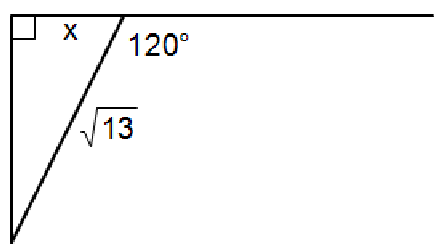
10. Use a trig ratio to find *x*.



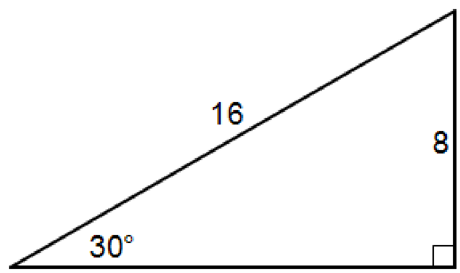
11. Use a trig ratio to find *x*.



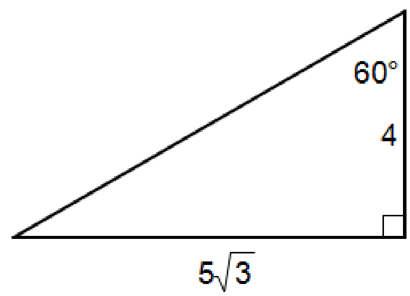
12. Use a trig ratio to find *x*.



13. Use a trig ratio to determine if the lengths of the sides shown for the 30-60-90 triangle are possible.



14. Use a trig ratio to determine if the sides shown for the 30-60-90 triangle are possible.



Unit 7: Lesson 05 — Trig ratios in right-triangles
Word problems using trig

The three main trig functions (ratios) – sine, cosine, & tangent – were given in the two previous lessons for 45°, 30°, and 60° angles.

The general definitions (for **any angle,** A) are:

$$\sin(A) = \frac{\text{opp}}{\text{hyp}}$$

$$\cos(A) = \frac{\text{adj}}{\text{hyp}}$$

$$\tan(A) = \frac{\text{opp}}{\text{adj}}$$

Please note that

- A is one of the acute angles of a right triangle,
- *opp* is the length of the side **opposite** angle A,
- *adj* is the length of the side **adjacent** to angle A,
- and *hyp* is the length of the hypotenuse.

In this course we will restrict ourselves to the three main trig functions above; however, there are three more trig functions. The other three are simply reciprocals of the main three. They are:

- secant (abbreviated, sec) - reciprocal of cosine
- cosecant (abbreviated csc) - reciprocal of sine,
- and cotangent (abbreviated cot) - reciprocal of tangent.

To find the trig function of any angle beside 30°, 45°, or 60°, use a scientific or graphing calculator.

For details on how to do this see **Calculator Appendix R** and an associated video.

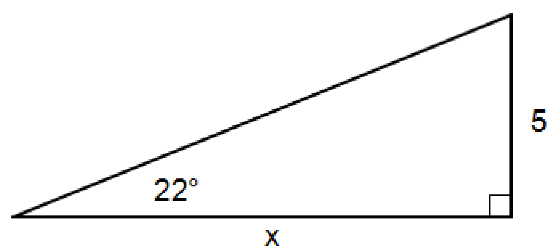
Use a calculator to find these trig functions in problems 1 – 4, rounded to 4 decimal places: (Be sure your calculator angle mode is set for degrees.)

Example 1: sin(23°)

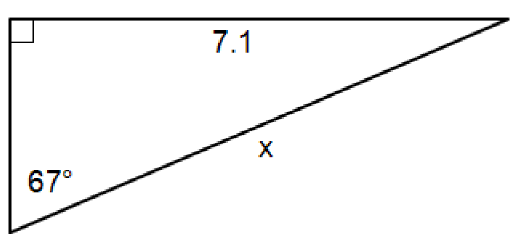
Example 2: cos(79.2°)

Example 3: tan(51°)

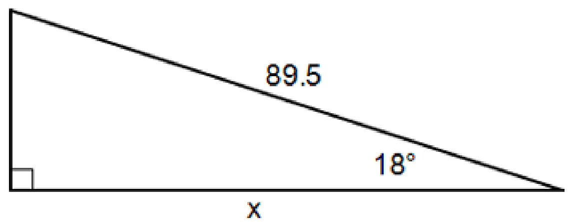
Example 4: Use a trig function to find *x*.



Example 5. Use a trig function to find *x*.



Example 6: Use a trig function to find *x*.



In word problems that involve a right triangle, draw the triangle and label the sides. Label the side asked for in the problem as some variable name, and then solve as in the examples above.

Example 7: An observer on the ground 300 ft from the base of a building measures from his position an angle of 36^o to the top of the building. If the angle is measured with respect to the ground, what is the height of the building?

The sine of an angle is equal to the cosine of its complement. Conversely, the cosine of an angle is equal to the sine of its complement.

Example 8: If sin(A) = .0736 what is the value of cos(90 – A)?

Assignment: In problems 1 – 4, use a calculator to find the indicated trig function's value to 4 decimal places.

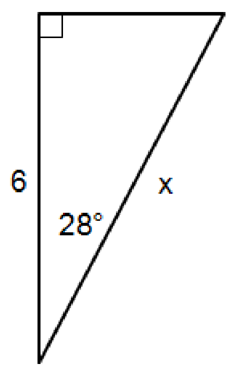
1. sin (29.22°)

2. cos (86°)

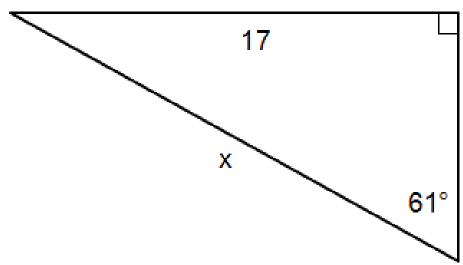
3. tan (9.72°)

4. sin (44°)

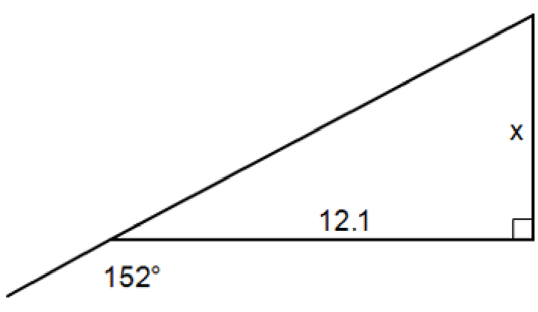
5. Use a trig function to find *x*.



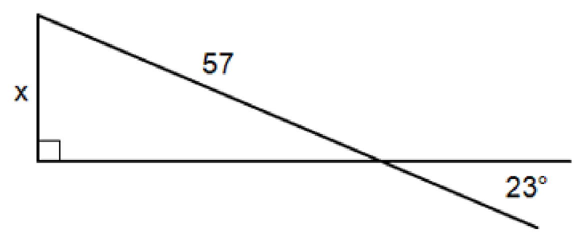
6. Use a trig function to find *x*.



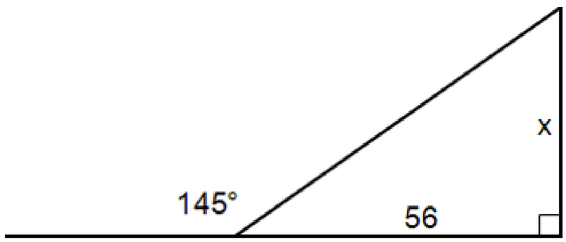
7. Use a trig function to find *x*.



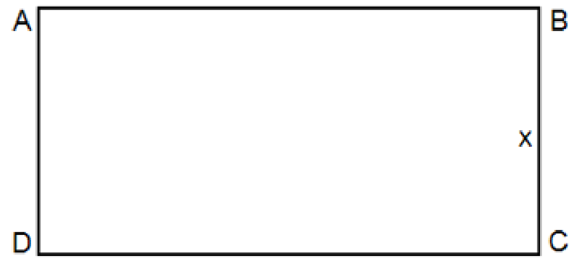
8. Use a trig function to find *x*.



9. Use a trig function to find *x*.



10. When diagonal $\overline{DB}$ is drawn in rectangle ABCD, $\angle BDC$ has a measure of 31.5^{o}. Find x if $BD = 13$.



11. An observer on the ground 500 m from the base of a building measures from his position an angle of 28^{o} to the top of the building. If the angle is measured with respect to the ground, what is the height of the building?

12. How far from the base of a building 200 ft high is an observer on the ground if from his position, he measures an angle of 31^{o} to the top of the building? (The angle is measured with respect to the horizontal ground.)

13. The sensors in a heat seeking missile at an altitude of 40,000 ft detect a target directly ahead and below at an altitude of 39,200 ft. How far away is the target if the angle from the missile to the target is 19^{o} with respect to an imaginary horizontal line?

---

14. The cosine of an angle is known to be .8772. What is the value of the sine of the complement of the angle?

Unit 7: Lesson 06

Solutions of non-right-triangles
Sine Law, Cosine Law, & a triangle area formula

Until this point in our studies we have been restricted to solutions of only right-triangles. No longer.

The **sine law** applies to **any type triangle**, not just right-triangles.

Each of the following proportions is an expression of the sine law. Notice that each ratio is made using an angle and a side that are **opposite each other** in the triangle.

For a derivation of the sine law see **Enrichment Topic H**.

Example 1: A = 66°, B = 32°, b = 19: Solve this triangle for the missing angles and sides.

There is a major pitfall when using the sine law: it is known as the "ambiguous case." This occurs when only **two sides and a non-included angle** are given. In this event, any one of the following is possible:

- No solution (an impossible situation)
- Exactly one solution (it's a right-triangle)
- Two different unique solutions

The sine law is unable to give a solution when **two sides and their included angle** is the only initial information given. In this case, application of the sine law always results in a single equation in two variables: all progress stops. **To solve such a problem the cosine law is needed**:

Since the angles and corresponding opposite sides could have been labeled differently, the **cosine law** should be learned as follows:

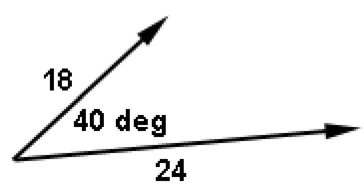
> ***The square of a side equals the sum of the squares of the other two sides minus twice the product of the other sides and the cosine of the angle between them.***

See **Enrichment Topic I** for a derivation of the cosine law.

Example 2: Solve the triangle ABC where $a = 5$, $c = 12$, $B = 28^{\circ}$.

Example 3: Two observers are stationed on an east-west line and are 2.3 miles apart. Observer A sees the steeple of the old church with a bearing of N 47^{o} E. Observer B reports a bearing of N 70^{o} W. How far is the church from B?

Example 4: Use the parallelogram method to add these vectors. Find both magnitude and direction of the resultant.



An important formula for finding the area of a triangle is:

> Area of a triangle = one-half the product of two sides times the sine of their included angle.

Example 5: In triangle ABC $a = 3$, $b= 2$, and $m\angle C = 95^o$. What is the area of the triangle?

Assignment: In problems 1 – 3, solve the triangle for the remaining sides and angles.

1. $A = 16^{o}$, $B = 57^{o}$, $a = 156.9$

2. $A = 46^{o}$, $C = 68^{o}$, $b = 11.04$

3. $B = 21^{o}$, $C = 61^{o}$, $c = 460$

4. Solve the triangle ABC where a = 200, b = 300, C = 130^{o}.

5. Find the angles of a triangle whose sides are: a = 20, b= 28, c = 21.

6. An observer in Dweeb City sights a UFO at a bearing of N 45^o E. Simultaneously, an observer in Nerdville sights the same UFO with a bearing of N 60^o W. How far is the UFO from Nerdville if Nerdville is 3.6 miles N 76^o E of Dweeb City?

7. Two forces of 250 lbs and 600 lbs are acting at the same point and make an angle of 65^o with each other. Find the magnitude of the resultant and its direction (the angle it makes with the 600 lb force).

8. In triangle PQR p = 7, q= 11, and m∠R = 38^{o}. What is the area of the triangle?

9. What is the exact (no calculator) area of an equilateral triangle in which the sides are of length 10?

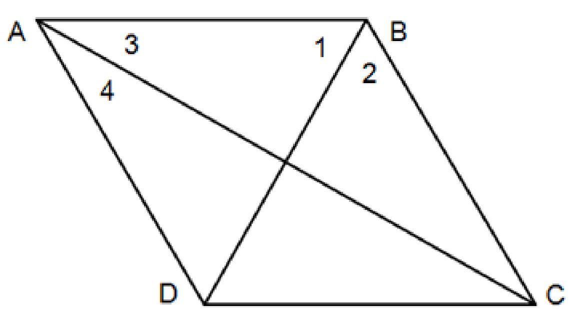
Unit 7: Cumulative Review

1. Draw line segment $\overline{\text{PV}}$ and point M as its midpoint. If PM = 5f + 2 and VM = 12 – f, what is the value of f?

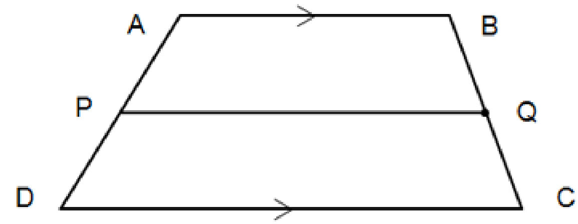
2. Use the information and results of problem 1 to determine PV.

3. Consider line segment $\overline{\text{AB}}$ of length 8 with point C being somewhere between A and B but not necessarily halfway in between. Draw $\overline{\text{AB}}$ and C and then determine if C is the midpoint of $\overline{\text{AB}}$ if AC = 8x – 12 and CB = 2x.

4. In rhombus ABCD m∠4 = 40° and m∠1 = (9x – 5)°. Find *x*.

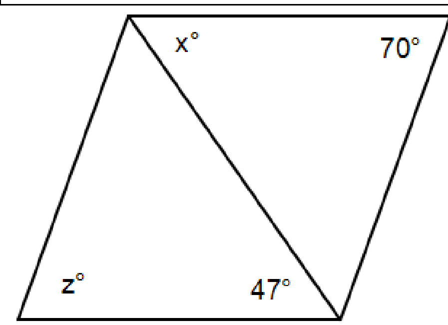


Use the trapezoid shown here in problems 5 & 6. $\overline{PQ}$ is a median.



5. If AB = 4x – 10, DC = 3x + 8, and the median has a length of 13, find *x*.	6. If AB = 9 and PQ = 12.5, what is DC?

Use the rhombus shown here in problems 7 - 10.

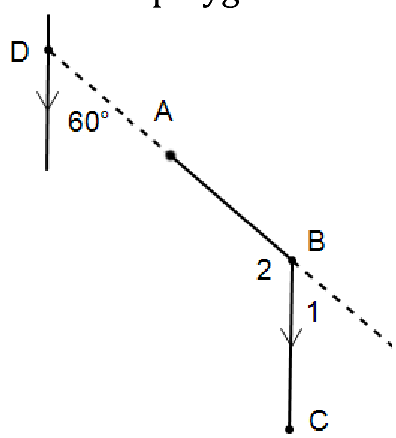


7. What is the relationship between the diagonal shown and the one not shown?	8. Find the value of *x*.

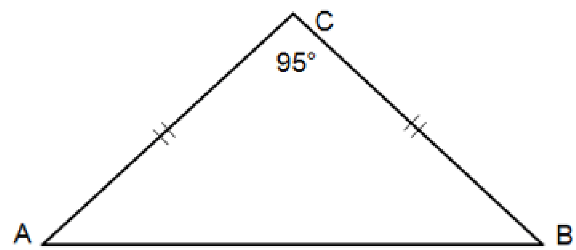
9. Find the measure of the angle adjacent to the angle whose measure is *x*.

10. Find the value of *z.*

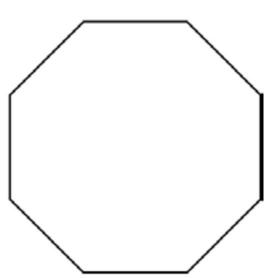
11. $\angle 2$ is one of the interior angles of a regular polygon. How many sides does this polygon have?



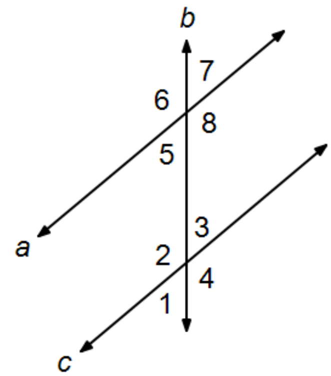
12. Find the measure of each of the base angles.



13. Find the sum of the interior angles of this regular polygon, the measure of each interior angle, the sum of the exterior angles, and the measure of each exterior angle.



Use this drawing for problems 14 & 15.
Assume a || c.



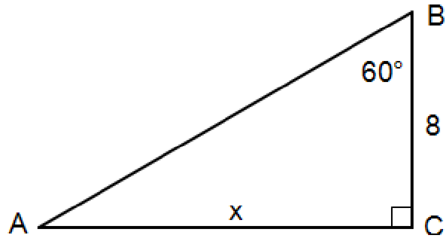
14. If $m\angle 4 = 141^{\circ}$, find $m\angle 7$.	15. If $m\angle 6 = 138^{\circ}$, find $m\angle 7$.

Unit 7: Review **Note to teachers:** This review does **not** include Lesson 6.

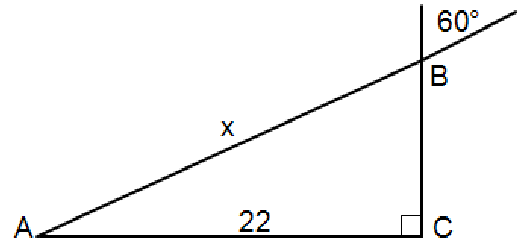
1. Draw a 45-45-90 triangle and label the lengths of the sides with the "standard" lengths.	2. Draw a 30-60-90 triangle and label the lengths of the sides with the "standard" lengths.
3. Using a ratio of sides in problem 1, give the sin(45^o).	4. Using a ratio of sides in problem 1, give the cos(45^o).
5. Using a ratio of sides in problem 1, give the tan(45^o).	6. Using a ratio of sides in problem 2, give the sin(30^o).
7. Using a ratio of sides in problem 2, give the cos(30^o).	8. Using a ratio of sides in problem 2, give the tan(30^o).

9. Use a trig ratio to find c in right-triangle ABC, when $m\angle A = 30^o$, and b = 11/2. Draw and label the triangle. (Do not use a calculator.)

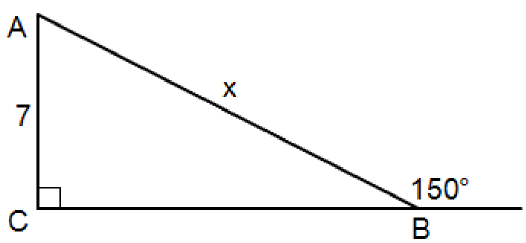
10. Use a trig ratio to find *x*.



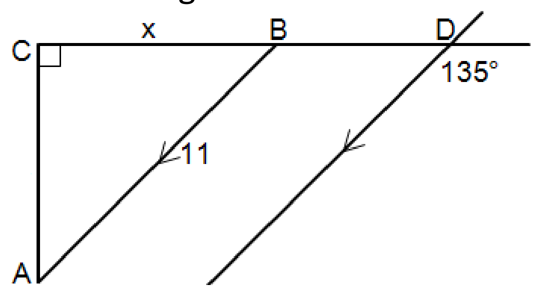
11. Use a trig ratio to find *x*.



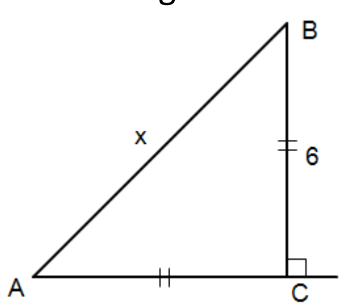
12. Use a trig ratio to find *x*.



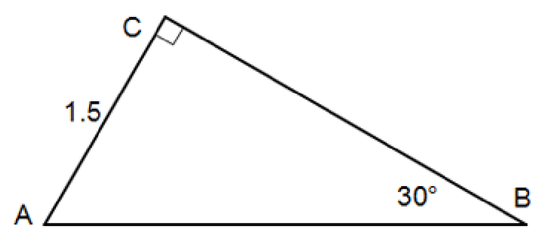
13. Use a trig ratio to find *x*.



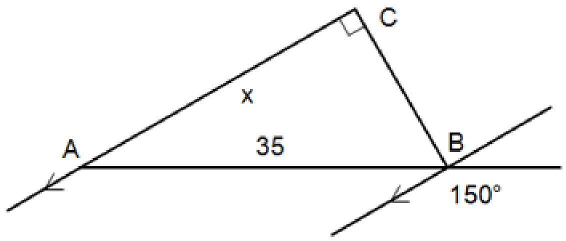
14. Use a trig ratio to find *x*.



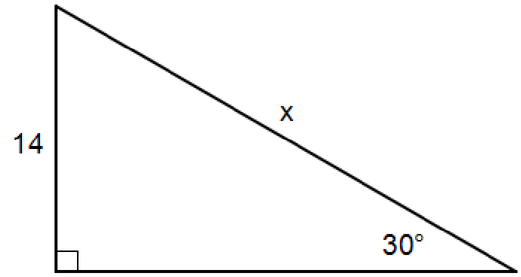
15. Use a trig ratio to find CB.



16. Use a trig ratio to find *x*.

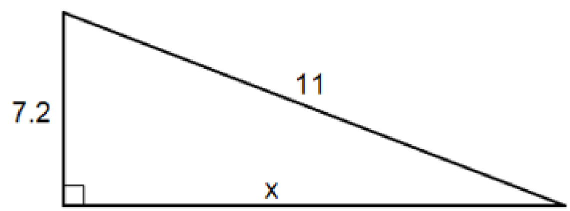


17. Use a trig ratio to find *x*. (Do not use a calculator on this problem.)

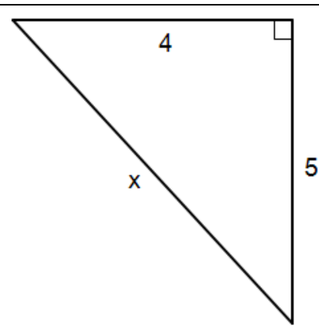


18. If the lengths of three sides of a triangle are 20, 35, & 40, determine if a triangle can be formed. Then determine if the triangle is right, acute, or obtuse.

19. Find *x*.

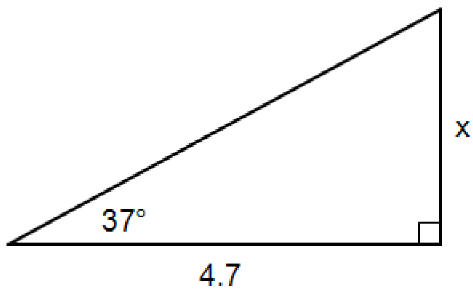


20. Find *x*.

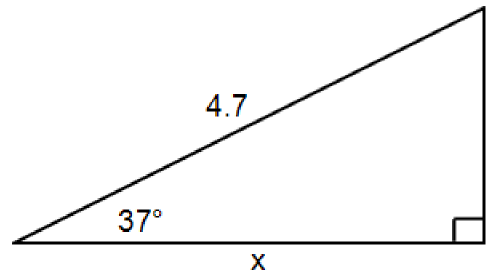


21. If the lengths of three sides of a triangle are 42, 18, & 36, determine if a triangle can be formed. Then determine if the triangle is right, acute, or obtuse.

22. Use a trig ratio to find *x*.



23. Use a trig ratio to find *x*.



24. Determine if 9, 40, and 41 constitute a Pythagorean triple.

25. Determine if 24, 45, 51 could be the sides of a right triangle.

Geometry, Unit 8

Ratios & Proportions
Parallel Lines and Proportional Parts
Similar Polygons

Unit 8: Lesson 01 — Practice with ratios and proportions
Associated word problems

Ratio: a comparison of two numbers by division.

The ratio of *a* to *b* can be written in either of these two ways:

Proportion: two ratios that are set equal to each other.

If the two ratios a/b and c/d are equal, then

Cross multiplying, we get

Example 1: Determine if the ratios 4/5 and 3/7 form a proportion.

Example 2: Determine if the ratios 11 : 5 and 33 : 15 form a proportion.

Proportions can be used to solve for a variable in the ratios.

Example 3: Solve for *x* in the proportion $3 : x = 5 : 4$.

Example 4: Solve for *x* when the ratio of $4x – 1$ to 5 is equal to $x : 4$.

In many word problems two variables can be related by $y = kx$ where *k* is a "constant of proportionality."

There are several ways to describe this relationship between *x* and *y*:

- *y* varies directly as *x*
- *y* varies as *x*
- *y* is directly proportional to *x*
- *y* is proportional to *x*
- *y* varies linearly with *x*

Such problems are easily solved with the proportion:

Example 5: If a Benedictine monk can copy 8 manuscripts in 5 days, how many days will it take him to copy 31 manuscripts?

Example 6: If the Blues are directly proportional to the Reds, and there are 5 Blues when there are 19 Reds, how many Reds can be expected when there are 15 Blues?

Example 7: In a poll just before the election, it was revealed that the ratio of those in favor of Senator Cheatum N. Steele to those against him was 3:5. At this rate, how many votes can he expect from 100,000 voters on election day?

Assignment:

1. What are two other ways to write the ratio of 4 to 145?

2. Determine if the ratios 17/9 and 121/ 57 form a proportion.

3. Determine if the ratios 18/7 and 108/42 form a proportion.

4. Solve for *x* in the proportion 7 : x = 11 : 4.

5. Solve for *x* when the ratio of 8x – 2 to 5 is equal to x : 4.

6. Solve the proportion $\frac{x+6}{5} = \frac{x}{3}$.

7. Solve the proportion $\frac{2}{x-2} = \frac{4}{x+6}$.

8. Write the proportion represented by "the ratio of boys to girls is 3 to 1."

9. The ratio of the poor to the rich is 19:2. If there are 38 rich people, how many are poor?

10. Two out of every nine students in Ms. Assignmore's classes have birthdays in February. If she has a total of 108 students, how many have birthdays in February?

11. *h* varies directly as *p*. If p = 9, h = 11, what will be the value of *p* when *h* is 121?

12. The profit varies linearly with demand. If the profit was $100 when the demand was 13, what would be the profit when the demand is 18?

---

13. A certain transversal is known to subdivide a line segment so that the ratio of the two parts is 11 : 2. If line segment $\overline{\mathrm{AB}}$ is cut by this transversal so that the longer part of the subdivided line is 22, how long is the shorter part?

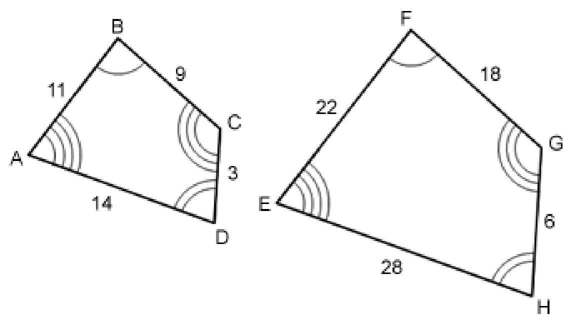
Unit 8: Lesson 02 Similar Polygons

Similar Polygons: polygons that have the same shapes but different sizes. For polygons to be similar, the following two things are **both** required.

- Corresponding angles of similar polygons are congruent.
- Corresponding sides of similar polygons are proportional.

(The mathematical symbol indicating similarity between polygons is the tilde, ~.)

Example 1: What angles must be congruent, and what are examples of the ratios of the sides that must all be the same (proportional)?



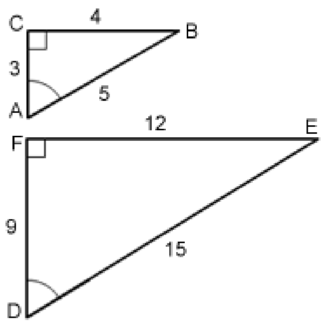
Scale factor: the ratio of the lengths of corresponding sides.

With similar polygons, the scale factor is consistently the same regardless of which ratio of corresponding sides is used. Even if only one ratio is inconsistent between two polygons, they are not similar.

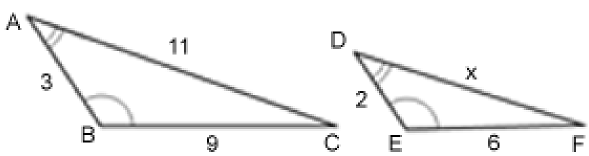
Example 2: Using the similar polygons of Example 1, determine the scale factor of quadrilateral EFGH to quadrilateral ABCD. Show that the scale factor is the same regardless of which ratio is used.

Example 3: Using the similar polygons of Example 1, determine the scale factor of quadrilateral ABCD to quadrilateral EFGH.

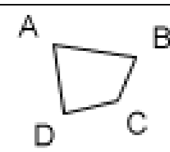
Example 4: Determine if these two triangles are similar.



Example 5: Triangle ABC ~ triangle DEF. Find the scale factor of triangle DEF to triangle ABC and determine *x*.



In discussions of similar polygons, the term "**included angle**" and "**included side**" are often used.



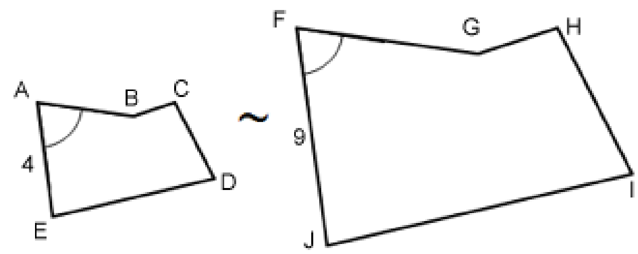
Example 6: Which angle is included between $\overline{AB}$ and $\overline{BC}$ in the drawing above?

Example 7: Which side is included between ∠B and ∠C in the drawing above?

Example 8: If the lengths of the sides of a triangle are in the ratio 4 : 5 : 6 and its perimeter is 150 ft, what is the length of the longest side?

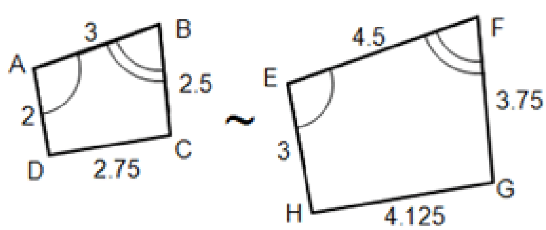
Assignment:

Use this figure for problems 1 – 6.

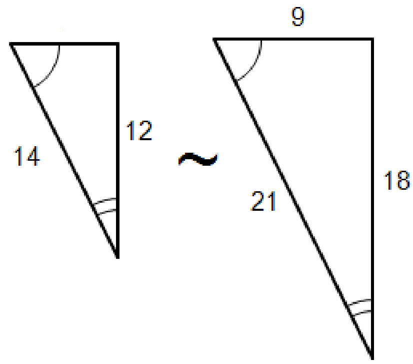


1. List the all congruent angle pairs.	2. List at least five ratios between sides that must all be equal to each other.
3. What is the scale factor of polygon ABCDE to polygon FGHIJ?	4. What is the scale factor of polygon FGHIJ to polygon ABCDE?
5. What side is included between ∠J and ∠F?	6. What angle is included between sides $\overline{BC}$ and $\overline{CD}$?

7. Show that the ratio of the perimeters is the same as the scale factor.



8. Find the missing side.

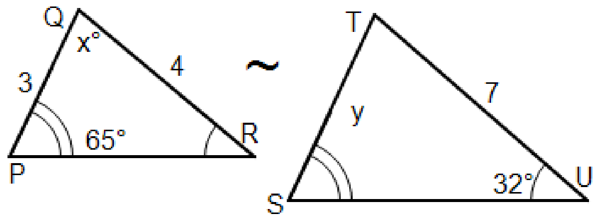


9. If the lengths of the sides of a triangle are in the ratio 3 : 7 : 8 and its perimeter is 162 ft, what is the length of the longest side? (Use the method of example 8 to work this problem.)

10. Work problem 9 again, this time using a scale factor. (Hint: The ratio of the perimeters is (3 + 7 + 8) : 162.)

11. If the lengths of the sides of a triangle are in the ratio 3 : 5 : 7 and its perimeter is 90, what is the length of the shortest side?

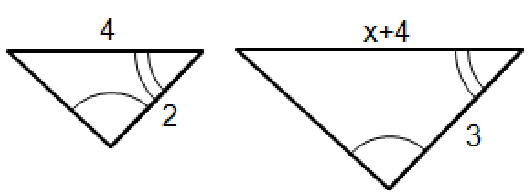
12. Find x and y in these two similar triangles.



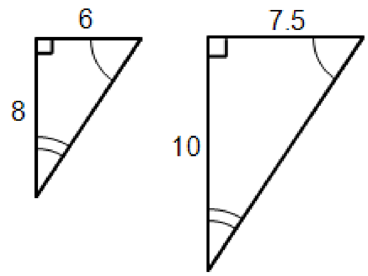
13. The ratio of the measures of two supplementary angles is 3 : 9. What are the measures of the angles?

14. The ratios of the measures of the interior angles of a pentagon are 4 : 6 : 7 : 9 : 10. What is the measure of the smallest interior angle?

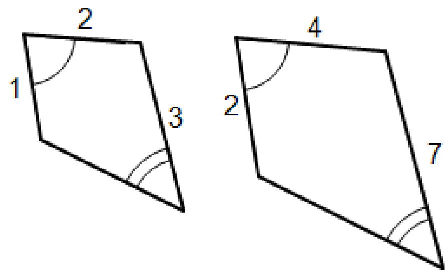
15. Find *x*.



16. Determine if these two triangles are similar.



17. Determine if these two polygons are similar.



Unit 8:
Lesson 03

Similar Triangles
AA, SAS, & SSS Similarity

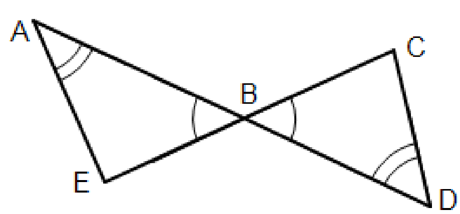
Recall that for polygons to be **similar**, there were two requirements:

- corresponding angles must be congruent, and
- corresponding sides must be proportional.

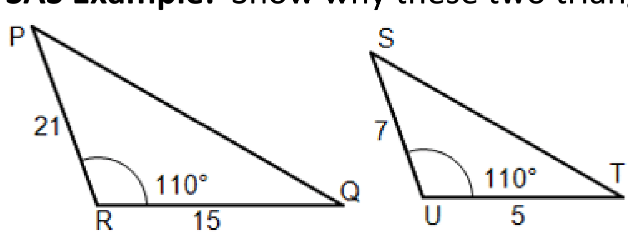
Triangles, being the simplest of all polygons, can additionally be shown to be similar by three simple methods:

- **AA** – If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.
- **SAS** –In a triangle, if two sides are proportional to the corresponding sides of another triangles, and the angles between the two sides are congruent, the triangles are similar.
- **SSS** – If all three pairs of corresponding sides of two triangles are proportional, then the triangles are similar.

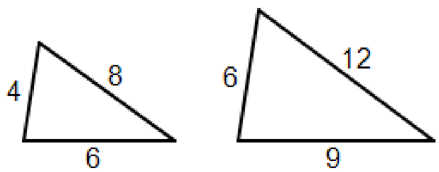
AA Example: Show why these two triangles are similar.



SAS Example: Show why these two triangles are similar.

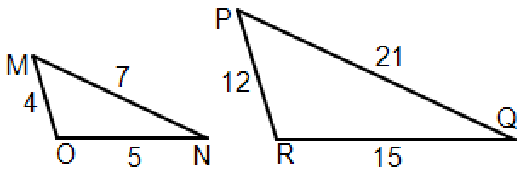


SSS Example: Show why these two triangles are similar. Give the scale factor of the first to the second triangle.

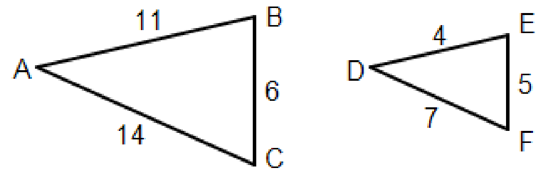


Assignment: In problems 1 – 8, determine if the two given triangles are similar. If they are similar, give the reason as AA, SAS, or SSS and give the scale factor (of the first to the second triangle) if possible.

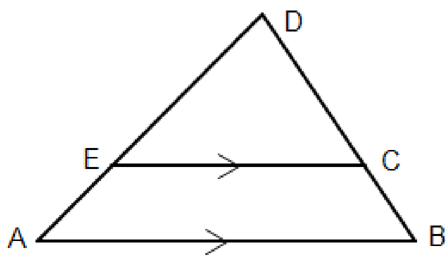
1. ΔMON, ΔPRQ



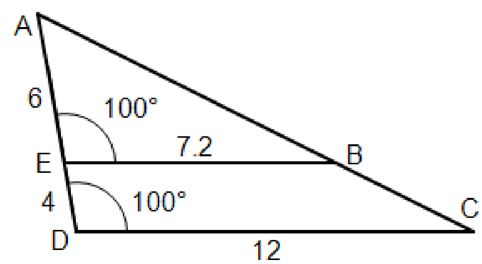
2. ΔABC, ΔDEF



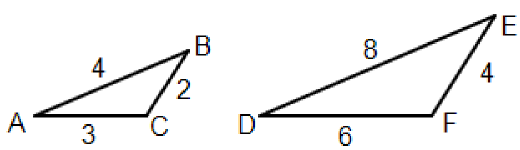
3. ΔADB, ΔEDC



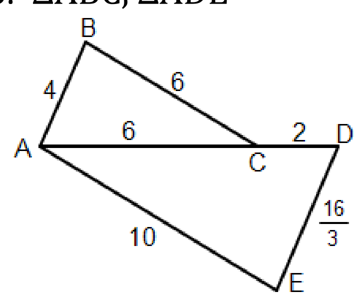
4. ΔABE, ΔACD



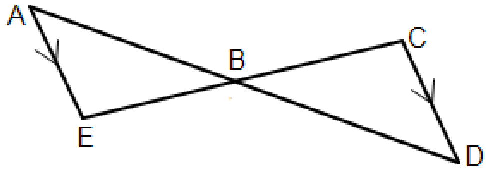
5. ΔABC, ΔDEF



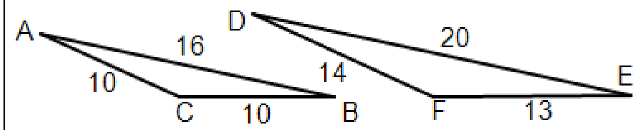
6. ΔABC, ΔADE



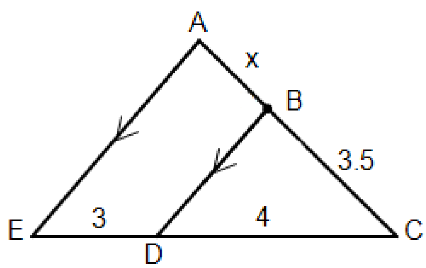
7. ΔABE, ΔBCD



8. ΔABC, ΔDEF



9. ΔACE, ΔBCD. Find the value of *x* that makes the triangles similar.



10. The measures of the sides of ΔABC are 3, 8, and 6 while ΔDEF has corresponding sides of measure 18, 48, and 36. Are these two triangles similar?

11. The measures of two sides of ΔPQR are 11 and 12 while their included angle is 18^o. The corresponding measures of two sides of ΔSTU are 55 and 62 while their included angle is also 18^o. Are these two triangles similar?

12. The measures of two sides of ΔPQR are 8 and 19 while their included angle is 20^o. The measures of two sides of ΔSTU are 40 and 95 while their included angle is also 20^o. Are these two triangles similar with SAS?

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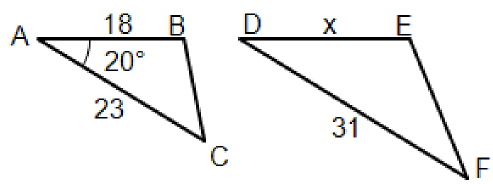
13. What is the value of *x* that would help make these two triangles similar? What else is needed for similarity?

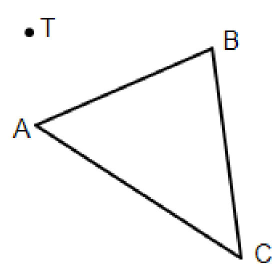


Unit 8: Lesson 04 Dilations

A **dilation** is an enlargement (or reduction) of a figure that is similar to the original figure.

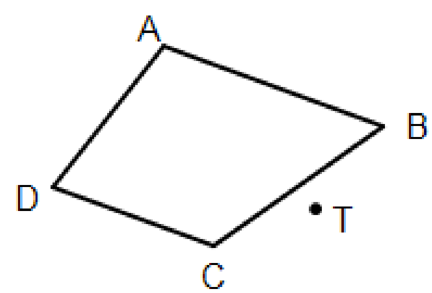
- For scale factors greater than 1, the dilation is an **enlargement**.
- For positive scale factors less than 1, the dilation is a **reduction.**
- A dilation is relative to a **point of dilation**. To locate a vertex of a dilated figure relative to the point of dialtion:
  - Draw a line segment from the point of dilation through the vertex of the original figure.
  - Measure the distance from the point of dilation to the vertex and multiply by the scale factor to produce a new distance *d*.
  - From the point of dilation, move a distance *d* along the line just produced to produce the new vertex of the dilated figure.

Example 1: Using a scale factor of 2 and point T as the point of dilation, dilate ΔABC to produce ΔA'B'C'.

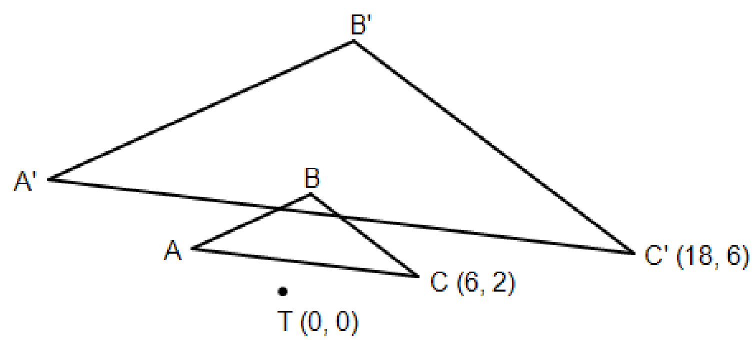


In Example 1, notice that the dilated figure's vertices are labeled with "primed" letters. For example, read A' as "A prime."

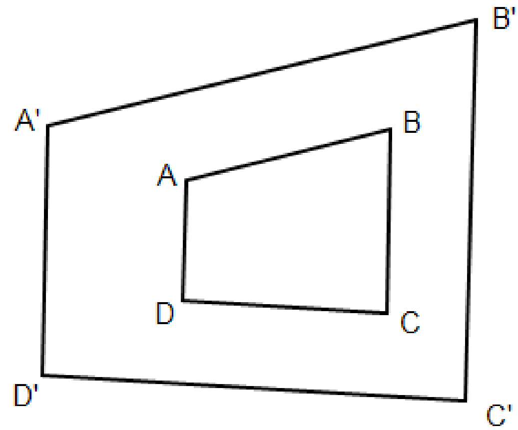
Example 2: Using a scale factor of 3 and point T as the point of dilation, dilate quadrilateral ABCD to produce quadrilateral A'B'C'D'.



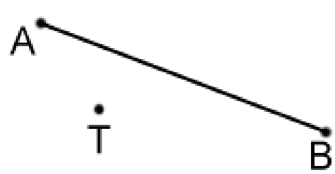
Example 3: If ΔA'B'C' is a dilation of ΔABC and point T is the point of dilation, find the scale factor.



Example 4: If quadrilateral A'B'C'D' is a dilation of quadrilateral ABCD, locate the point of dilation, T.

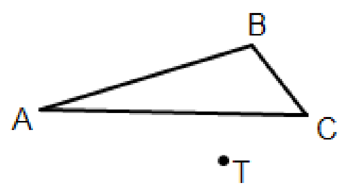


Example 5: Using a scale factor of 3 and T as the point of dilation, dilate line segment $\overline{AB}$ to produce line segment $\overline{A'B'}$. How does $\overline{A'B'}$ compare to $\overline{AB}$?

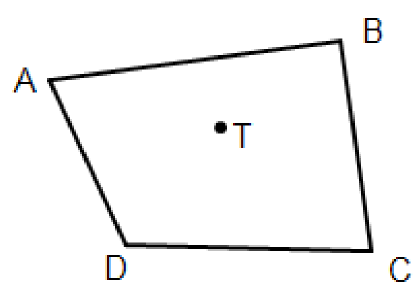


Assignment:

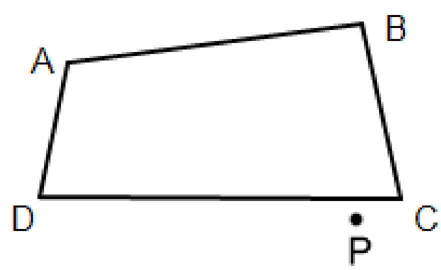
1. Dilate ΔABC using a scale factor of 3 and dilation point T to produce ΔA'B'C'.



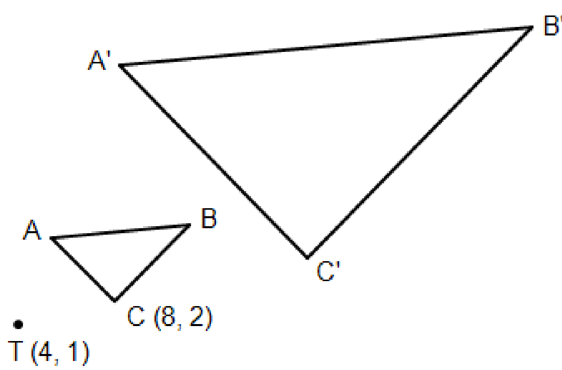
2. Dilate quadrilateral ABCD using a scale factor of 2 and dilation point T to produce quadrilateral A'B'C'D'.



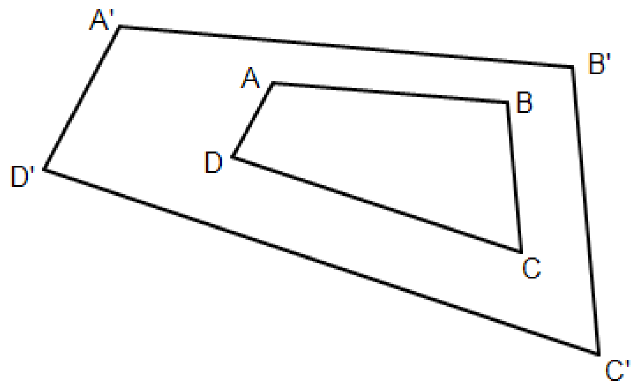
3. Dilate quadrilateral ABCD using a scale factor of 2 and dilation point P to produce quadrilateral A'B'C'D'.



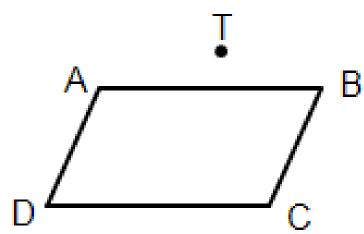
4. If ΔA'B'C' is a dilation of ΔABC and point T is the point of dilation, find the scale factor if the coordinates of C' are (16, 4).



5. If quadrilateral A'B'C'D' is a dilation of quadrilateral ABCD, locate the point of dilation T.



6. Dilate parallelogram ABCD using point T as the point of dilation to produce parallelogram A'B'C'D' if AB = 6 and A'B' = 18.



Unit 8: Lesson 05 Indirect measurement word problems

Many things in science and engineering are measured **indirectly** because a **direct** measurement is impractically complicated or expensive. For example:

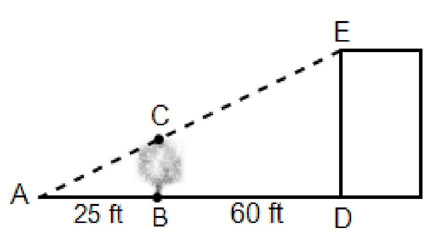
- To measure the temperature of a very hot furnace, a very expensive thermometer could be built that could withstand the intense heat of the furnace. That would be a **direct measurement** of the temperature.

  An indirect measurement of the temperature would be to heat an iron ball in the furnace, drop it in a bucket of water, and observe how much water boils away. A formula could be applied that predicts what the temperature of the iron ball was as a function of the amount of water boiled away.

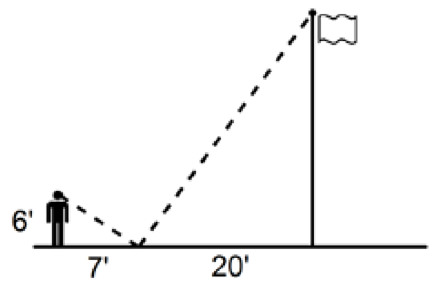
- To measure the height of a building directly, one could climb to the top of the building, tie a weight on a string and then lower it until it touched the ground. The length of the string could be then be measured with a tape measure. That would be a **direct measurement**.

  An **indirect measurement** of the height of building is illustrated below in Example 1. It uses ratios that result from similar triangles.

Example 1: A 32 ft tall tree is located near a building. How high is the building given the distances in the drawing? BC is the height of the tree and DE is the height of the building.

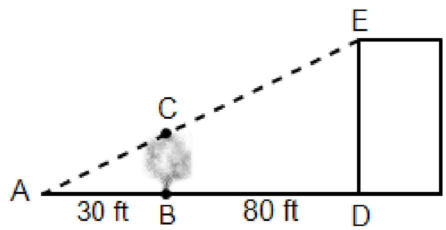


Example 2: Billy Bob, a 6 ft man, is standing 7 ft from a small mirror lying horizontally on the ground. Looking down at the mirror, he sees a reflection of the top of a flagpole whose base is 20 ft on the other side of the mirror. How tall is the flagpole? (Consider the mirror as a point, reflecting the same angle from the man's eye to the top of the flagpole.)



Assignment:

1. A tree located near a building is 25 ft high. How high is the building given the distances in the drawing? BC is the height of the tree and DE is the height of the building.



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2. A 30 ft tree is located 100 ft from the base of a building. An armadillo starts at the base of the tree and moves in a direction that is away from both the tree and the building. An observer on top of the building finally sees the armadillo when it is 38 ft from the base of the tree. How tall is the building?

---

3. A flagpole casts a shadow 12 ft long on the ground. At the same time, a 5-foot student's shadow is 3 ft long. How tall is the flag pole?

4. To find the height of a building, a 7-foot basketball player places a mirror on the ground 100 ft from the base of a building. Standing 8 ft on the other side of the mirror, he looks into the mirror and sees the top of the building. How tall is the building?

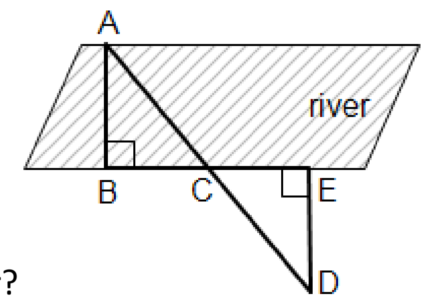
5. A 12-ft ladder is leaning against a vertical wall touching it at a point 6 feet above the ground. At what height above the ground would a 15 ft ladder touch the wall if it is leaning at the same angle as the first ladder?

6. The cities of Amesville, Furman, and Valentine are shown on a map. Lines connecting these cities form a triangle. If the distance on the map between Amesville and Furman is 3 inches and the distance between Amesville and Valentine is 7 inches on the map, how many miles is it from Amesville to Valentine if the distance from Amesville to Furman is 150 miles?

7. Standing side-by-side with her friend, 5-foot Hanna notices her shadow is 3 ft long while the friend's shadow is 4 ft long. How tall is the friend?

---

8. John doesn't have a boat to get to the other side of a river, yet he wants to know how wide the river is. He is on the side of the river with points B, C, D and E where he is able to measure the distances BC = 125 ft, CE = 50 ft, and DE = 60 ft.



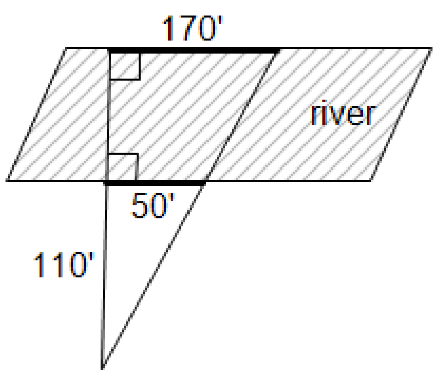
How wide is the river?

---

9. A kite string is tied to the ground and the wind blows the kite off at an angle. At a point on the string that is 2 meters along the string from the ground, it is 1.5 meters straight down to the ground. How high is the kite if the length of the kite string is 50 meters?

10. To measure the height of a building, Bob laid on the ground and looked at the top of a building while his twin brother Larry walked toward the building with a 7 ft pole. When Larry was 8 ft from Bob and 40 ft from the building, the tip of the pole was in Bob's line of sight as he looked at the top of the building. How high was the building?

11. Use this drawing to find how far it is across the river.



Unit 8: Lesson 06 Proportional parts produced by parallel lines

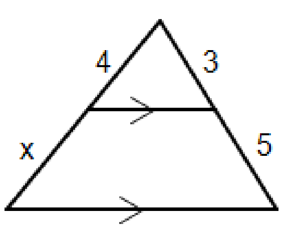
Triangle Proportionality Theorem:

Given a triangle and a line parallel to one side and intersecting the other two sides, the other two sides are separated into segments that are of proportional length. (See **Theorem Proof I**.)

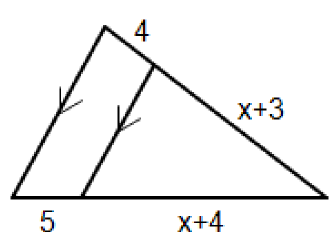
Other proportions include $\frac{CB}{BA} = \frac{CD}{DE}$ which is equivalent to $\frac{CB}{CD} = \frac{BA}{DE}$ the first proportion given above. Notice they give the **same result** when **cross multiplied**.

This leads to a rule that when cross multiplying proportions, elements on a common diagonal can be **interchanged**.

Example 1: Find the value of *x*.



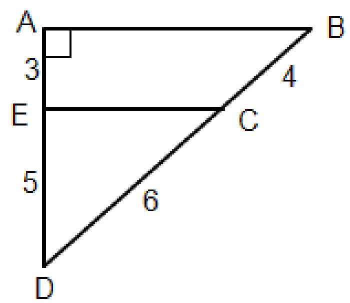
Example 2: Find the value of *x*.



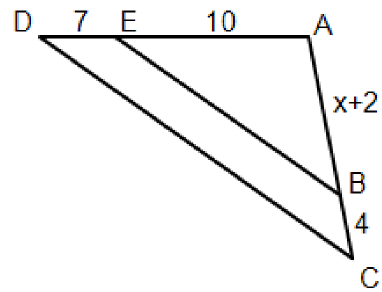
The converse of the theorem on the previous page is:

> If a line intersects two sides of a triangle such that the two sides are separated into corresponding segments of proportional length, then the line is parallel to the third side.

Example 3: Determine if $\overline{\mathrm{EC}}$ is parallel to $\overline{\mathrm{AB}}$.

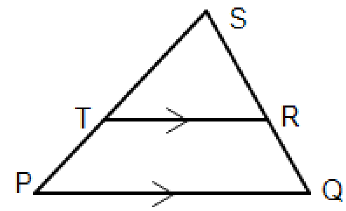


Example 4: What must be the value of *x* so that $\overline{\mathrm{DC}} \parallel \overline{\mathrm{BE}}$?



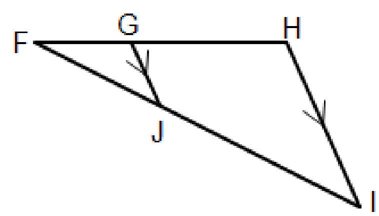
Assignment:

Use this drawing for problems 1- 4 and find the missing element of each proportion.



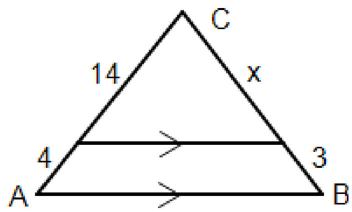
1. $\frac{ST}{TP} = \frac{SR}{?}$	2. $\frac{?}{RS} = \frac{TP}{RQ}$
3. $\frac{SQ}{RQ} = \frac{?}{TP}$	4. $\frac{TP}{?} = \frac{SP}{SQ}$

Use this drawing for problems 5-8 and find the missing element of each proportion.

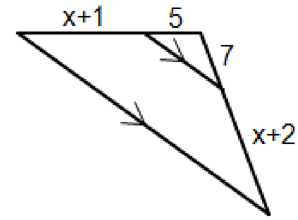


5. $\frac{FJ}{JI} = \frac{FG}{?}$	6. $\frac{?}{FI} = \frac{GH}{JI}$
7. $\frac{GH}{JI} = \frac{?}{FJ}$	8. $\frac{FI}{?} = \frac{FJ}{FG}$

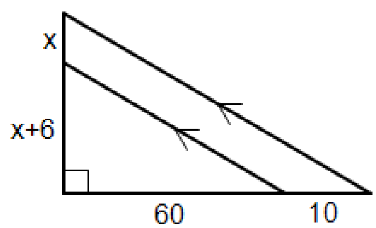
9. Find the value of *x*.



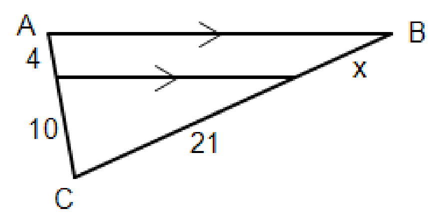
10. Find the value of *x*.



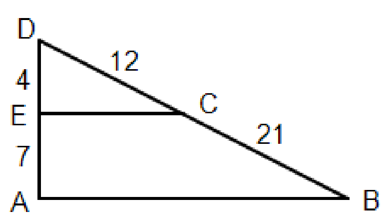
11. Find the value of x.



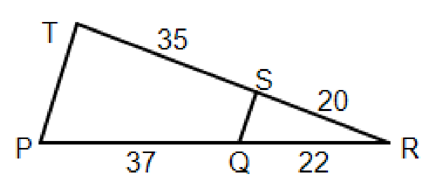
12. Find the length of $\overline{BC}$.



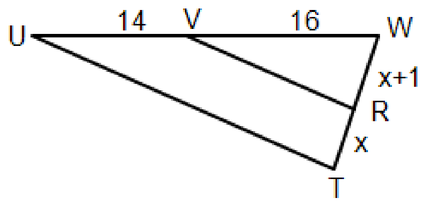
13. Determine if AB || EC.



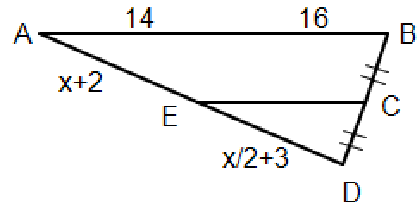
14. Determine if PT || QS.



15. What must be the value of *x* so that $\overline{UT} \parallel \overline{VR}$?



16. If BD = 20, what must be the value of *x* so that $\overline{AB} \parallel \overline{EC}$?



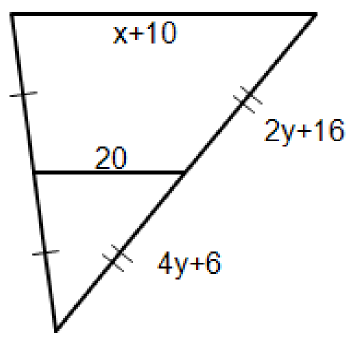
Unit 8:
Lesson 07

More parallel lines and proportional segments
Line joining midpoints of triangle sides

Theorem:

A line segment that joins the midpoints of two sides of a triangle is parallel to the third side, and its length is half the length of the third side. (See **Triangle Proof E**.)

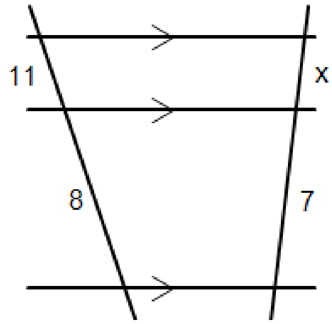
Example 1: Find the values of *x* and *y*.



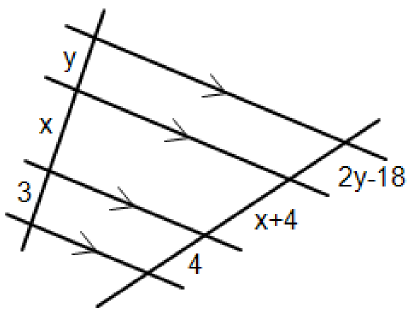
Theorem:

If three or more parallel lines intersect two transversals, then the transversals are cut into proportional segments.

Example 2: Find the value of *x*.



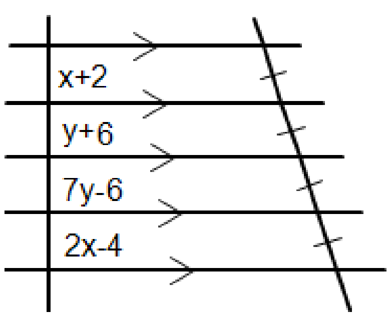
Example 3: Find the values *x* and *y*.



Theorem:

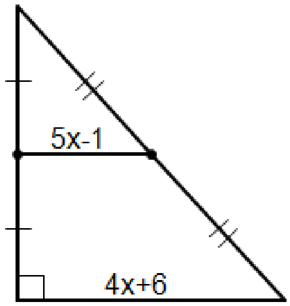
If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example 4: Find *x* and *y*.

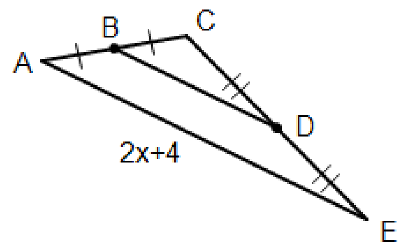


Assignment:

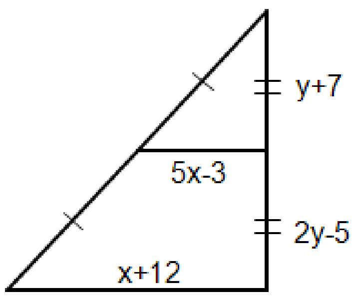
1. Find *x*.



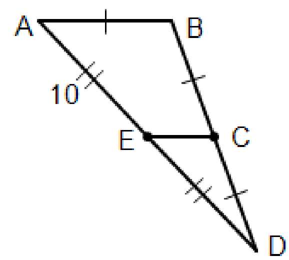
2. Find *x* if BD = 10.



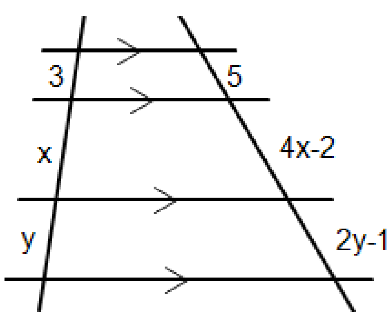
3. Find *x* and *y*.



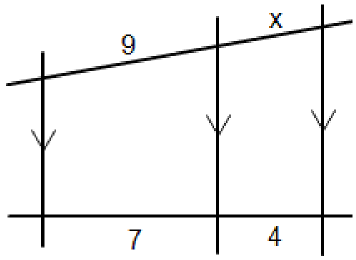
4. If AB = 6, find the lengths BC, CD, ED, and CE.



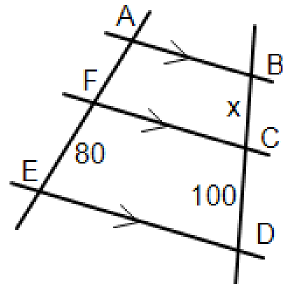
5. Find *x* and *y*.



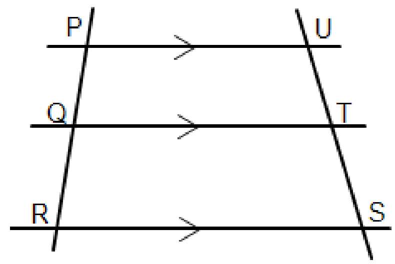
6. Find *x*.



7. Find *x* when AE = 140.



Use this drawing for problems 8 – 11.
Find the missing part of the proportion.



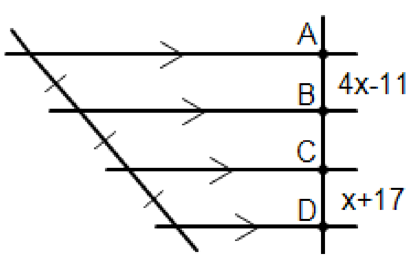
8. $\frac{?}{QR} = \frac{US}{TS}$

9. $\frac{PQ}{?} = \frac{UT}{TS}$

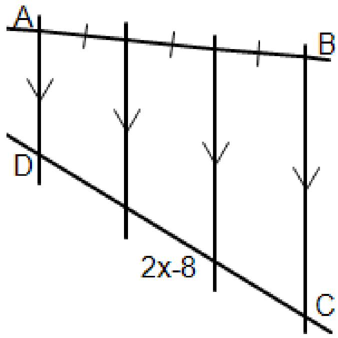
10. $\frac{PQ}{UT} = \frac{?}{TS}$

11. $\frac{US}{PR} = \frac{UT}{?}$

12. Find *x* and AD.



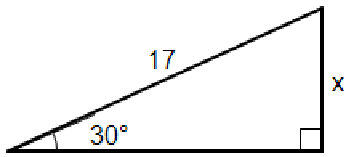
13. Find x if DC = 18.



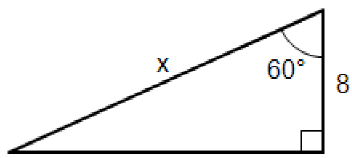
Unit 8: Cumulative Review

In problems 1-4, find *x* **without** using a calculator.

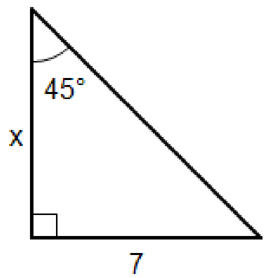
1.



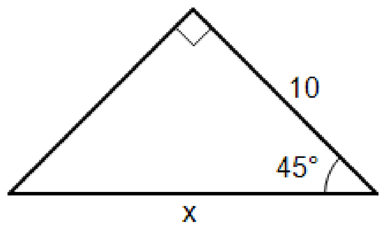
2.



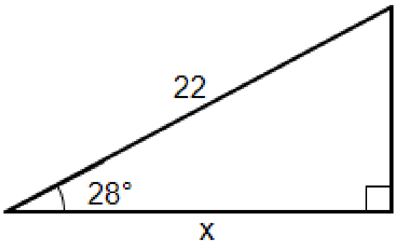
3.



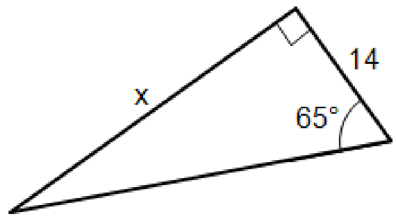
4.



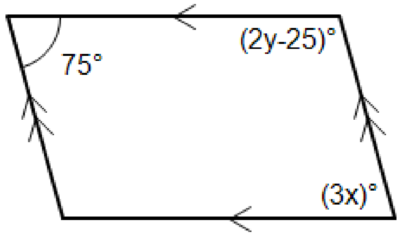
5. Find *x*.



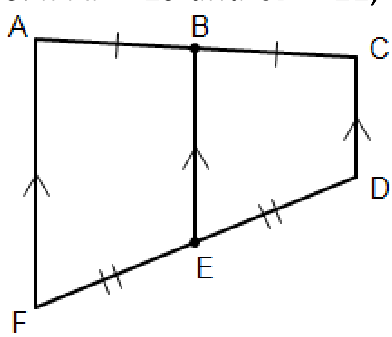
6. Find *x*.



7. Find *x* and *y*.



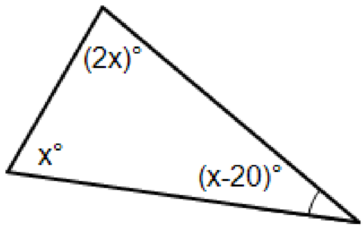
8. If AF = 19 and CD = 11, what is BE?



9. Use the drawing in problem 8 to find *x* when AF = x + 2, BE = 9, and CD = x.

10. In what kind(s) of parallelogram(s) do the diagonals bisect each other?	11. In what kind(s) of parallelogram(s) are the diagonals perpendicular bisectors of each other?

12. Find *x*.

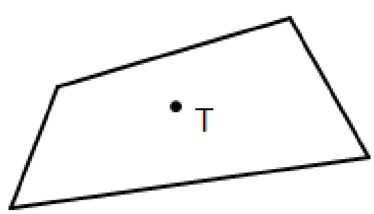


13. What is the measure of each individual interior angle of a regular octagon?

14. Is it possible for a triangle to have sides 18, 20, and 40?

15. What is the measure of each individual exterior angle of a regular 36-gon?

16. Dilate quadrilateral ABCD using point T as the point of dilation and a scale factor of 3.



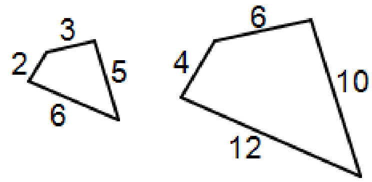
17. Find the measure of an angle if its measure is 24^{o} more than 1/3 its supplement.

Unit 8: Review

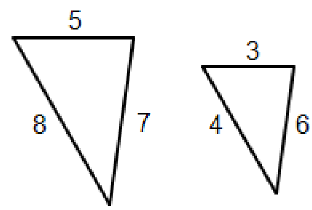
1. Determine if these two ratios form a proportion: 5/7, 15/23

2. Determine if these two ratios form a proportion: 20/9, 60/27

3. Determine if these polygons are similar.

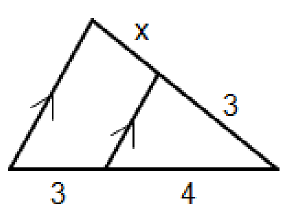


4. Determine if these two triangles are similar.

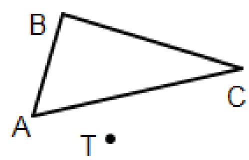


5. Describe three ways of determining if triangles are similar.

6. Find *x*.



7. Dilate triangle ABC so as to produce triangle A’B’C’. The point of dilation is T and the scale factor is 3.



8. A 32 ft tree is 63 ft from the base of a building. A point on the ground another 21 ft beyond the tree is in perfect alignment with the top of the tree and the top of the building. How tall is the building?

9. A 5’ 6’’ woman gazes down at a mirror lying on the ground 10 ft away from her and sees the reflection of the top of a tree whose base is 30 ft on the other side of the mirror. How tall is the tree?

10. Cities A, B, and C form a triangle on a map. On the map the distance from A to B is 3.5” and the distance from A to C is 6”. The actual distance from A to C is 620 miles. What is the actual distance in miles from A to B?

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11. At 10:00 AM in the morning a 6’ man’s shadow is 3.2’ long while a nearby flag pole’s shadow is 11.5’ long. How tall is the flag pole?

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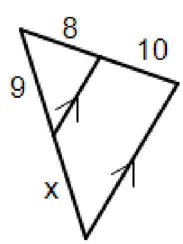
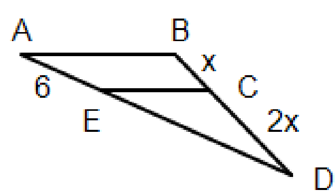
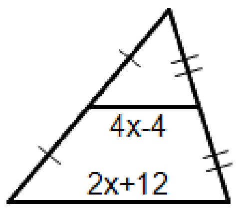
12. Find *x*.



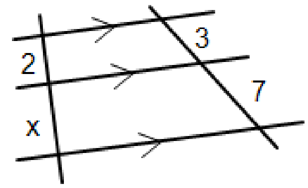
13. If AD = 18, determine if AB || EC.



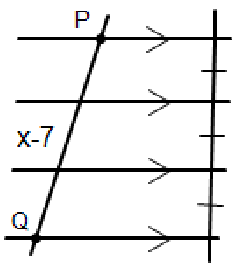
14. Find *x*.



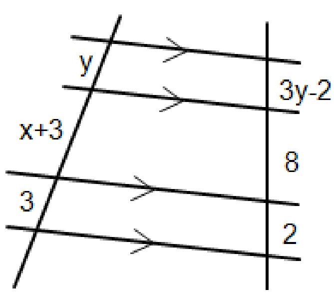
15. Find *x*.



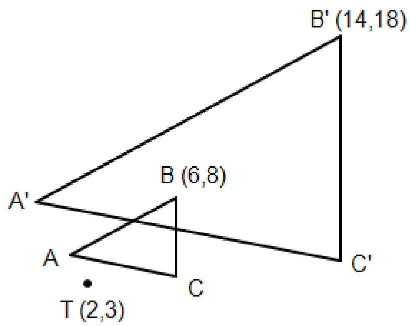
16. Find *x* if PQ = 24.



17. Find *x* and *y*.



18. Triangle A'B'C' is a dilation of triangle ABC using T as the point of dilation. What is the scale factor?



Geometry, Unit 9

Area and Perimeter

Unit 9:
Lesson 01

Rectangle area, perimeter, and diagonal

A **rectangle**:

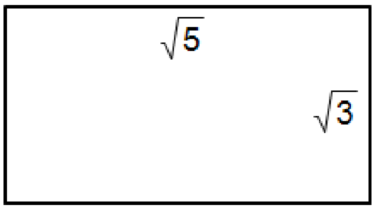
- is a parallelogram
- that has four right angles
- and diagonals of equal length.

The area of a rectangle is obtained by multiplying its length and width.

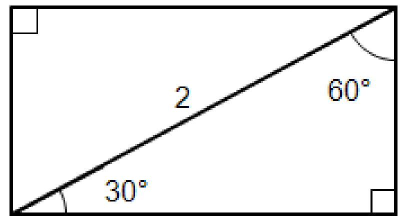
The perimeter of a rectangle is obtained by finding the sum of the lengths of all four sides.

The length of a diagonal of a rectangle is found using the Pythagorean Theorem.

Example 1: Find the area, perimeter, and length of the diagonal of this rectangle.



Example 2: Find the area, perimeter, and length of the diagonal of this rectangle.



Example 3: A certain rectangle has a width of 36 while the length of its diagonal is 48. What is its length?

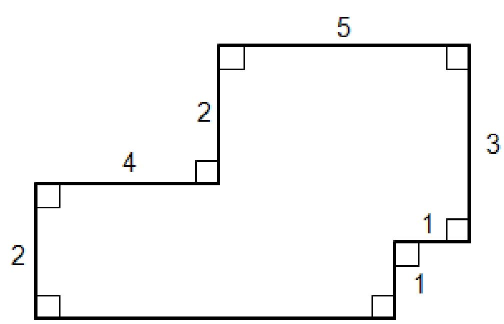
A **square** is a special rectangle in which

- all sides are of equal length,
- and where the diagonals are perpendicular (because it is also a rhombus).

Example 4: If a diagonal of a square measures 5, how long are the sides of the square? What is its area and perimeter?

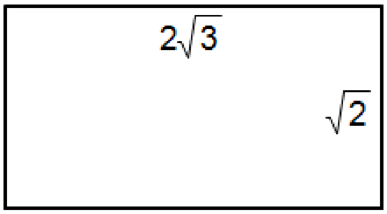
Example 5: The length of a rectangle is 6 more than twice its width while the perimeter is 60 meters. What are the dimensions of the rectangle? What is its area?

Example 6: What is the area of this concave polygon? What is its perimeter?

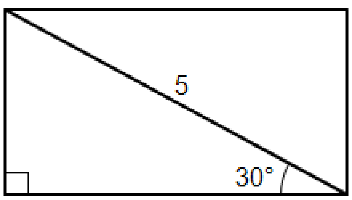


Assignment: In problems 1-4, find the area, perimeter, and diagonal length of the given rectangles.

1.



2.



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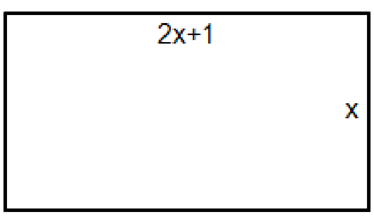

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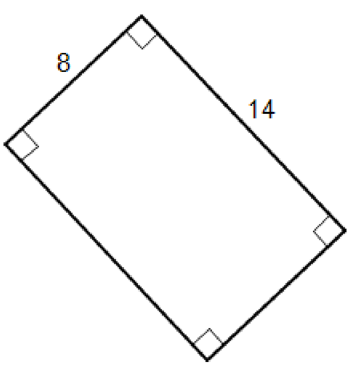


5. Rectangle ABCD has a width of 11 while its diagonal is 18. Find the area and perimeter.

6. Rectangle MNOP has a length of 20 and a perimeter of 80. Find the area and diagonal length.

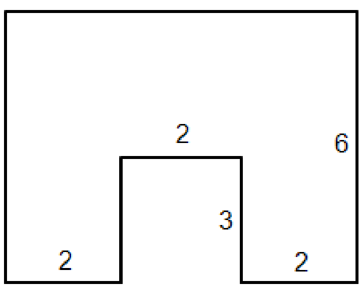
7. What is the length of a side of a square whose diagonal length is 25?

8. What is the area and perimeter of a square whose diagonal measures 15?

9. The length of a rectangle is 4 more than twice its width while the perimeter is 72. What is the area, perimeter, and diagonal length of this rectangle?

---

10. What is the area of this figure assuming all right angles? What is the perimeter?



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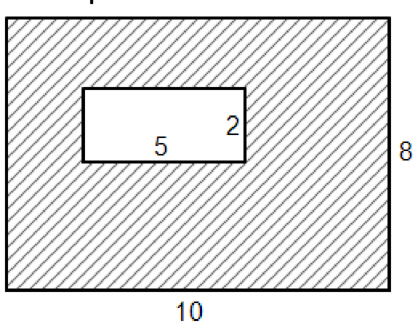
11. What is the shaded area of this figure assuming all right angles? What is the outer perimeter?



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12. What is the length of a diagonal of a square whose perimeter is 24 inches?

13. If the area of a square is 36, what is the length of a side of the square? What is its perimeter and diagonal length?

---

14. The width of a rectangle is 22 meters less than its length while its perimeter is 200 meters. Find the dimensions, area, and diagonal length of the rectangle.

Unit 9: Lesson 02 Parallelogram area and perimeter

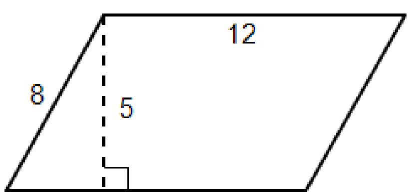
To find the **area of a parallelogram**, begin by showing the height (also called the altitude) of the triangle (in two places).

Now think of removing triangle CED from the right side and moving it to the left side. This creates rectangle ACEE' from which we can easily compute the area.

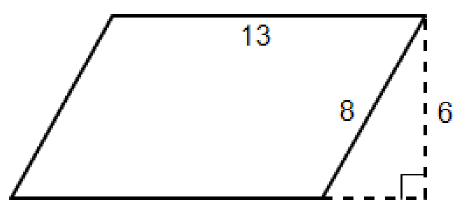
Thus, for a parallelogram the area is the base of the parallelogram in the original drawing times the height.

The perimeter of a parallelogram is simply the sum of the 4 sides.

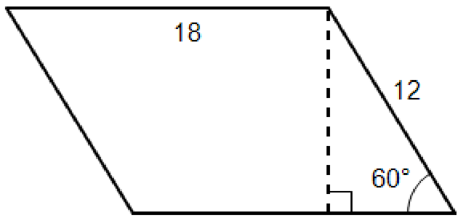
Example 1: Identify the base and height of this parallelogram. Then use them to find the area. What is the perimeter?



Example 2: Identify the base and height of this parallelogram. Then use them to find the area. What is the perimeter?



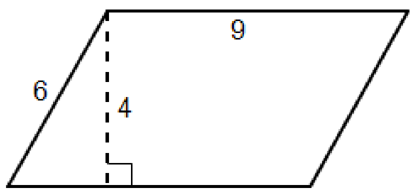
Example 3: Identify the base and height of this parallelogram. Then use them to find the area. What is the perimeter?



Example 4: If the area of a parallelogram is 96 in^2, and the base is 20 in, what is its height?

Assignment: In problems 1 – 4, find the area and perimeter of the given parallelograms.

1.



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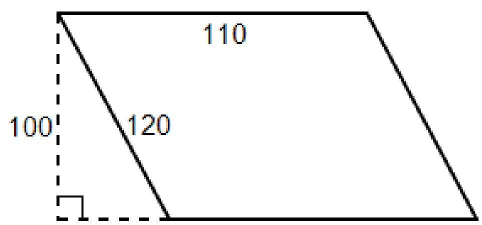

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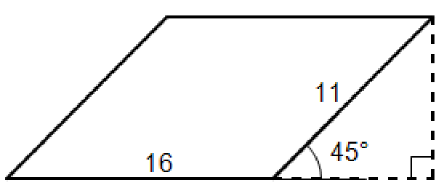

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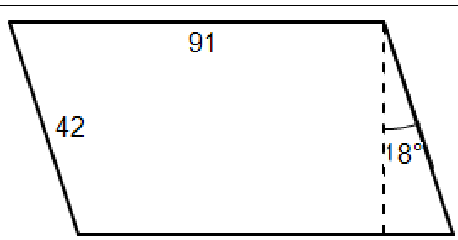


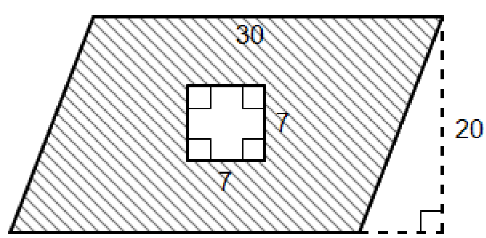
5. Find the area and perimeter of a parallelogram that has one side of length 24 ft and the other side of 68 ft. One of the interior angles is 63°.

---

6. The area of a parallelogram is 280 m^2 and its altitude is 22 m. Find the length of the base.

---

7. Find the shaded area.



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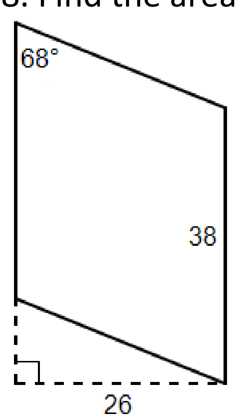
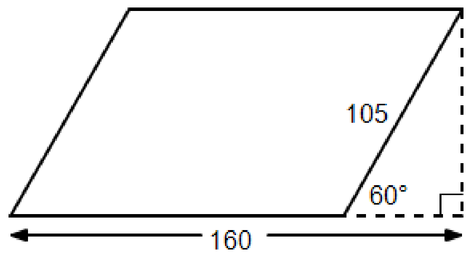
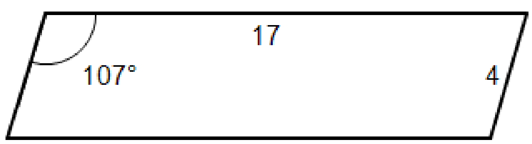
8. Find the area and perimeter of this parallelogram.



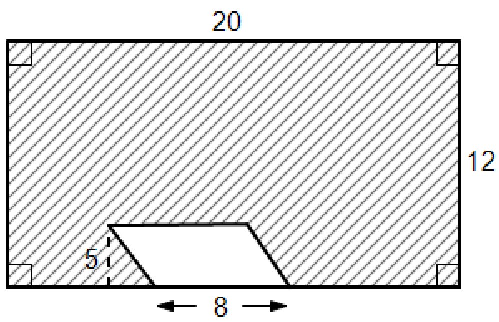
9. Find the area and perimeter of this parallelogram.



10. Find the area and perimeter of this parallelogram.



11. Find the shaded area.



Unit 9: Lesson 03 Triangle area and perimeter

To find the **area of triangle** ABC, begin by drawing in the height (also called the altitude as shown by the dotted line).

Duplicate this triangle by flipping ABC vertically and then again horizontally to produce triangle A'B'C'.

Move the two triangles together to form a parallelogram.

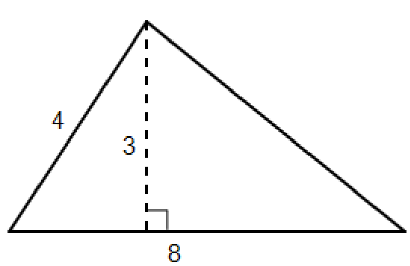
Discard the vertex labels B' and C' and we are left with parallelogram ABA'C with an area of the two triangles.

Recall from the previous lesson that the area of a parallelogram is the base times the height, so it follows that the area of the triangle is:

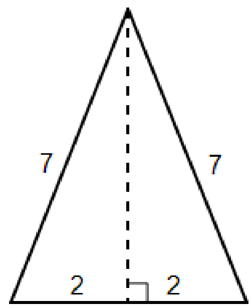
The perimeter of a triangle is the sum of the lengths of the three sides.

In examples 1 – 3, find the base, height, area, and perimeter of each triangle.

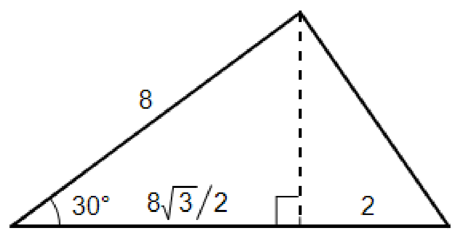
Example 1:



Example 2:

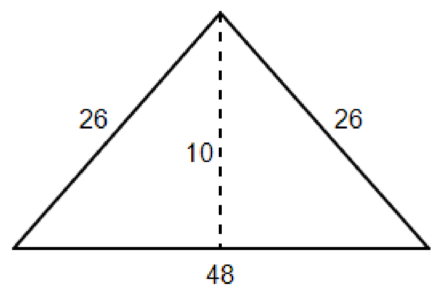


Example 3:

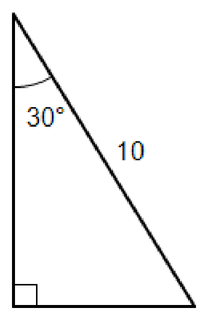


Assignment: In problems 1 – 6, find the base, height, area, and perimeter of each triangle.

1.



2.



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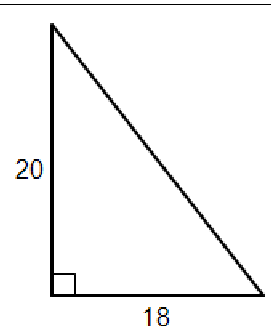

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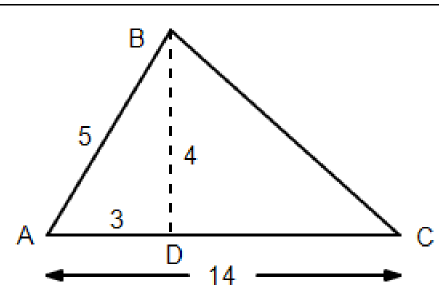

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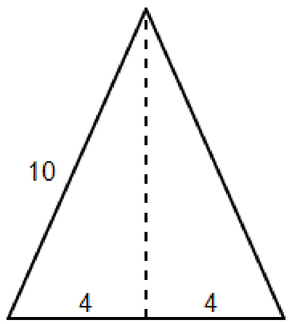
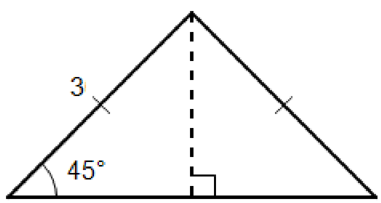
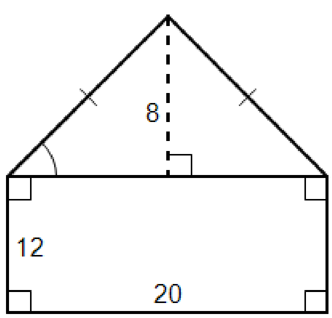


6.

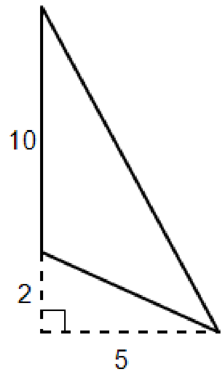


7. Find the area and perimeter.

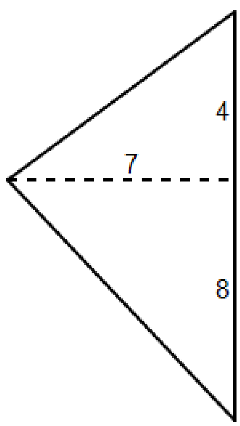


8. If the area of a triangle is 22 and its altitude is 4.5, how long is the base?

9. Find the area and perimeter.



10. Find the area and perimeter of this triangle.



Unit 9:
Lesson 04

Rhombus area and perimeter

Recall that a **rhombus** is a quadrilateral with 4 congruent sides. We also know a rhombus:

- is a parallelogram,
- has diagonals that bisect each other,
- and has diagonals that are perpendicular to each other.

To find a formula for the area of a rhombus, draw a rhombus and show one of the diagonals (call it D_1).

Draw the other diagonal and call it d_2. Since the diagonals bisect each other, label the top half of this diagonal as $(1/2)D_2$.

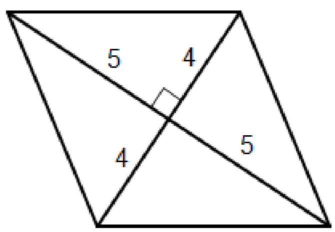
Find the area of the top triangle using the triangle area formula, (1/2)(base)(height).

Since the rhombus is comprised of two such triangles, the **area of the rhombus is twice the area of the triangle**:

Since all sides of a rhombus have equal lengths, it follows that:

Example 1: What is the area, side length, and perimeter of the rhombus having diagonal lengths of 12 and 16?

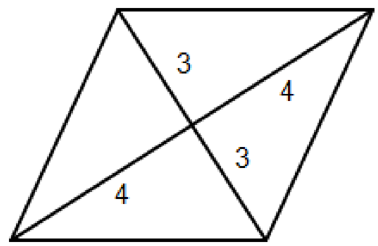
Example 2: For the rhombus shown here, find the lengths of both diagonals, the area, side length, and perimeter.



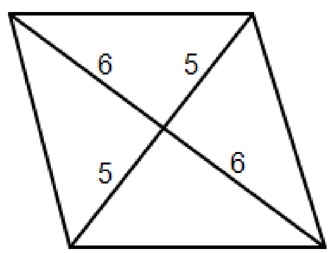
Example 3: A certain rhombus has an area of 100 m^2. If the longer of the two diagonals is 17 m, how long is the shorter diagonal?

Assignment: For the rhombi shown in problems 1-3, find the lengths of both diagonals, the area, side length, and perimeter.

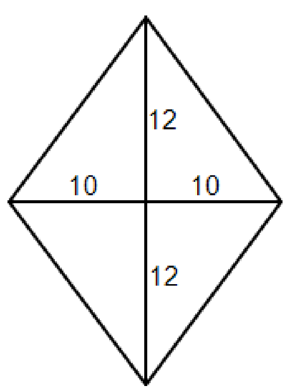
1.



2.



3.



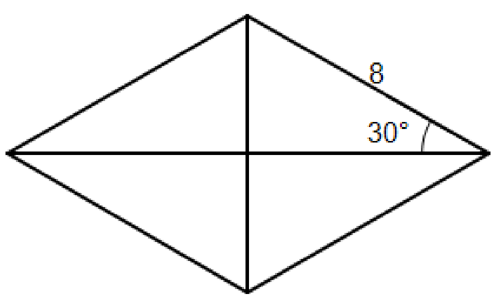
4. What is the area, side length, and perimeter of the rhombus having diagonal lengths of 20 and 28?

---

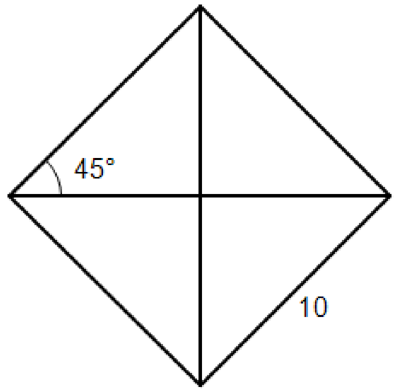
5. A certain rhombus has an area of 120 m^2. If the longer of the two diagonals is 20 m, how long is the shorter diagonal?

---

6. What is the area and perimeter of this rhombus?



7. What is the area and perimeter of this rhombus?



Unit 9:
Lesson 05

Trapezoid area and perimeter

Recall that a trapezoid is a quadrilateral with exactly two parallel sides.

- The two parallel sides are called the bases
- The two non-parallel sides are called the legs.

To find the **area of a trapezoid**, begin with trapezoid ABCD and produce a new trapezoid A'B'C'D' by flipping ABCD both vertically and horizontally:

Move the two together and discard the B' and C' labels. The new figure AD'A'D is a parallelogram.

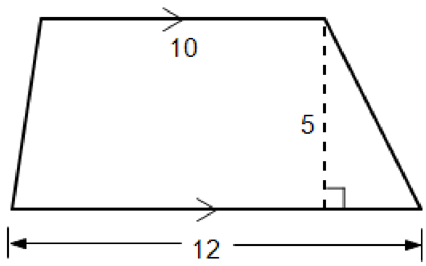
Recall that the area of a parallelogram is the base of the parallelogram ($b_1 + b_2$ in this case) times the height:

The parallelogram we have produced here is comprised of two trapezoids, so the area of one trapezoid is half that of the parallelogram:

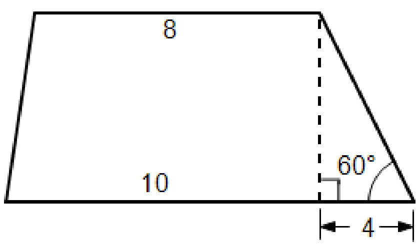
To find the perimeter of a trapezoid, simply add all four sides.

In the following examples, find the lengths of the two bases, the height, and the area of the given trapezoids.

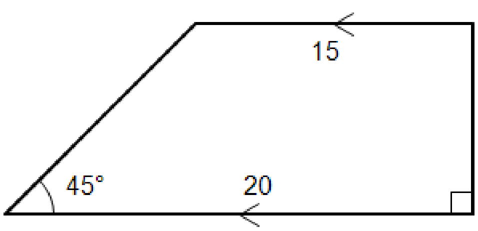
Example 1:



Example 2:



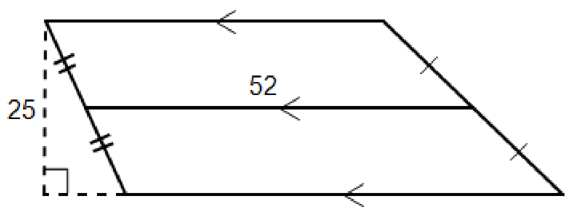
Example 3: Also find the perimeter.



The median of a trapezoid is the line segment joining the midpoints of the two legs. Its length is:

Since the area and median formulas both contain $(1/2)(b_1 + b_2)$, an alternate formula for the area of a trapezoid is:

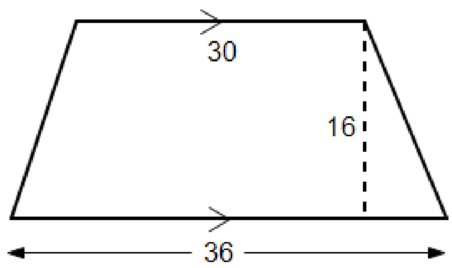
Example 4: Find the area of this trapezoid.



Example 5: If a trapezoid has an area of 79.2 m^2 and a height of 4 m, what is the length of its median?

Assignment: In problems 1-4, find the lengths of the two bases, the height, and the area of the given trapezoids.

1.



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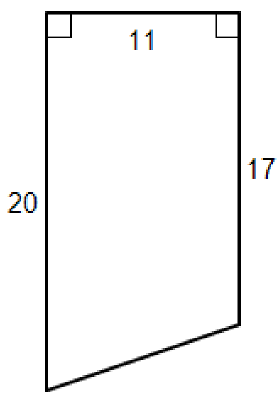

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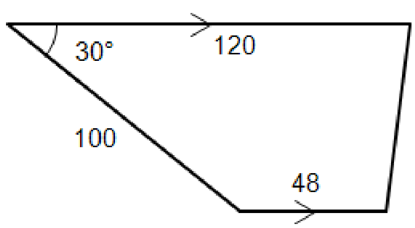

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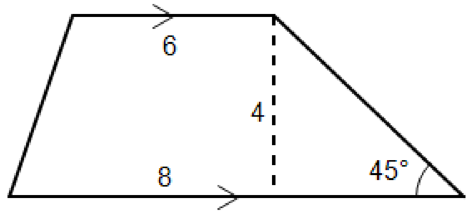
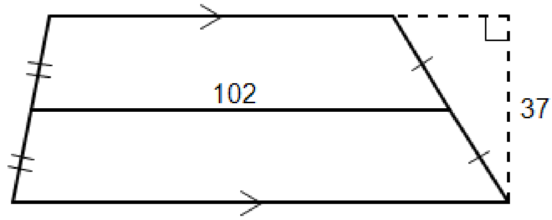


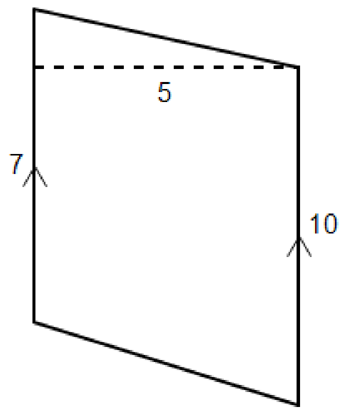
5. Find the area of this trapezoid.



6. If a trapezoid has an area of 500 ft^2 and a height of 22′, what is the length of its median?

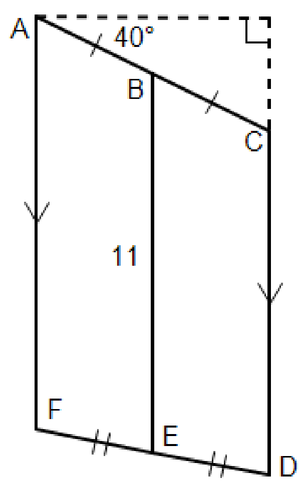
7. A certain trapezoid has a median that is two-thirds the length of the longer base. If the shorter base (b_2) has a length 6 cm, what is the length (b_1) of the longer base?

8. Find the area of this trapezoid.



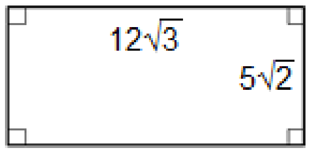
9. Find the area and perimeter of an isosceles trapezoid with 42° base angles if the bases have lengths 16 and 20.

10. Find the area of this trapezoid if AC = 9 and $\overline{BE}$ is the median.



Unit 09: Review

1. Find the area, perimeter, and the length of the diagonal of this rectangle.



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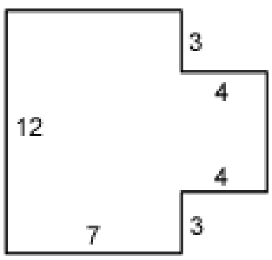
2. What is the area and perimeter of this figure assuming all intersecting segments are perpendicular?



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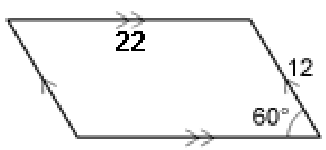
3. Find the base and height of this parallelogram and then use them to find the area and perimeter.



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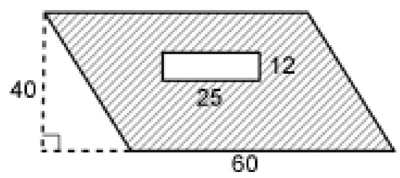
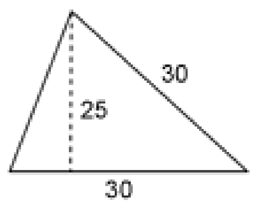
4. If the area of a parallelogram is 210 ft^2 and the base is 35 ft, what is its height?

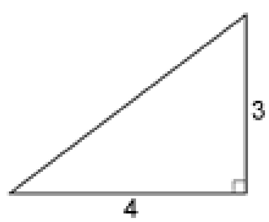
5. Find the shaded area.



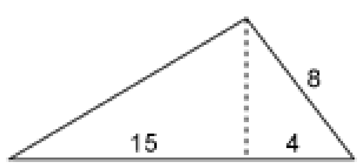
6. Find the base, height, area, and perimeter of this triangle.



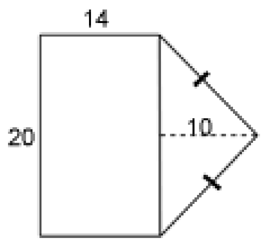
7. Find the base, height, area, and perimeter of this triangle.



8. Find the base, height, and area of this triangle.



9. Find the area and perimeter of this figure.



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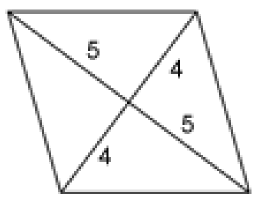
10. Find the lengths of both diagonals, the area, side length, and perimeter for this rhombus.



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11. What is the area, side length, and perimeter of the rhombus having diagonal lengths of 20 and 30?

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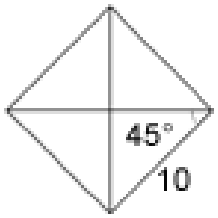
12. A certain rhombus has an area of 110 m^2. If the longer of the two diagonals is 20 m, how long is the shorter diagonal?

13. What is the area and perimeter of this rhombus?



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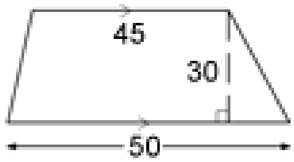
14. Find the lengths of the two bases, the height, and the area of this trapezoid.



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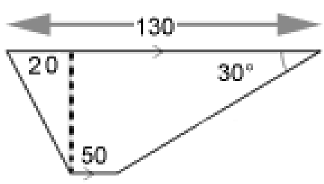
15. Find the lengths of the two bases, the height, and the area of this trapezoid.



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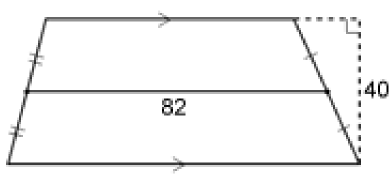
16. What is the area of this trapezoid?



17. If a trapezoid has an area of 600 ft^2 and a height of 25 ft, what is the length of its median?

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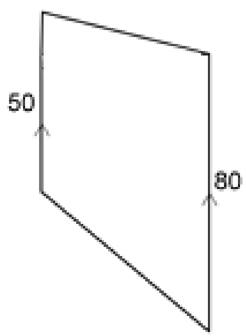
18. Find the area of this figure if the distance between parallel sides is 47.



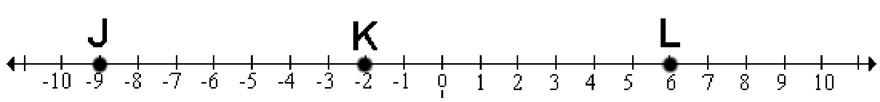
Sem 1: Review

Comprehensive Review

1. Factor $4x^2 + x - 14$.

2. Solve $3x^2 - 5x + 1 = 0$ using the quadratic formula.

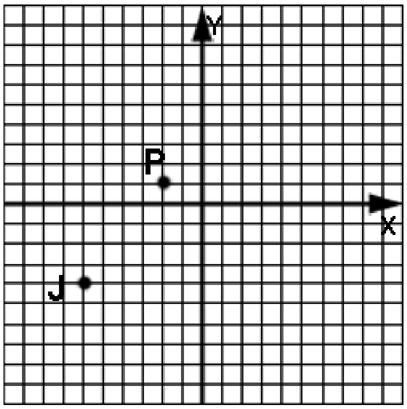
Use this drawing for problems 3 and 4.



3. Find JL.

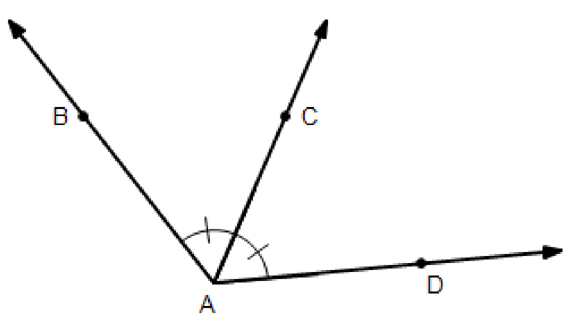
4. Find the midpoint of JK.

5. What are the coordinates of K if J is the midpoint of $\overline{PK}$?

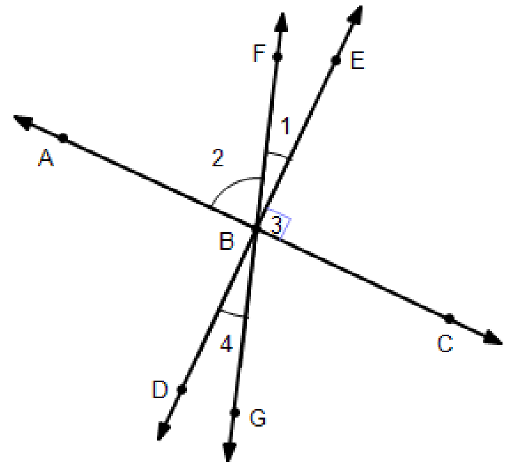


6. Use the points given in problem 5 to find the midpoint of line segment $\overline{PJ}$.

7. If m∠BAC = $(4x - 2)^{o}$ and m∠BAD = 126^{o}, what is m∠CAD?



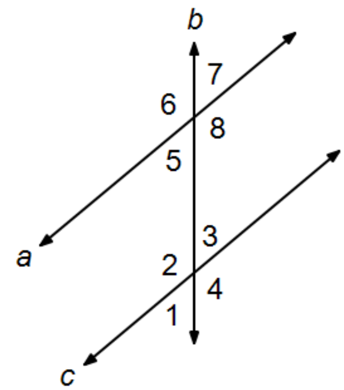
Use this drawing for problems 8-11.
Assume $\overleftrightarrow{AC} \perp \overleftrightarrow{DE}$.



8. Name the angle that is the complement to $\angle 1$.	9. Name an angle that is the supplement to $\angle 4$.
10. If $m\angle 2 = 70^o$, what is $m\angle 1$?	11. If $m\angle 2 = 70^o$, what is $m\angle 4$?

12. If twice the measure of an angle is 6 more that its complement, what is the measure of the angle?

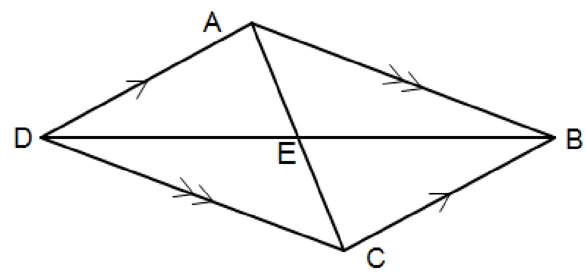
Use this drawing for problems 13 – 16.



13. If m∠7 = 45°, find m∠1.	14. If m∠4 = 145°, find m∠7.
15. If m∠5 =(5x – 10)° and m∠3 = (3x + 10)°, find *x*.	16. If m∠6 = 140°, find m∠7.

17. Construct a perpendicular bisector of $\overline{AB}$.

Use this drawing in problems 18– 21.



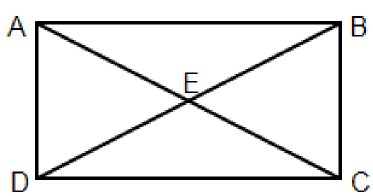
18. If m∠ABC = 70^{o}, what is m∠ADC

19. If AD = 37, what is the length of $\overline{BC}$?

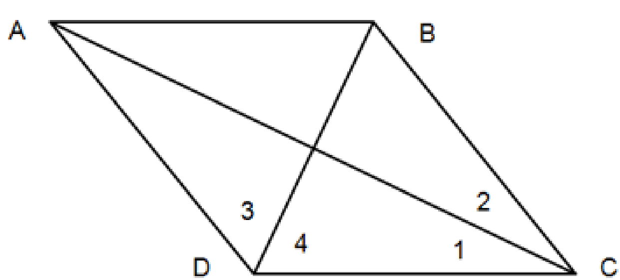
20. If m∠DAB = 112^{o}, what is m∠ADC?

21. If CA = 20, what is EC?

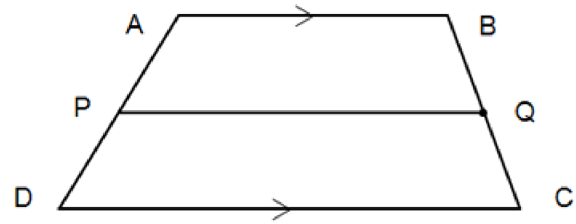
22. Parallelogram ABCD is a rectangle. If EC = 8x – 2 and DB = 22, what is *x*?



23. In rhombus ABCD m∠4 = $(25x)^{o}$ and m∠1 = $(10x – 5)^{o}$. Find *x*.



Use this trapezoid in problems 24 & 25.



24. If AB = 4x – 10, DC = 3x + 8, and the median has a length of 13, find *x*.

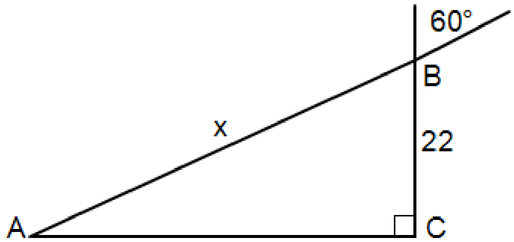
25. If AB = 10 and PQ = 13.5, what is DC?

26. Draw a 30-60-90 triangle and label the lengths of the sides with the "standard" lengths.

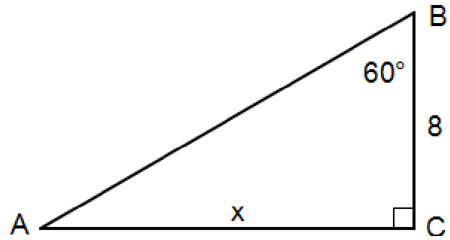
27. Draw a 45-45-90 triangle and label the lengths of the sides with the "standard" lengths.

28. Define the sine, cosine, and tangent of angle A in terms of opp, adj, and hyp.

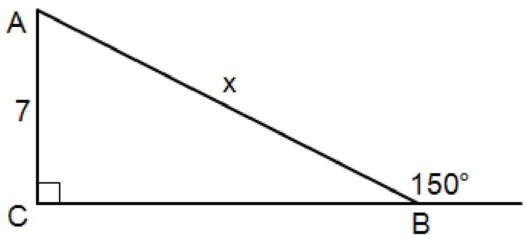
29. Without using a calculator, find *x*.



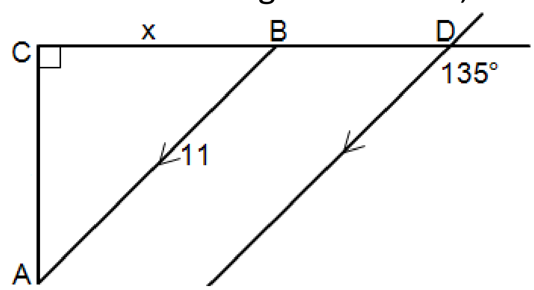
30. Without a calculator, find *x*.



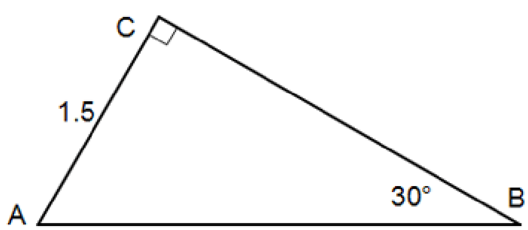
31. Without using a calculator, find *x*.



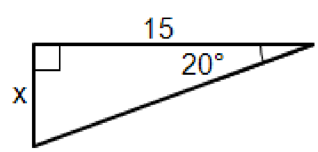
32. Without using a calculator, find *x*.



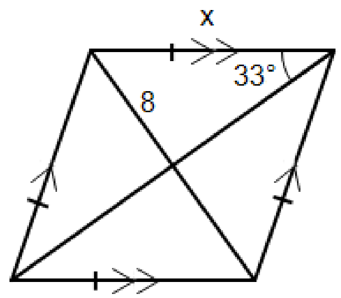
33. Without using a calculator, find CB.



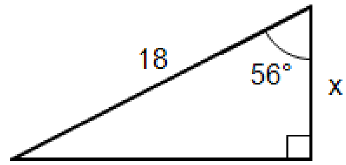
34. Use a calculator to find *x*.



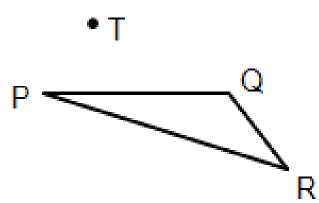
35. Use a calculator to find *x.*



36. Use a calculator to find *x*.



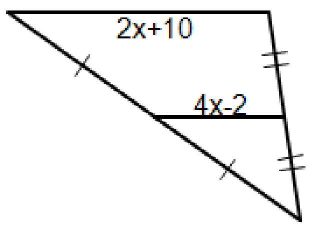
37. Dilate traingle PQR so as to produce triangle P'Q'R'. The point of dilation is T and the scale factor is 2.



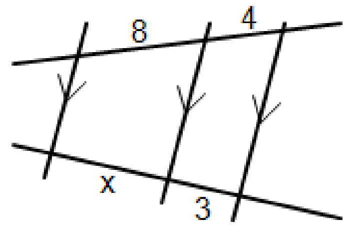
38. A monkey whose head is only 2' off the ground looks at a mirror on the ground that is 4' in front of him. He sees a banana in a tree that is another 15' on the other side of the mirror. How high is the banana?

39. Close to noon, shadows become very short. At 11:15 AM a 6' man's shadow is only 1.2' long while a nearby flagpole's shadow is 3.5' long. How tall is the flagpole?

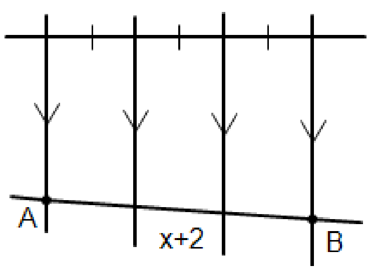
40. Find *x*.



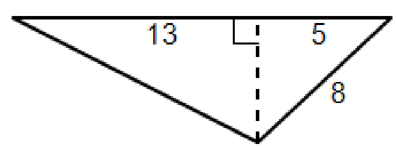
41. Find *x*.



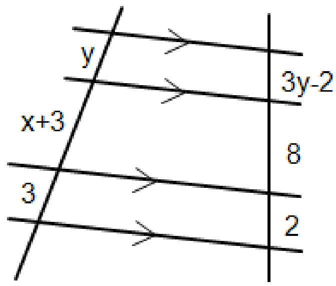
42. Find *x* if AB = 30.



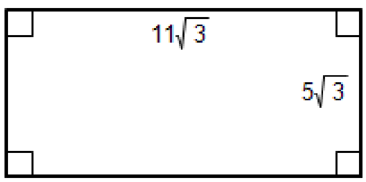
43. Find the base, height, and area of this triangle.



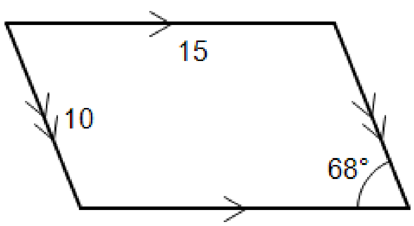
44. Find *x* and *y*.



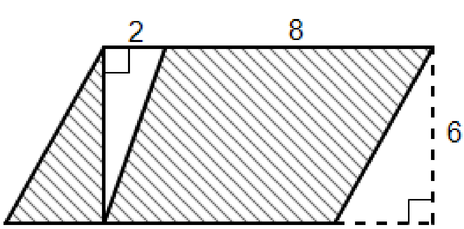
45. Find the area, perimeter, and the length of the diagonal of this rectangle.



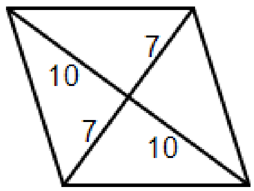
46. Identify the base and height of this parallelogram and then use them to find the area and perimeter.



47. Find the shaded area.



48. Find the lengths of both diagonals, the area, side length, and perimeter for this rhombus.



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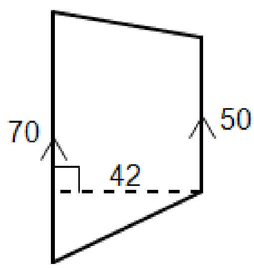
49. Find the lengths of the two bases, the height, and the area of this trapezoid.



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50. What is the area of this trapezoid?