Blue Pelican Geometry

First Semester

Teacher Version 1.01

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Geometry Syllabus (First Semester)

Unit 1: Algebra review

Lesson 01: Solving linear equations and inequalities

Lesson 02: Solving systems of two linear equations

Lesson 03: Trinomial factoring

Lesson 04: Special factoring formulas
$a^2 – b^2$; $a^2 \pm 2ab + b^2$

Lesson 05: Solving quadratic equations

Unit 1 review
Unit 1 test

Unit 2: Basic definitions & concepts (points, lines, and planes)

Lesson 01: Definitions & conventions

Lesson 02: Postulates concerning points, lines, & planes
Practice with points, lines, and planes

Lesson 03: Distance on a number line
Length of a line segment

Lesson 04: Midpoint of a line segment (midpoint formula)

Lesson 05: Line segment bisectors

Unit 2 review
Unit 2 test

Unit 3: Angles

Lesson 01: Angle fundamentals

Lesson 02: Special angle pairs, perpendicular lines
Supplementary and complementary angles

Lesson 03: Angle word problems

Lesson 04: Construction fundamentals
Copying segments & angles; bisecting segments & angles

Lesson 4: Trapezoids

Cumulative review
Unit 6 review
Unit 6 test

Unit 7: Right triangles
Trigonometric ratios (sine, cosine, & tangent)

Lesson 1: The Pythagorean Theorem

Lesson 2: Pythagorean triples
Converse of the Pythagorean Theorem

Lesson 3: A special triangle (45-45-90)
Introduction to trig ratios

Lesson 4: Another special triangle (30-60-90)

Lesson 5: Trig ratios in right-triangles
Word problems using trig

Lesson 6: Solutions of non-right-triangles
Sine Law, Cosine Law, and Area Formula

Cumulative review
Unit 7 review
Unit 7 test

Unit 8: Ratios, Proportional Parts
Similar Polygons, Dilations

Lesson 1: Practice with ratios and proportions
Associated word problems

Lesson 2: Similar polygons

Lesson 3: Similar triangles
AA, SAS, & SSS similarity

Lesson 4: Dilations

Lesson 5: Indirect measurement word problems

Lesson 6: Proportional parts produced by parallel lines

Lesson 7: More parallel lines and proportional segments
Line joining midpoints of triangle sides

Cumulative review
Unit 8 review
Unit 8 test

Unit 9: Area and perimeter

Lesson 1: Rectangle area, perimeter, and diagonal

Lesson 2. Parallelogram area and perimeter

Lesson 3: Triangle area and perimeter

Lesson 4: Rhombus area and perimeter

Lesson 5: Trapezoid area and perimeter

Unit 9 review
Unit 9 test

Semester summary

Semester review
Semester test

In-depth Topics

Topic A: Sign rules
Topic B: Derivation of the quadratic formula
Topic C: Conic section applications and equation derivations
Topic D: Euclidean/non-Euclidean geometry
Topic E: Constructions
Topic F: Exterior Angle Sum Theorem
Topic G: Interior Angle Sum Theorem
Topic H: Derivation of the Sine Law
Topic I: Derivation of the Cosine Law
Topic J: Derivation of a triangle area formula
Topic K: Analytic Geometry and the use of equations in geometry
Topic L: Area & volume density and associated unit conversions
Topic M: Deductive and inductive reasoning
Topic N: Area of a regular polygon by apothem and perimeter
Topic O: Tessellations
Topic P: Fractals

Geometry, Unit 1

Algebra Review

Unit 1: Lesson 01 Solving linear equations and inequalities

Example 1: Solve x + 2 = 19

$$x + 2 = 19$$
$$x = 19 - 2$$
$$x = \boxed{17}$$

Example 2: Solve 2x = 46

$$2x = 46$$
$$x = \frac{46}{2}$$
$$x = \boxed{23}$$

Example 3: Solve 4x – 8 = 40

$$4x - 8 = 40$$
$$4x = 40 + 8$$
$$4x = 48$$
$$x = \frac{48}{4} = \boxed{12}$$

Example 4: 4x – x + 12 = 0

$$4x - x + 12 = 0$$
$$3x = -12$$
$$x = \frac{-12}{3} = \boxed{-4}$$

Example 5: Solve 11h – 7 = 2h + 1

$$11h - 7 = 2h + 1$$
$$11h - 2h = 7 + 1$$
$$9h = 8$$
$$h = \boxed{\frac{8}{9}}$$

Example 6: 4(x – 3) = 8

$$4(x - 3) = 8$$
$$4x - 12 = 8$$
$$4x = 12 + 8 = 20$$
$$x = \frac{20}{4} = \boxed{5}$$

Example 7: Solve -3y – 4(6y + 2) = y – 9

$$-3y - 4(6y + 2) = y - 9$$
$$-3y - 24y - 8 = y - 9$$
$$-27y - y = -9 + 8$$
$$-28y = -1$$
$$y = \boxed{\frac{1}{28}}$$

Example 8: Solve (1/6)g + 1/3 = (1/2)g – 1

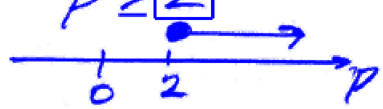
$$\frac{1}{6}g + \frac{1}{3} = \frac{1}{2}g - 1$$
$$6\left(\frac{1}{6}g + \frac{1}{3}\right) = 6\left(\frac{1}{2}g - 1\right)$$
$$1g + 2 = 3g - 6$$
$$1g - 3g = -6 - 2$$
$$-2g = -8$$
$$g = \frac{-8}{-2} = \boxed{4}$$

Inequalities are solved exactly like equations with this exception:

> If both sides of the inequality are either multiplied or divided by a **negative** quantity, the **inequality symbol must be reversed**.

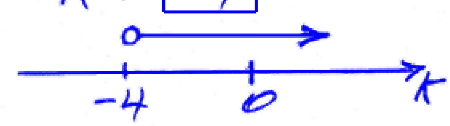
Example 9: Solve and graph 5p – 8 ≥ 2.

$$5p - 8 \geq 2$$
$$5p \geq 8 + 2$$
$$5p \geq 10$$
$$p \geq \frac{10}{5}$$
$$p \geq \boxed{2}$$



Example 10: Solve and graph
4k + 11 < 6k + 19.

$$4k + 11 < 6k + 19$$
$$4k - 6k < 19 - 11$$
$$-2k < 8$$
$$k > \frac{8}{-2}$$
$$k > \boxed{-4}$$



Fundamental to all of algebra is knowledge and immediate recall of all sign rules. See **In-Depth Topic A** for practice with the sign rules.

Assignment: Solve for the variable in each equation or inequality. Graph the inequalities.

1. 13h – 4 = 22

$$13h - 4 = 22$$
$$13h = 22 + 4$$
$$13h = 26$$
$$h = \frac{26}{13} = \boxed{2}$$

2. 4x – 9 = -3

$$4x - 9 = -3$$
$$4x = -3 + 9$$
$$4x = 6$$
$$x = \frac{6}{4} = \boxed{\frac{3}{2}}$$

3. 5(3 – 2e) + e = 11

$$5(3-2e) + e = 11$$
$$15 - 10e + e = 11$$
$$-9e = -15 + 11$$
$$-9e = -4$$
$$e = \frac{-4}{-9} = \boxed{\frac{4}{9}}$$

4. (1/2)x + 1 = 12

$$\frac{1}{2}x + 1 = 12$$
$$\frac{1}{2}x = 12 - 1$$
$$\frac{1}{2}x = 11$$
$$\cancel{2}\left(\frac{1}{\cancel{2}}x\right) = 11(2)$$
$$x = \boxed{22}$$

5. 114 = (x + 2)15 – 3x

$$114 = (x+2)15 - 3x$$
$$114 = 15x + 30 - 3x$$
$$114 - 30 = 12x$$
$$84 = 12x$$
$$\frac{84}{12} = x$$
$$x = \boxed{7}$$

6. p – 12 = 4p + 21

$$p - 12 = 4p + 21$$
$$p - 4p = 21 + 12$$
$$-3p = 33$$
$$p = \frac{33}{-3}$$
$$p = \boxed{-11}$$

7. -3f + 2 = 3(f – 2) – 8f

$-3f+2=3(f-2)-8f$

$-3f+2=3f-6-8f$

$-3f=-5f-6-2$

$-3f+5f=-8$

$2f=-8$

$f=\frac{-8}{2}=\boxed{-4}$

8. $\dfrac{4x+1}{3} - 1 = 12$

$\frac{4x+1}{3}-1=12$

$\frac{4x+1}{3}=12+1$

$3\cdot\frac{4x+1}{3}=13\cdot 3$

$4x+1=39$

$4x=39-1=38$

$x=\frac{38}{4}$

$x=\boxed{\frac{19}{2}}$

9. 8x < x – 14

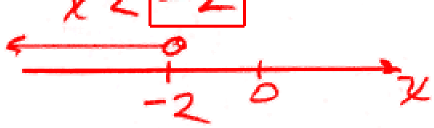
$8x<x-14$

$8x-x<-14$

$7x<-14$

$x<\frac{-14}{7}$

$x<\boxed{-2}$



10. -x + 1 ≥ 6x + 22

$-x+1\geq 6x+22$

$-x-6x\geq 22-1$

$-7x\geq 21$

$x\leq\frac{21}{-7}$

$x\leq -3$

11. -2(x – .5) < x – 1

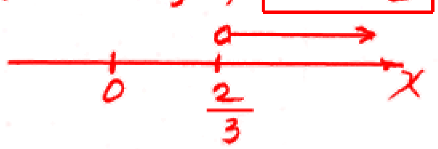
$-2(x-.5)<x-1$

$-2x+1<x-1$

$-2x-x<-1-1$

$-3x<-2$

$x>\frac{-2}{-3}$; $\boxed{x>\frac{2}{3}}$



12. 4(u – 3) ≤ 5(u – 6)

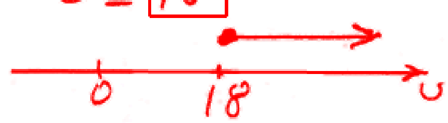
$4(u-3)\leq 5(u-6)$

$4u-12\leq 5u-30$

$4u-5u\leq -30+12$

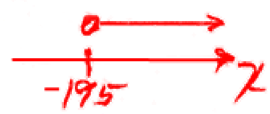
$-u\leq -18$

$u\geq\boxed{18}$



13. $.3(x – 5) > -60$

$$.3(x-5) > -60$$
$$.3x - 1.5 > -60$$
$$.3x > -60 + 1.5$$
$$.3x > -58.5$$
$$x > -58.5/.3$$
$$x > \boxed{-195}$$



14. $2b + 3b – 1 = 8b – 13$

$$2b + 3b - 1 = 8b - 13$$
$$5b - 1 = 8b - 13$$
$$5b - 8b = -13 + 1$$
$$-3b = -12$$
$$b = \frac{-12}{-3} = \boxed{4}$$

Unit 1: Lesson 02 Solving systems of two linear equations

A system of linear equations that we will consider here consists of two linear equations whose graphs (lines) generally intersect.

The (x, y) **point of intersection** is considered the **solution** of the system.

We will consider two techniques for solving such a system:

- The **elimination method** (sometimes called the addition method)
- The **substitution method**

Example 1: Solve this system using the elimination method.

$-2x + 3y = 11; \quad 2x + y = 1$

$$\cancel{-2x}+3y=11$$
$$\cancel{2x}+y=1$$
$$4y=12$$
$$y=\frac{12}{4}$$
$$y=\boxed{3}$$

$$2x+y=1$$
$$2x+3=1$$
$$2x=1-3$$
$$2x=-2$$
$$x=\frac{-2}{2}=\boxed{-1}$$

or (-1, 3)

Example 2: Solve this system using the elimination method.

$2x - 3y = 4 ; \ x + 4y = -9$

$$2x-3y=4 \longrightarrow \cancel{2x}-3y=4$$
$$-2(x+4y)=-9(-2) \longrightarrow \cancel{-2x}-8y=18$$
$$-11y=22$$
$$y=\frac{22}{-11}=\boxed{-2}$$

$$x+4y=-9$$
$$x+4(-2)=-9$$
$$x-8=-9$$
$$x=-9+8$$
$$x=\boxed{-1}$$

or (-1, -2)

Example 3: Solve this system using the substitution method.

$y = 3x + 4$
$x – y = 2$

$y = 3x + 4$ $\qquad$ $x - y = 2$

$y = 3(-3) + 4$ $\qquad$ $x - (3x+4) = 2$

$y = -9 + 4$ $\qquad$ $x - 3x - 4 = 2$

$y = \boxed{-5}$ $\qquad$ $-2x = 4+2$

$x = \frac{6}{-2} = \boxed{-3}$

or (-3, -5)

Example 4: Solve this system using the substitution method.

$x – 3y = 4$
$2x + 7y = -5$

$x - 3y = 4$

$x = 3y + 4$ $\qquad$ $2x + 7y = -5$

$x = 3(-1) + 4$ $\qquad$ $2(3y+4) + 7y = -5$

$x = -3 + 4$ $\qquad$ $6y + 8 + 7y = -5$

$x = \boxed{1}$ $\qquad$ $13y = -8 - 5$

$13y = -13$

$y = \frac{-13}{13} = \boxed{-1}$

or (1, -1)

Assignment: Solve the following linear systems using the substitution method.

1. x + y = 8 ; y = 3x

$y = 3x$ $\quad x + y = 8$

$y = 3\cdot 2$ $\quad x + 3x = 8$

$4x = 8$

$y = \boxed{6}$ $\quad x = \frac{8}{4} = \boxed{2}$

or (2, 6)

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2. y = 3x – 8 ; x + y = 4

$y = 3x - 8$ $\quad x + y = 4$

$x + 3x - 8 = 4$

$y = 3\cdot 3 - 8$ $\quad 4x = 4 + 8$

$y = 9 - 8$ $\quad 4x = 12$

$x = \frac{12}{4} = \boxed{3}$

$y = \boxed{1}$

or (3, 1)

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3. 3x – 5y = 11 ; x = 3y + 1

$x = 3y + 1$ $\quad 3x - 5y = 11$

$x = 3\cdot 2 + 1$ $\quad 3(3y + 1) - 5y = 11$

$x = 6 + 1$ $\quad 9y + 3 - 5y = 11$

$4y = 11 - 3 = 8$

$x = \boxed{7}$ $\quad y = \frac{8}{4} = \boxed{2}$

or (7, 2)

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4. x + 4y = 1 ; 2x + y = 9

$x + 4y = 1$ $\quad 2x + y = 9$

$x = 1 - 4y$ $\quad 2(1 - 4y) + y = 9$

$2 - 8y + y = 9$

$x = 1 - 4(-1)$ $\quad -7y = 9 - 2 = 7$

$x = 1 + 4$

$x = \boxed{5}$ $\quad y = \frac{7}{-7} = \boxed{-1}$

or (5, -1)

5. 2a + 7b = 3 ; a = 1 – 4b

$a = 1-4b$ $\quad$ $2a + 7b = 3$

$a = 1-4(-1)$ $\quad$ $2(1-4b)+7b = 3$

$a = 1+4$ $\quad$ $2-8b+7b = 3$

$a = \boxed{5}$ $\quad$ $-1b = 3-2 = 1$

$b = \frac{1}{-1} = \boxed{-1}$

or (a, b) = (5, -1)

---

6. p – 5q = 2 ; 2p + q = 4

$p-5q = 2$ $\quad$ $2p+q = 4$

$p = 5q+2$ $\quad$ $2(5q+2)+q = 4$

$p = 5\cdot 0+2$ $\quad$ $10q+4+q = 4$

$p = 0+2$ $\quad$ $11q = 4-4 = 0$

$p = \boxed{2}$ $\quad$ $q = \frac{0}{11} = \boxed{0}$

or (p, q)=(2, 0)

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7. -4a + 5b = 17 ; 5a – b = 5

$5a-b = 5$ $\quad$ $-4a+5b = 17$

$-b = -5a+5$ $\quad$ $-4a+5(5a-5) = 17$

$-1(-b) = -1(-5a+5)$ $\quad$ $-4a+25a-25 = 17$

$b = 5a-5$ $\quad$ $21a = 17+25 = 42$

$b = 5\cdot 2-5$ $\quad$ $a = \frac{42}{21} = \boxed{2}$

$b = 10-5$

$b = \boxed{5}$

or (a,b)= (2, 5)

Solve the following linear systems using the elimination method.

8. 4x – 3y = -2 ; 2x + 3y = 26

$$\begin{aligned} 4x - \cancel{3y} &= -2 \\ 2x + \cancel{3y} &= 26 \\ \hline 6x &= 24 \\ x &= \frac{24}{6} = \boxed{4} \end{aligned}$$

$$\begin{aligned} 2x + 3y &= 26 \\ 2\cdot 4 + 3y &= 26 \\ 8 + 3y &= 26 \\ 3y &= 26 - 8 = 18 \\ y &= \frac{18}{3} = \boxed{6} \end{aligned}$$

or, (4, 6)

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9. a – b = 4 ; a + b = 8

$$\begin{aligned} a - \cancel{b} &= 4 \\ a + \cancel{b} &= 8 \\ \hline 2a &= 12 \\ a &= \frac{12}{2} = \boxed{6} \end{aligned}$$

$$\begin{aligned} a + b &= 8 \\ 6 + b &= 8 \\ b &= 8 - 6 \\ b &= \boxed{2} \end{aligned}$$

or (a, b) = (6, 2)

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10. 2x – 5y = -6 ; 2x – 7y = -14

$$2x - 5y = -6 \longrightarrow \cancel{2x} - 5y = -6$$

$$-1(2x - 7y) = -14(-1) \longrightarrow \underline{-\cancel{2x} + 7y = 14}$$

$$\begin{aligned} 2y &= 8 \\ y &= \frac{8}{2} = \boxed{4} \end{aligned}$$

$$\begin{aligned} 2x - 5y &= -6 \\ 2x - 5(4) &= -6 \\ 2x &= 20 - 6 = 14 \\ x &= \frac{14}{2} = \boxed{7} \end{aligned}$$

or (7, 4)

11. 3x + y = 4 ; 5x – y = 12

$$3x + \cancel{y} = 4$$
$$5x - \cancel{y} = 12$$
$$8x = 16$$
$$x = \frac{16}{8} = \boxed{2}$$

$$3x + y = 4$$
$$3 \cdot 2 + y = 4$$
$$y = 4 - 6$$
$$y = \boxed{-2}$$

or (2, -2)

12. 5p + 2q = 6 ; 9p + 2q = 22

$$-1(5p + 2q) = 6(-1) \longrightarrow -5p - \cancel{2q} = -6$$
$$9p + 2q = 22 \longrightarrow 9p + \cancel{2q} = 22$$
$$4p = 16$$
$$p = \frac{16}{4} = \boxed{4}$$

$$5p + 2q = 6$$
$$5 \cdot 4 + 2q = 6$$
$$20 + 2q = 6$$
$$2q = -20 + 6 = -14$$
$$q = \frac{-14}{2} = \boxed{-7}$$

or (p, q) = (4, -7)

13. 5x + 12y = -1 ; 8x + 12y = 20

$$-1(5x + 12y) = -1(-1) \longrightarrow -5x - \cancel{12y} = 1$$
$$8x + 12y = 20 \longrightarrow 8x + \cancel{12y} = 20$$
$$3x = 21$$
$$x = \frac{21}{3} = \boxed{7}$$

$$5x + 12y = -1$$
$$5 \cdot 7 + 12y = -1$$
$$12y = -35 - 1 = -36$$
$$y = \frac{-36}{12} = \boxed{-3}$$

or (7, -3)

14. $3x – 4y = 8$; $4x + 3y = 19$

$$3(3x - 4y) = 8 \cdot 3 \longrightarrow 9x - \cancel{12y} = 24$$
$$4(4x + 3y) = 19 \cdot 4 \longrightarrow \underline{16x + \cancel{12y} = 76}$$
$$25x = 100$$
$$x = \frac{100}{25} = \boxed{4}$$

$$3x - 4y = 8$$
$$3 \cdot 4 - 4y = 8$$
$$-4y = 8 - 12$$
$$y = \frac{-4}{-4} = \boxed{1}$$

or (4, 1)

Unit 1: Lesson 03 Trinomial factoring

Example 1: Use the box method to find the factors of $x^2 + 2x - 48$. Begin by placing the x^2 and -48 terms in the box and producing a product.

x^2	
	-48

$\text{product} = -48x^2$

Next, find two terms whose product is that given above ($-48x^2$) and whose sum is 2x, and then fill in the other two positions of the box with these two terms.

x^2	$-6x$
$8x$	-48

$8x(-6x) = -48x^2$
$8x - 6x = 2x$

Now place the GCF of each row to the left of the row. Place the GCF of each column above the column. Finally, use these GCFs to produce the factors as shown here:

	x	-6
x	x^2	$-6x$
8	$8x$	-48

$x^2 + 2x - 48 = \boxed{(x-6)(x+8)}$

Example 2: Use the box method to find the factors of $6x^2 - 17x + 5$. Specify the product and sum that were used in arriving at the answer.

	$2x$	-5
$3x$	$6x^2$	$-15x$
-1	$-2x$	5

Product: $30x^2$

Sum: $-17x$

$6x^2 - 17x + 5 = \boxed{(2x-5)(3x-1)}$

Example 3: Use the box method to find the factors of $3x^2 + 13x - 10$. Specify the product and sum that were used in arriving at the answer.

	x	5
$3x$	$3x^2$	$15x$
-2	$-2x$	-10

Product: $-30x^2$

Sum: $13x$

$$3x^2 + 13x - 10 = \boxed{(3x-2)(x+5)}$$

Important sign rule!

If the item in either cell indicated here (or both) is negative, then the corresponding GCF adjacent to it will be **negative**; otherwise the GCF is positive.

		$-GCF$
		$-\#$
$-GCF$	$-\#$	

Assignment: Use the box method to find the factors of the given trinomial. Specify the product and sum that were used in arriving at the answer.

1. $3x^2 - 2x - 5$

	$3x$	-5
x	$3x^2$	$-5x$
1	$3x$	-5

Product: $-15x^2$

Sum: $-2x$

$3x^2 - 2x - 5 = \boxed{(3x-5)(x+1)}$

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2. $y^2 + 5y - 24$

	y	8
y	y^2	$8y$
-3	$-3y$	-24

Product: $-24y^2$

Sum: $5y$

$y^2 + 5y - 24 = \boxed{(y+8)(y-3)}$

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3. $5m^2 - 13m - 6$

	m	-3
$5m$	$5m^2$	$-15m$
2	$2m$	-6

Product: $-30m^2$

Sum: $-13m$

$5m^2 - 13m - 6 = \boxed{(m-3)(5m+2)}$

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4. $x^2 + 7x + 12$

	x	4
x	x^2	$4x$
3	$3x$	12

Product: $12x^2$

Sum: $7x$

$x^2 + 7x + 12 = \boxed{(x+4)(x+3)}$

5. $2z^2 + 5z - 12$

	z	4
$2z$	$2z^2$	$8z$
-3	$-3z$	-12

Product: $-24z^2$

Sum: $5z$

$2z^2+5z-12=\boxed{(z+4)(2z-3)}$

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6. $x^2 + 6x + 5$

	x	5
x	x^2	$5x$
1	$1x$	5

Product: $5x^2$

Sum: $6x$

$x^2+6x+5=\boxed{(x+5)(x+1)}$

---

7. $p^2 - p - 12$

	p	-4
p	p^2	$-4p$
3	$3p$	-12

Product: $-12p^2$

Sum: $-p$

$p^2-p-12=\boxed{(p-4)(p+3)}$

---

8. $x^2 + 3x - 18$

	x	6
x	x^2	$6x$
-3	$-3x$	-18

Product: $-18x^2$

Sum: $3x$

$x^2+3x-18=\boxed{(x+6)(x-3)}$

---

9. $k^2 + 3k + 2$

	k	2
k	k^2	$2k$
1	$1k$	2

Product: $2k^2$

Sum: $3k$

$k^2+3k+2=\boxed{(k+2)(k+1)}$

10. $10x^2 - 7x + 1$

	$2x$	-1
$5x$	$10x^2$	$-5x$
-1	$-2x$	1

Product: $10x^2$

Sum: $-7x$

$10x^2 - 7x + 1 = \boxed{(2x-1)(5x-1)}$

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11. $4x^2 + 19x + 21$

	x	3
$4x$	$4x^2$	$12x$
7	$7x$	21

Product: $84x^2$

Sum: $19x$

$4x^2 + 19x + 21 = \boxed{(x+3)(4x+7)}$

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12. $t^2 - 3t - 70$

	t	-10
t	t^2	$-10t$
7	$7t$	-70

Product: $-70t^2$

Sum: $-3t$

$t^2 - 3t - 70 = \boxed{(t-10)(t+7)}$

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13. $p^2 + 12p + 32$

	p	4
p	p^2	$4p$
8	$8p$	32

Product: $32p^2$

Sum: $12p$

$p^2 + 12p + 32 = \boxed{(p+4)(p+8)}$

---

14. $2x^2 - 7x + 6$

	$2x$	-3
x	$2x^2$	$-3x$
-2	$-4x$	6

Product: $12x^2$

Sum: $-7x$

$2x^2 - 7x + 6 = \boxed{(2x-3)(x-2)}$

Unit 1: Lesson 04

Special factoring formulas
$a^2 - b^2$, $a^2 \pm 2ab + b^2$

Recall from Algebra 1 the shortcut for factoring $a^2 \pm 2ab + b^2$:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Example 1: Multiply $(x - 5)^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(x - 5)^2 = x^2 - 2x5 + 5^2$

$= \boxed{x^2 - 10x + 25}$

Example 2: Multiply $(3y + 2b)^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(3y + 2b)^2 = (3y)^2 + 2(3y)(2b) + (2b)^2$

$= \boxed{9y^2 + 12yb + 4b^2}$

Example 3: Factor $x^2 - 8x + 16$

$a^2 - 2ab + b^2 = (a - b)^2$

$x^2 - 2x4 + 4^2 = \boxed{(x - 4)^2}$

Example 4: Factor $m^2 + 18m + 81$

$a^2 + 2ab + b^2 = (a + b)^2$

$m^2 + 2m9 + 9^2 = \boxed{(m + 9)^2}$

Recall from Algebra 1 the shortcut for factoring $a^2 - b^2$ (difference of squares):

$$a^2 - b^2 = (a - b)(a + b)$$

Example 5: Multiply (p – 7y)(p + 7y)

$(a - b)(a + b) = a^2 - b^2$

$(p - 7y)(p + 7y) = \boxed{p^2 - 49y^2}$

Example 6: Factor $k^2 - 100$

$(a)^2 - (b)^2 = (a - b)(a + b)$

$(k)^2 - (10)^2 = \boxed{(k - 10)(k + 10)}$

Assignment:

1. Multiply $(x - 8)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(x-8)^2 = x^2 - 2x8 + 64$

$= \boxed{x^2 - 16x + 64}$

2. Multiply $(5 + b)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(5+b)^2 = 5^2 + 2 \cdot 5b + b^2$

$= \boxed{25 + 10b + b^2}$

3. Multiply $(v - 8)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(v-8) = v^2 - 2v8 + (8)^2$

$= \boxed{v^2 - 16v + 64}$

4. Multiply $(2f + g)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(2f+g)^2 = (2f)^2 + 2 \cdot 2f \cdot g + g^2$

$= \boxed{4f^2 + 4fg + g^2}$

5. Factor $r^2 - 64$

$r^2 - 64$

$= (r)^2 - (8)^2 = \boxed{(r-8)(r+8)}$

$= (a)^2 - (b)^2$

6. Factor $t^2 - 22t + 121$

$t^2 - 22t + 121$

$= t^2 - 2(11)t + (11)^2$

$= \boxed{(t-11)^2}$

7. Factor $4v^2 - 12v + 9$

$4v^2 - 12v + 9$

$= (2v)^2 - 2(2v)3 + 3^2$

$(a)^2 - 2a\,b + b^2$

$= \boxed{(2v-3)^2}$

8. Factor $r^2 + 24r + 144$

$r^2 + 24r + 144$

$= r^2 + 2 \cdot 12r + (12)^2$

$= \boxed{(r+12)^2}$

9. Factor $4b^2 - 49$

$$4b^2 - 49$$
$$= (2b)^2 - (7)^2$$
$$= \boxed{(2b-7)(2b+7)}$$

10. Factor $4 - 4p + p^2$

$$4 - 4p + p^2$$
$$= 4 - 2\cdot 2p + p^2$$
$$= \boxed{(2-p)^2}$$

11. Factor $81z^2 - 121$

$$(a)^2 - (b)^2 = (a-b)(a+b)$$
$$(9z)^2 - (11)^2 = \boxed{(9z-11)(9z+11)}$$

*12. Factor $h^4 - 25$

$$(a)^2 - (b)^2 = (a-b)(a+b)$$
$$(h^2)^2 - (5)^2 = \boxed{(h^2-5)(h^2+5)}$$

13. Factor $16x^2 - 8x + 1$

$$a^2 - 2ab + b^2 = (a-b)^2$$
$$(4x)^2 - 2(4x)1 + 1^2 =$$
$$\boxed{(4x-1)^2}$$

14. Factor $p^2 + 22p + 121$

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$p^2 + 2\cdot 11p + 11^2 = \boxed{(p+11)^2}$$

15. Factor $j^2x^2 - 12jx + 36$

$$(a)^2 - 2ab + (b)^2 = (a-b)^2$$
$$(jx)^2 - 2jx6 + (6)^2 = \boxed{(jx-6)^2}$$

16. Factor $16 + 8x + x^2$

$$(a)^2 + 2ab + (b)^2 = (a+b)^2$$
$$4^2 + 2\cdot 4x + (x)^2 = \boxed{(4+x)^2}$$

Unit 1: Lesson 05 Solving quadratic equations

A quadratic equation is an equation that can be put in this form:

$$ax^2 + bx + c = 0$$

where *x* is a variable and *a*, *b*, and *c* are constants.

In this lesson we will review the following techniques for solving a quadratic equation.

- Factoring
- Quadratic formula
- Graphing calculator

Example 1: Solve $x^2 + 4x - 12 = 0$ by factoring.

$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x+6=0 \qquad x-2=0$$
$$\boxed{x=-6} \qquad \boxed{x=2}$$

Example 2: Solve $8x^2 - 2x - 3 = 0$ by factoring.

	$4x$	-3
$2x$	$8x^2$	$-6x$
1	$4x$	-3

Product: $-24x^2$

Sum: $-2x$

$$8x^2 - 2x - 3 = 0$$
$$(4x-3)(2x+1) = 0$$

$$4x-3=0 \qquad 2x+1=0$$
$$4x=3 \qquad 2x=-1$$
$$x=\boxed{\tfrac{3}{4}} \qquad x=\boxed{-\tfrac{1}{2}}$$

For quadratics that can't be factored, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

See **In-Depth Topic B** for a derivation of the quadratic formula.

Example 3: Solve $x^2 + 10x + 24 = 0$ using the quadratic formula.

$a = 1 \quad b = 10 \quad c = 24$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 1 \cdot 24}}{2 \cdot 1}$$

$$x = \frac{-10 \pm \sqrt{100 - 96}}{2}$$

$$x = \frac{-10 \pm \sqrt{4}}{2}$$

$$x = \frac{-10 \pm 2}{2}$$

$$x = \frac{-10 + 2}{2} \qquad x = \frac{-10 - 2}{2}$$

$$x = \frac{-8}{2} \qquad x = \frac{-12}{2}$$

$$x = \boxed{-4} \qquad x = \boxed{-6}$$

Example 4: Solve $2x^2 + 3x – 2 = 0$ using the quadratic formula

$2x^2 + 3x - 2 = 0$

$a = 2 \quad b = 3 \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2 \cdot 2}$$

$$x = \frac{-3 \pm \sqrt{9 + 16}}{4}$$

$$x = \frac{-3 \pm \sqrt{25}}{4}$$

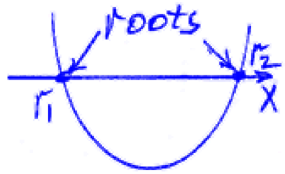
$$x = \frac{-3 \pm 5}{4}$$

$$x = \frac{-3 + 5}{4} \qquad x = \frac{-3 - 5}{4}$$

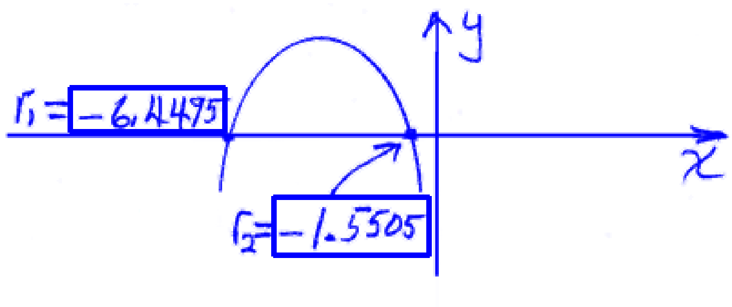
$$x = \boxed{\frac{1}{2}} \qquad x = \frac{-8}{4}$$

$$x = \boxed{-2}$$

Quadratic equations are easily solved using a graphing calculator by first graphing the quadratic function and then finding the zeros (where it crosses the x-axis).

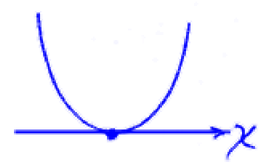


Example 5: Use a graphing calculator to find the roots (zeros) of $y = -x^2 - 8x - 10$. Make a sketch of the calculator display and label the roots.

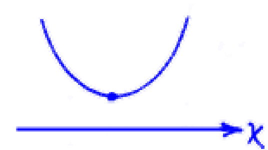


Special cases:

- **Double root:** The parabola is tangent to the x-axis (only touches in one place).



- **No solution:** The parabola never crosses the x-axis. There are no real roots; however, there **are** two roots (imaginary).



Assignment:

1. Solve $x^2 + 7x - 8 = 0$ by factoring.

$x^2+7x-8=0$

$(x+8)(x-1)=0$

$x+8=0 \qquad x-1=0$

$x=\boxed{-8} \qquad x=\boxed{1}$

2. Solve $x^2 - 29x + 180 = 0$ by factoring.

$x^2-29x+180=0$

$(x-20)(x-9)=0$

$x-20=0 \qquad x-9=0$

$x=\boxed{20} \qquad x=\boxed{9}$

3. Solve $x^2 + x - 90 = 0$ by factoring.

$x^2+x-90=0$

$(x+10)(x-9)=0$

$x+10=0 \qquad x-9=0$

$x=\boxed{-10} \qquad x=\boxed{9}$

4. Solve $x^2 + 14x + 33 = 0$ by factoring.

$x^2+14x+33=0$

$(x+11)(x+3)=0$

$x+11=0 \qquad x+3=0$

$x=\boxed{-11} \qquad x=\boxed{-3}$

5. Solve $3x^2 + x - 14 = 0$ by factoring.

	$3x$	7
x	$3x^2$	$7x$
-2	$-6x$	-14

Product: $-42x^2$

Sum: $1x$

$3x^2+x-14=0$

$(3x+7)(x-2)=0$

$3x+7=0 \qquad x-2=0$

$3x=-7 \qquad x=\boxed{2}$

$x=\boxed{\frac{-7}{3}}$

6. Solve $6p^2 - 17p + 12 = 0$ by factoring.

	$3p$	-4
$2p$	$6p^2$	$-8p$
-3	$-9p$	12

Product: $72p^2$

Sum: $-17p$

$6p^2-17p+12=0$

$(3p-4)(2p-3)=0$

$3p-4=0 \qquad 2p-3=0$

$3p=4 \qquad 2p=3$

$p=\boxed{4/3} \qquad p=\boxed{3/2}$

7. Solve $x^2 + 8x - 9 = 0$ using the quadratic formula.

$a=1 \quad b=8 \quad c=-9$

$x = \dfrac{-8 \pm \sqrt{8^2-4(1)(-9)}}{2(1)}$

$x = \dfrac{-8 \pm \sqrt{64+36}}{2}$

$x = \dfrac{-8 \pm \sqrt{100}}{2}$

$x = \dfrac{-8 \pm 10}{2}$

$x = \dfrac{-8+10}{2} \qquad x = \dfrac{-8-10}{2}$

$x = \dfrac{2}{2} = \boxed{1} \qquad x = \dfrac{-18}{2} = \boxed{-9}$

8. Solve $2x^2 - 3x + 1 = 0$ using the quadratic formula.

$a=2 \quad b=-3 \quad c=1$

$x = \dfrac{3 \pm \sqrt{(-3)^2-4(2)1}}{2 \cdot 2}$

$x = \dfrac{3 \pm \sqrt{9-8}}{4}$

$x = \dfrac{3 \pm \sqrt{1}}{4}$

$x = \dfrac{3 \pm 1}{4}$

$x = \dfrac{3+1}{4} \qquad x = \dfrac{3-1}{4}$

$x = \dfrac{4}{4} = \boxed{1} \qquad x = \dfrac{2}{4} = \boxed{\dfrac{1}{2}}$

9. Solve $10x^2 - 9x - 7 = 0$ using the quadratic formula.

$a=10 \quad b=-9 \quad c=-7$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{9 \pm \sqrt{(-9)^2-4(10)(-7)}}{2(10)}$

$x = \dfrac{9 \pm \sqrt{81+280}}{20}$

$x = \dfrac{9 \pm \sqrt{361}}{20}$

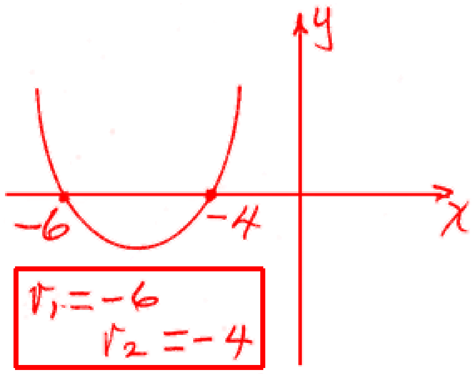
$x = \dfrac{9 \pm 19}{20}$

$x = \dfrac{9+19}{20} \qquad x = \dfrac{9-19}{20}$

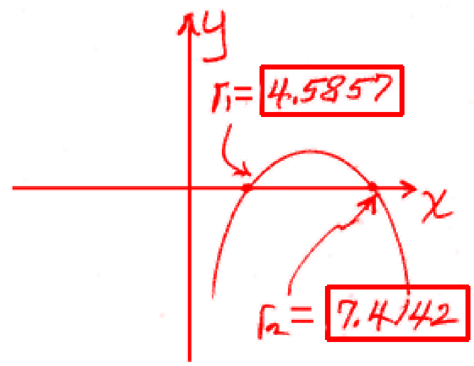
$x = \dfrac{28}{20} \qquad x = \dfrac{-10}{20}$

$x = \boxed{\dfrac{7}{5}} \qquad x = \boxed{-\dfrac{1}{2}}$

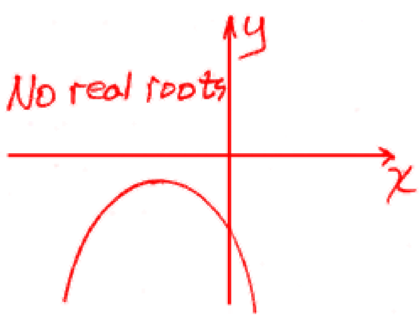
10. Solve $2x^2 + 20x + 48 = 0$ using a graphing calculator. Make a sketch of the calculator display.



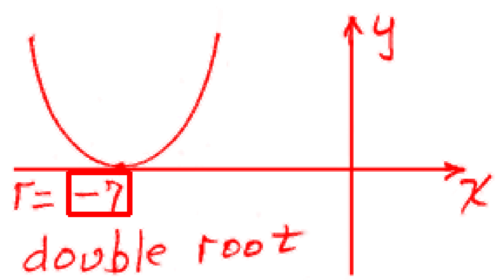
11. Solve $-x^2 + 12x - 34 = 0$ using a graphing calculator. Make a sketch of the calculator display.



12. Solve $-x^2 - 4x - 5 = 0$ using a graphing calculator. Make a sketch of the calculator display.



13. Solve $x^2 + 14x + 49 = 0$ using a graphing calculator. Make a sketch of the calculator display.



Unit 1: Review

1. Solve 11p – 7 = 2p + 1

$11p-7=2p+1$

$11p-2p=1+7$

$9p=8$

$p=\boxed{8/9}$

2. Solve -3y – 4(6y + 2) = y – 9

$-3y-4(6y+2)=y-9$

$-3y-24y-8=y-9$

$-27y-y=-9+8$

$-28y=-1$

$y=-1/(-28)=\boxed{1/28}$

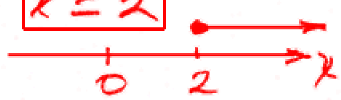
3. Solve and graph 5x – 8 ≥ 2.

$5x-8\ge 2$

$5x\ge 2+8$

$5x\ge 10$

$\boxed{x\ge 2}$



4. Solve and graph 4k + 11 ≤ 6k + 19.

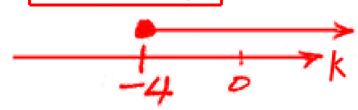
$4k+11\le 6k+19$

$4k-6k\le 19-11$

$-2k\le 8$

$k\ge 8/(-2)$

$\boxed{k\ge -4}$



5. Solve this system using the substitution method.

– 2x + 3y = 11; 2x + y = 1

$2x+y=1$

$y=-2x+1$

$-2x+3y=11$

$-2x+3(-2x+1)=11$

$-2x-6x+3=11$

$-8x=11-3$

$-8x=8$; $x=8/(-8)=\boxed{-1}$

$y=-2(-1)+1$

$y=2+1=\boxed{3}$

6. Solve this system using the elimination method.

$y = 3x + 4$
$x - y = 2$

$$-3x + \cancel{y} = 4$$
$$x - \cancel{y} = 2$$
$$-2x = 6$$
$$x = 6/(-2) = \boxed{-3}$$

$$y = 3x + 4$$
$$y = 3(-3) + 4$$
$$y = -9 + 4$$
$$y = \boxed{-5}$$

---

7. Factor $2x^2 + 7x - 30$

	x	6
$2x$	$2x^2$	$12x$
-5	$-5x$	-30

Product: $-60x^2$

Sum: $7x$

$$2x^2 + 7x - 30 = \boxed{(x+6)(2x-5)}$$

---

8. Factor $x^2 - x - 12$

	x	-4
x	x^2	$-4x$
3	$3x$	-12

Product: $-12x^2$

Sum: $-1x$

$$x^2 - x - 12 = \boxed{(x-4)(x+3)}$$

---

9. Factor $10v^2 - 7v + 1$

	$5v$	-1
$2v$	$10v^2$	$-2v$
-1	$-5v$	1

Product: $10v^2$

Sum: $-7v$

$$10v^2 - 7v + 1 = \boxed{(5v-1)(2v-1)}$$

10. Multiply $(h - 3)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(h-3)^2 = h^2 - 2h3 + 3^2$

$= \boxed{h^2 - 6h + 9}$

11. Multiply $(2x + 5)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(2x+5)^2 = (2x)^2 + 2 \cdot 2x \cdot 5 + 5^2$

$= \boxed{4x^2 + 20x + 25}$

12. Multiply $(3x - 7)(3x + 7)$

$(a-b)(a+b) = a^2 - b^2$

$(3x-7)(3x+7) = (3x)^2 - 7^2$

$= \boxed{9x^2 - 49}$

13. Factor $x^2 - 18x + 81$

$a^2 - 2ab + b^2 = (a-b)^2$

$= x^2 - 2x9 + 9^2$

$= \boxed{(x-9)^2}$

14. Factor $y^2 - 25$

$a^2 - b^2 = (a-b)(a+b)$

$y^2 - (5)^2 = \boxed{(y-5)(y+5)}$

15. Factor $c^2 + 4c + 4$

$a^2 + 2ab + b^2 = (a+b)^2$

$c^2 + 2c2 + 2^2 = \boxed{(c+2)^2}$

16. Solve $x^2 + 4x - 12 = 0$ by factoring.

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x + 6 = 0 \quad x - 2 = 0$

$x = \boxed{-6} \quad x = \boxed{2}$

17. Solve $2x^2 - 7x + 1 = 0$ using the quadratic formula.

$$2x^2 - 7x + 1 = 0$$

$a = 2 \quad b = -7 \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2 \cdot 2}$$

$$x = \frac{7 \pm \sqrt{49 - 8}}{4}$$

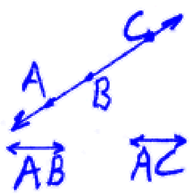
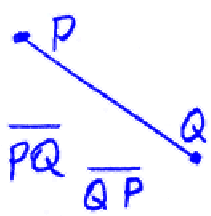
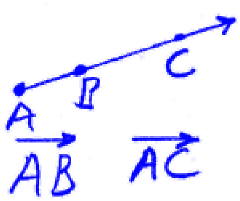
$$x = \boxed{\frac{7 \pm \sqrt{41}}{4}}$$

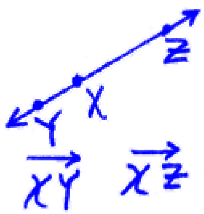
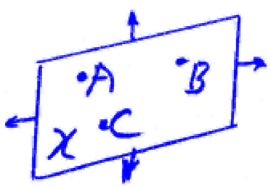
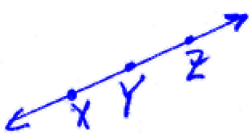
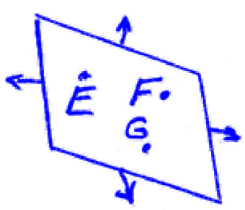
Geometry, Unit 2

Basic Definitions & Concepts
Points, Lines, and Planes

Unit 2: Lesson 01 Basic definitions (points, lines, & planes)

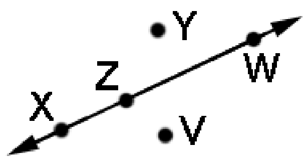
For the study of geometry, we must first establish a vocabulary. This means word **definitions** along with their corresponding **symbols**.

Term / Symbol	**Definition**
Point 	An exact location in space. A point has no size and is infinitely small. Denote points with a capital letter. (A, B, X, Y) Multiple points can be classified as collinear, coplanar. See those definitions below.
Line 	An object with no thickness that extends to infinity in two opposite directions. There are infinitely many points on the line. Denote lines with a bar that arrows on both ends. ($\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$) Lines have only one dimension, length.
Line Segment 	A portion of a line having two endpoints (and all points in between). ($\overline{PQ}$, $\overline{QP}$)
Ray 	A piece of a line having only one end point and extending infinitely far in one direction. ($\overrightarrow{AB}$, $\overrightarrow{AC}$) The arrow indicates the direction in which the ray extends.

Opposite rays 	Two rays that share the same end point but go in opposite directions.
Plane 	A flat surface that extends infinitely far in all directions within that flat surface. A plane contains an infinite number of points. The plane shown here is denoted with a capital letter, X.
Collinear points 	Points that lie on the same line. Any two points are always collinear because they lie on the line joining the two points.
Coplanar points 	Points that lie on the same plane. Any three points are always coplanar.

Assignment:

1. Give three points that are all collinear.



X, Z, and W

2. Which of the following set(s) of points from the drawing in problem 1 are **not** all collinear?

A. Y, X, and Z
B. X, W, and Z
C. Y, W, and V
D. X and V

A and C

3. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the front wall?

All of them

4. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the ceiling?

Upper right and upper left corners.

5. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the floor?

Lower right and lower left corners

6. Consider the four corners of the front wall of a rectangular classroom (upper right, upper left, lower right, and lower left). Which of these points are coplanar with the plane of the left wall?

Upper left and lower left corners

7. Where would be the **end point** of a **ray** of sunshine?

The sun

8. Would a goal line on a football field be described as a point, a ray, a line, a line segment, or a plane?

A line segment

9. Would the surface of a playing field be described as a point, a ray, a line, a line segment, or a plane? *A plane*	10. Would the place where the 50 yard-line and the side-line meet on a football field be described as a point, a ray, a line, a line segment, or a plane? *A point*
11. Would a knot in a rope most likely be described as a point, a ray, a line, a line segment, or a plane? *A point*	12. Would the top of your kitchen table most likely be described as a point, a ray, a line, a line segment, or a plane? *A plane*
13. Would a wall of your bedroom most likely be described as a point, a ray, a line, a line segment, or a plane? *A plane*	14. Would the colored dots (pixels) on a computer screen most likely be described as a point, a ray, a line, a line segment, or a plane? *A point*
15. Would a star in the nighttime sky most likely be described as a point, a ray, a line, a line segment, or a plane? *A point*	16. Would a flashlight beam most likely be described as a point, a ray, a line, a line segment, or a plane? *A ray*
17. Would a chocolate chip cookie most likely be described as a point, a ray, a line, a line segment, or a plane? *A plane*	18. Would the speck of chocolate in a chocolate chip cookie most likely be described as a point, a ray, a line, a line segment, or a plane? *A point*
19. Would a crease in a folded piece of paper most likely be described as a point, a ray, a line, a line segment, or a plane? *A line segment*	20. Would the path of a bullet most likely be described as a point, a ray, a line, a line segment, or a plane? *A ray*

21. Would a trampoline most likely be described as a point, a ray, a line, a line segment, or a plane?

A plane

22. Suppose you live on the same street as the school. Would the path from your house to the school likely be described as a point, a ray, a line, a line segment, or a plane?

A line segment

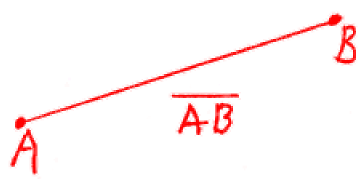
23. Would a fly caught in a spider web most likely be described as a point, a ray, a line, a line segment, or a plane?

A point

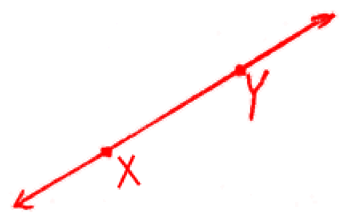
24. Suppose a rubber band is stretched forever in both directions (assume it never breaks). Would this most likely be described as a point, a ray, a line, a line segment, or a plane?

A line

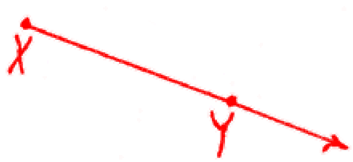
25. Draw a line segment and label the endpoints as A and B.



26. Draw and label $\overleftrightarrow{XY}$.



27. Draw and label $\overrightarrow{XY}$.



28. Draw and label $\overline{YZ}$.

Unit 2: Lesson 02

Postulates concerning points, lines, and planes
Practice with points, lines, and planes

A **postulate** is a statement that is **assumed to be true without requiring proof**. Following are some postulates related to points, lines, and planes.

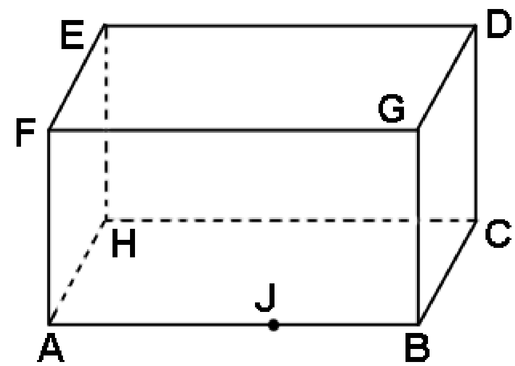
- A line contains at least two points.
- Through any two points there is exactly one line.
- If two lines intersect, then they intersect in exactly one point.
- A plane contains at least three non-collinear points.
- Planes through three points:
  - Through any three points there is **at least** one plane. If the points are collinear there are an infinite number of planes.
  - Through any three non-collinear points there is **exactly** one plane.
- If two points are in a plane, then the line that contains the points is also in the plane.
- If two different planes intersect, then their intersection is a line.

A **theorem** is a statement that must be proved.

Examples of theorems that we will encounter later:

- Vertical angles formed as the result of two intersecting lines are equal.
- The sum of the interior angles of a triangle is 180^o.
- The diagonals of a parallelogram bisect each other.
- The diagonals of a rhombus are perpendicular.
- and many more.

Use this drawing to answer the questions in the following examples. When possible, give one of the postulates on the preceding page to support your answer.



Example 1: Considering all the surfaces of the rectangular box, how many planes are shown? *6*	**Example 2:** Name the intersection of planes EFD and DCG. $\overleftrightarrow{GD}$ *If two different planes intersect, then their intersection is a line.*
Example 3: Are points E, J, and C coplanar? *Yes, through any three non-collinear points there is exactly one plane.*	**Example 4:** Do points A and J determine a line? *Yes, through any two points there is exactly one line.*
Example 5: Name the intersection of plane FGB and $\overleftrightarrow{AH}$. *Point A*	**Example 6:** How many lines are there passing through points A and D? *Just one. Through any two points there is exactly one line.*
Example 7: How many planes are there passing through points A, J, and B? *Infinitely many. If the points are collinear there are an infinite number of planes.*	**Example 8:** Name the intersection of $\overleftrightarrow{AJ}$ and $\overleftrightarrow{BC}$. *Point B. If two lines intersect, then they intersect in exactly one point.*

Example 9: Is $\overleftrightarrow{AE}$ in plane AHF?

Yes, if two points are in a plane, then the line that contains the points is also in the plane.

Example 10: Which plane(s) contain both $\overleftrightarrow{HC}$ and $\overleftrightarrow{CB}$?

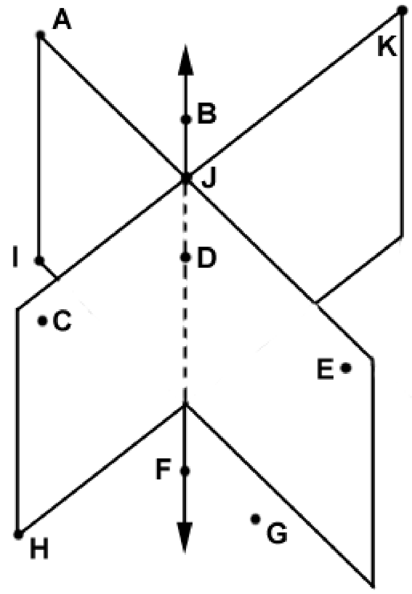
Plane ABC

Example 11: Does $\overrightarrow{JA}$ point toward the left, right, up, down, front, or back?

Left

Assignment:

Use this drawing to answer the questions in problems 1 - 8.



1. What are at least four possible names of the plane that slants from upper left to lower right? *AIJ, ADI, AJD, JDI, JDE, EIJ, EDI, etc.*	2. Name the line that is the intersection of the two planes. $\overleftrightarrow{BF}$ *or any other combination of two letters along this line.*
3. Name all the points that lie in the plane that slants from lower left to upper right. *H, C, J, D, K, B, F*	4. Name a set of at least three collinear points that lie in the plane that slants from upper right to lower left. *B, J, D, F*
5. Name all of the points in the plane that slants from upper left to lower right that are coplanar. *A, J, D, I, E, B, F*	6. In which plane does the line $\overleftrightarrow{KH}$ lie? *The plane that slants from lower left to upper right.*

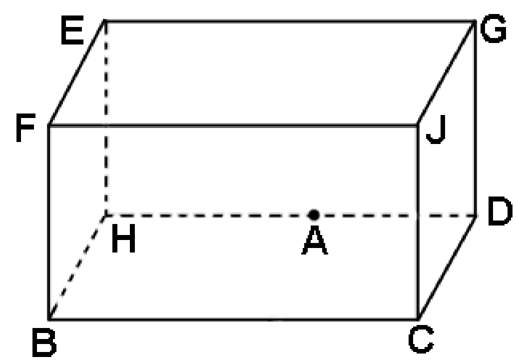
7. Are points K, D, and G coplanar?

Yes, any three points lie in a plane.

8. Are points C and E collinear?

Yes, two points are automatically collinear because a line can be drawn between them.

Use this drawing to answer the questions in problems 9 - 18.



9. Name three points that are collinear.

H, A, and D

10. Name the intersection of planes HBF and GJF.

Line $\overleftrightarrow{FE}$

11. How many planes make up the sides of the box? (Don't count the top or bottom.)

4

12. Are points H, A, and C coplanar? If possible, describe the plane in terms of a surface of the box.

Yes, it's the bottom of the box.

13. Are points B, D, and E coplanar? If possible, describe the plane in terms of a surface of the box.

Yes, three points always form a plane; however, it's not a surface of the box.

14. Does $\overrightarrow{JC}$ point toward the left, right, up, down, front, or back?

Down

15. Do points A and F form a line? If so, is it an edge of the box?

Yes, two points always form a line. No, it's not an edge of the box.

16. Name the intersection of plane GFJ and $\overleftrightarrow{EH}$.

Point E

17. How many planes pass through points H, A, and D?

Infinitely many. These points are collinear.

18. How many points are in plane JGD?

Infinitely many. A plane contains an infinite number of points.

19. A statement that is assumed to be true without proof is called a _______.

postulate

20. A statement that must be proven is called a _________.

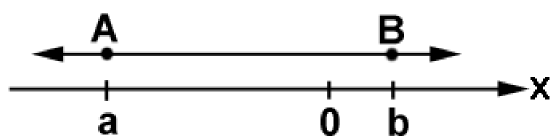
theorem

Unit 2: Lesson 03 Distance on a number line (length of a line segment)

Recall that a **line segment** consists of two end points and all "in-between" points on the line connecting them.

In this lesson we are concerned with the **length** of line segments.

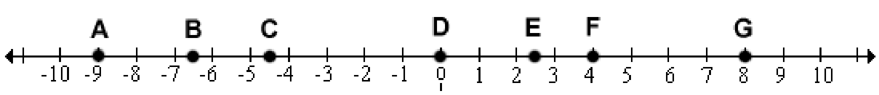
If a line segment lies on a number line and the end points A and B are at coordinates *a* and *b* on the number line, then the **length of the line segment** is simply the **distance between the two points**.



$$AB = |a - b|$$

Notice that **AB** now symbolizes the length of $\overline{AB}$ while $\overline{\mathbf{AB}}$ symbolizes the line segment itself.

Use the points on this number line to find the line segment lengths in examples 1 - 6:



Example 1: AB = ?

$$AB = |-9 - (-6.5)|$$
$$= |-9 + 6.5|$$
$$= |-2.5| = \boxed{2.5}$$

Example 2: GD = ?

$$GD = |8 - 0| = |8|$$
$$= \boxed{8}$$

Example 3: FB = ?

$FB = |4 - (-6.5)|$
$= |4 + 6.5|$
$= |10.5| = \boxed{10.5}$

Example 4: CE = ?

$CE = |-4.5 - 2.5|$
$= |-7|$
$= \boxed{7}$

Example 5: What is the difference between XY and $\overline{XY}$?

XY is the length of line segment $\overline{XY}$ while $\overline{XY}$ symbolizes the line segment itself.

Example 6: What is the length of $\overleftrightarrow{FD}$?

$\overleftrightarrow{FD}$ is a line that has infinite length.

For line segments that do not necessarily lie on a number line and for which the end points (x_1, y_1) & (x_2, y_2) are given, the length of the line segment is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

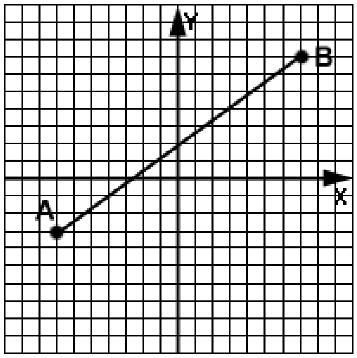
This is known as the **distance formula**.

Example 7: Find the length of the line segment whose end points are (4, -2) and (-8, 6).

$(x_1, y_1) = (4, -2)$
$(x_2, y_2) = (-8, 6)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-8 - 4)^2 + (6 - (-2))^2}$
$d = \sqrt{(-12)^2 + (8)^2} = \sqrt{144 + 64}$
$d = \sqrt{208} = \boxed{14.4222}$

Example 8: Find the length of line segment $\overline{AB}$.



A(−7, −3) B(7, 7)

$d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$

$d = \sqrt{(-7-7)^2 + (-3-7)^2}$

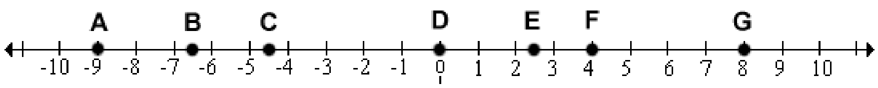
$d = \sqrt{(-14)^2 + (-10)^2}$

$d = \sqrt{196 + 100}$

$d = \sqrt{296} = \boxed{17.20465}$

Assignment:

Use the points on this number line to find the line segment lengths in problems 1 - 6:



1. EB = ?

$$EB = |2.5 - (-6.5)|$$
$$= |2.5 + 6.5|$$
$$= |9| = \boxed{9}$$

2. AE = ?

$$AE = |-9 - 2.5|$$
$$= |-11.5|$$
$$= \boxed{11.5}$$

3. GA = ?

$$GA = |8 - (-9)|$$
$$= |8 + 9| = |17|$$
$$= \boxed{17}$$

4. FC = ?

$$FC = |4 - (-4.5)|$$
$$= |4 + 4.5|$$
$$= |8.5| = \boxed{8.5}$$

5. AD = ?

$$AD = |-9 - 0|$$
$$= |-9| = \boxed{9}$$

6. EG = ?

$$EG = |2.5 - 8|$$
$$= |-5.5|$$
$$= \boxed{5.5}$$

7. What is the length of line segment $\overline{WV}$ where the coordinates of W are (101, -22) and the coordinates of V are (-8, 4)?

$$d = \sqrt{(101-(-8))^2+(-22-4)^2}$$
$$d = \sqrt{(101+8)^2+(-26)^2}$$
$$d = \sqrt{109^2+26^2}$$
$$d = \sqrt{12557} = \boxed{112.0580}$$

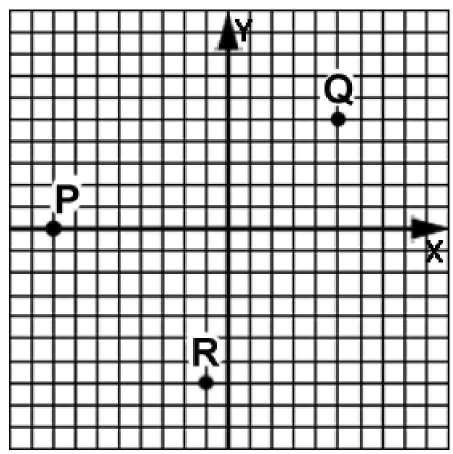
8. How far is it between (15.2, 8.6) and (9, -10.11)?

$$d = \sqrt{(15.2-9)^2+(8.6+10.11)^2}$$
$$d = \sqrt{6.2^2+18.71^2}$$
$$= \sqrt{388.5041}$$
$$= \boxed{19.7105}$$

9. Find PQ where P is located at (10,0) and Q at (-5, -12).

$$d = \sqrt{(10-(-5))^2+(0-(-12))^2}$$
$$= \sqrt{15^2+12^2}$$
$$= \sqrt{225+144}$$
$$= \sqrt{369} = \boxed{19.20937}$$

Use these points to find the indicated lengths in problems 10 & 11.



10. PQ = ?

P(-8, 0) Q(5, 5)

$$d = \sqrt{(-8-5)^2+(0-5)^2}$$
$$= \sqrt{(-13)^2+(-5)^2}$$
$$= \sqrt{169+25} = \sqrt{194}$$
$$= \boxed{13.9284}$$

11. QR = ?

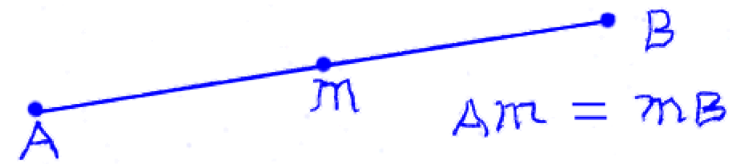
Q(5, 5) R(-1, -7)

$$d = \sqrt{(5-(-1))^2+(5-(-7))^2}$$
$$d = \sqrt{6^2+12^2}$$
$$d = \sqrt{180} = \boxed{13.4164}$$

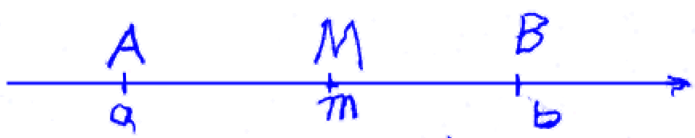
Unit 2: Lesson 04 Midpoint of a line segment (midpoint formula)

The **midpoint of a line segment** is the point that is equidistant from both endpoints.

For line segment $\overline{AB}$, midpoint M is located such that AM = MB.

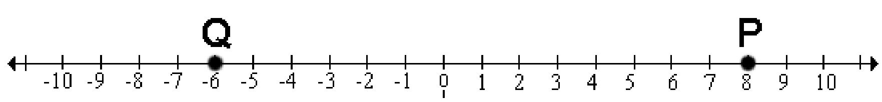


If the line segment $\overline{AB}$ is on a number line where the coordinate of point A is *a* and point B is *b*, then the coordinate of the midpoint *m* is the **average** of *a* and *b*.



$$m = \frac{a+b}{2}$$

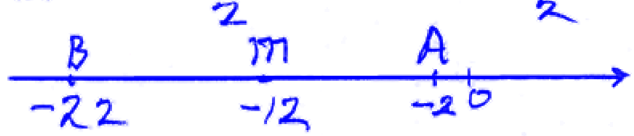
Example 1: Find the midpoint of $\overline{PQ}$.



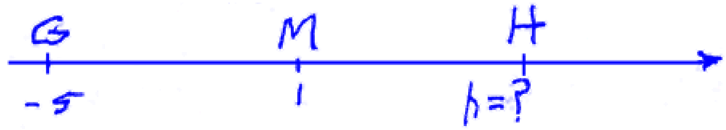
$$m = \frac{-6+8}{2} = \frac{2}{2} = \boxed{1}$$

Example 2: Find the midpoint of $\overline{AB}$ on a number line where A is at -2 and B is at -22.

$$m = \frac{-2+(-22)}{2} = \frac{-24}{2} = \boxed{-12}$$



Example 3: Consider line segment $\overline{GH}$ that lies on a number line. If G is located at -5 and the midpoint at 1, what is the coordinate of H?

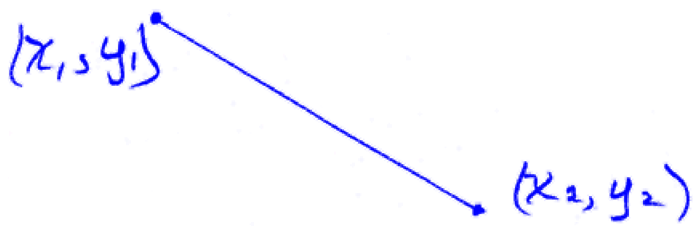


$$1 = \frac{-5+h}{2}$$

$$2 \cdot 1 = \frac{-5+h}{\cancel{2}} \cancel{2}$$

$$2 = -5+h \longrightarrow 2+5 = h \rightsquigarrow \boxed{7} = h$$

Now consider a line segment that does not lie on a number line.



The midpoint coordinates are still found by averaging.

$$x_m = \frac{x_1+x_2}{2}$$

$$y_m = \frac{y_1+y_2}{2}$$

$$(x_m, y_m) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

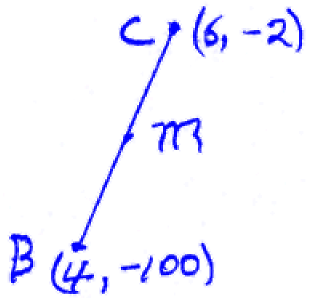
Example 4: If A has coordinates (2,-8) and B has coordinates (-2, 1), find the coordinates of the midpoint of $\overline{AB}$.

$$x_m = \frac{2+(-2)}{2} = 0$$

$$y_m = \frac{-8+1}{2} = \frac{-7}{2}$$

$$(x_m, y_m) = \boxed{\left(0, -\frac{7}{2}\right)}$$

Example 5: If B has coordinates (4,-100) and C has coordinates (6, -2), find the coordinates of the midpoint of $\overline{BC}$.

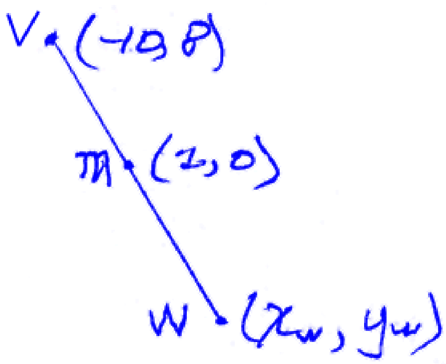


$$x_m = \frac{4+6}{2} = \frac{10}{2} = 5$$

$$y_m = \frac{-100-2}{2} = \frac{-102}{2} = -51$$

$$(x_m, y_m) = \boxed{(5, -51)}$$

Example 6: What are the coordinates of W if the midpoint of $\overline{WV}$ is at (2, 0) and the coordinates of V are (-10, 8)?



$$2 = \frac{-10 + x_w}{2}$$

$$4 = -10 + x_w$$

$$4 + 10 = x_w$$

$$14 = x_w$$

$$0 = \frac{8 + y_w}{2}$$

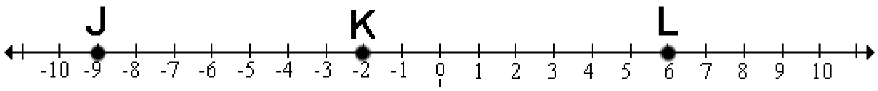
$$0 = 8 + y_w$$

$$-8 = y_w$$

$$(x_w, y_w) = \boxed{(14, -8)}$$

Assignment:

Use the points shown here in problems 1 and 2.



1. Find the midpoint of $\overline{JL}$.

$$m = \frac{-9+6}{2}$$
$$= \boxed{\frac{-3}{2}}$$

2. Find the midpoint of $\overline{JK}$.

$$m = \frac{-9+(-2)}{2}$$
$$m = \boxed{\frac{-11}{2}}$$

3. Find the midpoint of $\overline{AB}$ when A is located at -18.2 on a number line and B is located at 9.

$$m = \frac{-18.2+9}{2}$$
$$= \frac{-9.2}{2} = \boxed{-4.6}$$

4. Point P is located at the origin of a number line and Q is 18 units to the left of the origin. Find the midpoint of $\overline{PQ}$.

$$p = 0 \quad q = -18$$
$$m = \frac{0+(-18)}{2}$$
$$= \frac{-18}{2} = \boxed{-9}$$

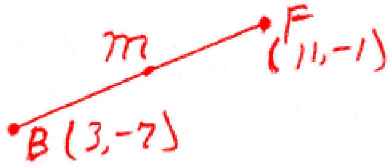
5. The midpoint of line segment $\overline{PL}$ is located at -4.2 on a number line while L is located at -.5 on the number line. What is the coordinate of P?

$$-4.2 = \frac{-.5 + x_P}{2}$$
$$-4.2(2) = \frac{-.5 + x_P}{\cancel{2}} \cancel{2}$$
$$-8.4 = -.5 + x_P$$
$$-8.4 + .5 = x_P$$
$$\boxed{-7.9} = x_P$$

6. Line segment $\overline{RL}$ lies on a number line with point L located at 17. What is the coordinate of R if the midpoint of $\overline{RL}$ is at -4.6?

$$-4.6 = \frac{17 + x_R}{2}$$
$$-9.2 = 17 + x_R$$
$$-9.2 - 17 = x_R$$
$$\boxed{-26.2} = x_R$$

7. Point B has coordinates (3, -7) while point F has coordinates (11, -1). What are the coordinates of the midpoint of $\overline{BF}$?

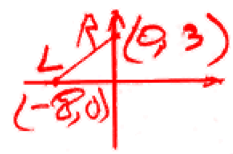


$x_m = \frac{3+11}{2} = \frac{14}{2} = 7$

$y_m = \frac{-7+(-1)}{2} = \frac{-8}{2} = -4$

$(x_m, y_m) = \boxed{(7, -4)}$

8. Where is the midpoint of $\overline{RL}$ if R is located 3 units above the origin and L is located 8 units to the left of the origin?

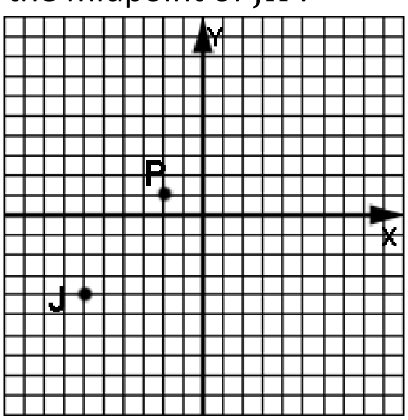


$x_m = \frac{-8+0}{2} = \frac{-8}{2} = -4$

$y_m = \frac{0+3}{2} = \frac{3}{2}$

$(x_m, y_m) = \boxed{\left(-4, \frac{3}{2}\right)}$

9. What are the coordinates of K if P is the midpoint of $\overline{JK}$?



J (-6, -4)
P (-2, 1)

$-2 = \frac{-6+x_K}{2}$; $-4 = -6 + x_K$
$-4+6 = x_K$
$2 = x_K$

$1 = \frac{-4+y_K}{2}$; $2 = -4 + y_K$
$2+4 = y_K$
$6 = y_K$

$(x_K, y_K) = \boxed{(2, 6)}$

10. If the midpoint of $\overline{AB}$ is (4, 10) and B is located at (21, -5), what are the coordinates of A?

$(x_m, y_m) = (4, 10)$
$(x_B, y_B) = (21, -5)$

$4 = \frac{21 + x_A}{2}$
$8 = 21 + x_A$
$8 - 21 = x_A$
$-13 = x_A$

$10 = \frac{-5 + y_A}{2}$
$20 = -5 + y_A$
$20 + 5 = y_A$
$25 = y_A$

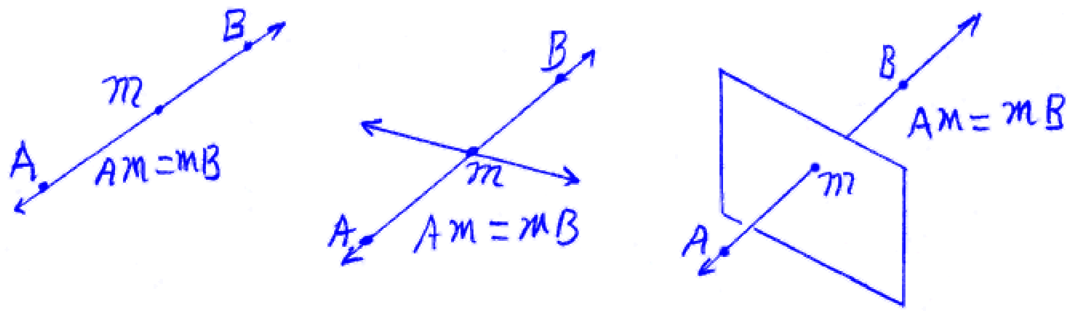
$(x_A, y_A) = \boxed{(-13, 25)}$

Unit 2: Lesson 05 Line segment bisectors

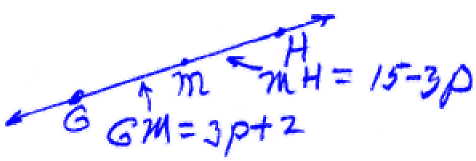
A line segment **bisector** can be a

- point,
- line (or line segment), or
- plane

that intersects a line segment at its midpoint.



Example 1: Line segment $\overline{\mathrm{GH}}$ is bisected at point M. If GM = 3p + 2 and MH = 15 – 3p, find the value of *p*.



GM = MH because M is the bisector

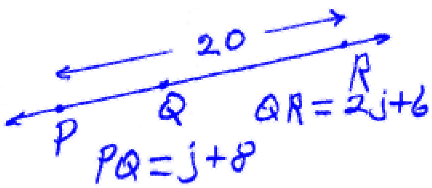
$3p + 2 = 15 - 3p$

$3p + 3p = 15 - 2$

$6p = 13$

$p = \boxed{\frac{13}{6}}$

***Example 2:** P, Q, and R are collinear with Q being somewhere between P and R (but not necessarily halfway in between.) If PR = 20, PQ = j + 8, and QR = 2j + 6, determine if Q is the bisector of $\overline{\text{PR}}$. (Begin by drawing $\overline{\text{PR}}$ and Q.)



$PQ + QR = 20$

$j+8+2j+6 = 20$

$3j + 14 = 20$

$3j = 20 - 14$

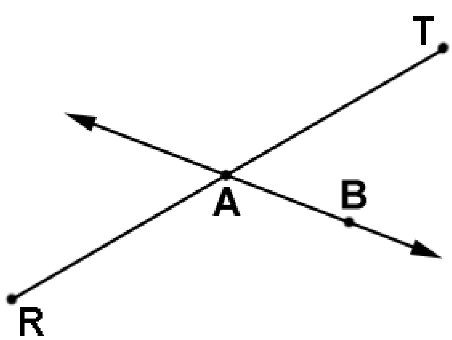
$j = \frac{6}{3} = 2$

$PQ = j+8 = 2+8 = 10$

$QR = 2j+6 = 2\cdot 2+6 = 10$

$PQ = QR$, $\boxed{\text{so Q is the bisector of } \overline{PR}}$

Example 3: Line $\overleftrightarrow{\text{AB}}$ bisects $\overline{\text{RT}}$ at A. If AT = 3z + 6 and RA = 11z – 18, find the value of *z*.



$AT = RA$

$3z+6 = 11z-18$

$3z - 11z = -18 - 6$

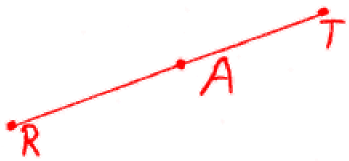
$-8z = -24$

$z = \boxed{3}$

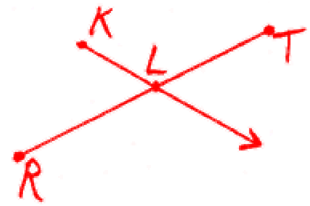
Now that we have a little geometry experience, it is appropriate to discuss the different types of geometry. In this course, we will study **Euclidean geometry**. See **In-Depth Topic D** for a discussion of both Euclidean and non-Euclidean geometry.

Assignment:

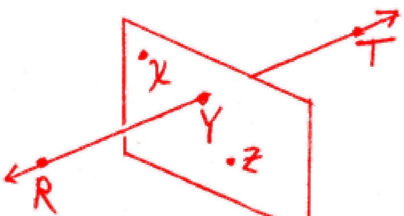
1. Draw an example of line segment $\overline{RT}$ being bisected by point A.



2. Draw an example of line segment $\overline{RT}$ being bisected at L by ray $\overrightarrow{KL}$.



3. Draw an example of line segment $\overline{RT}$ being bisected at Y by plane XYZ.



4. Given that $\overline{AB}$ has length 20 and C lies between A and B, draw $\overline{AB}$ and C. What is CB if AC = 9?

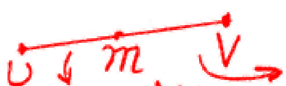
$AB = AC + CB$

$20 = 9 + CB$

$20 - 9 = CB$

$\boxed{11} = CB$

5. Draw line segment $\overline{UV}$ and point M as its midpoint. If UM = 4f + 2 and VM = 11 – f, what is the value of f?



$VM = 11 - f$

$UM = 4f + 2$

$UM = VM$

$4f + 2 = 11 - f$

$4f + f = 11 - 2$

$5f = 9$

$f = \boxed{\frac{9}{5}}$

6. Draw line segment $\overline{PQ}$ and point M as its midpoint. If PM = 46 and MQ = 3h – 2, what is the value of h?

$PM = 46 \quad MQ = 3h - 2$

$PM = MQ$

$46 = 3h - 2$

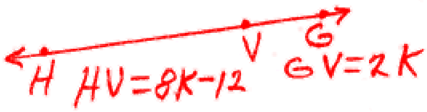
$46 + 2 = 3h$

$48 = 3h$

$\frac{48}{3} = h$

$h = \boxed{16}$

*7. Consider line segment $\overline{HG}$ of length 8 with point V being somewhere between H and G but not necessarily halfway in between. Draw $\overline{HG}$ and V and then determine if V is the midpoint of $\overline{HG}$ if HV = 8k – 12 and GV = 2k.



$HV + GV = HG$

$8K - 12 + 2K = 8$

$10K = 8 + 12$

$10K = 20$

$K = 2$

$HV = 8K - 12 = 8 \cdot 2 - 12$

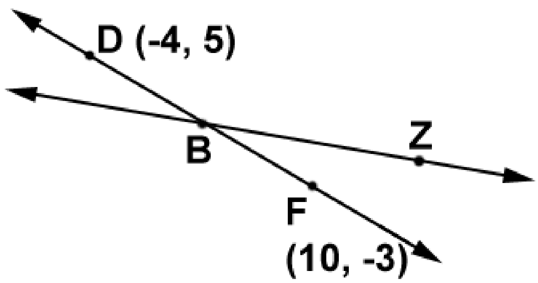
$= 4$ same

$GV = 2k = 2 \cdot 2 = 4$

Since $HV = GV$, $\boxed{\text{V is the midpoint of } \overline{HG}}$

---

8. If $\overline{DF}$ is bisected by line $\overleftrightarrow{BZ}$, what are the coordinates of B?



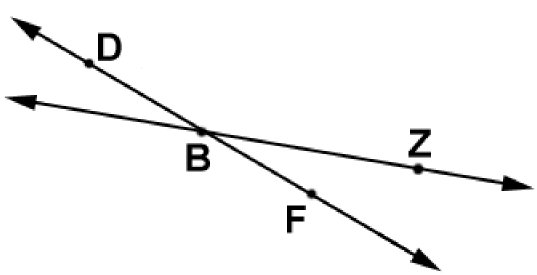
$x_B = \frac{-4+10}{2} = \frac{6}{2} = 3$

$y_B = \frac{5-3}{2} = \frac{2}{2} = 1$

$(x_B, y_B) = \boxed{(3, 1)}$

---

9. Determine the value of *c* if DB = 44c – 2 and BF = 2c + 82. (B is the midpoint of $\overline{DF}$.)



$DB = BF$

$44c - 2 = 2c + 82$

$44c - 2c = 82 + 2$

$42c = 84$

$c = \boxed{2}$

10. Using the information in problem 9, what are the lengths of $\overline{DB}$ and $\overline{BF}$?

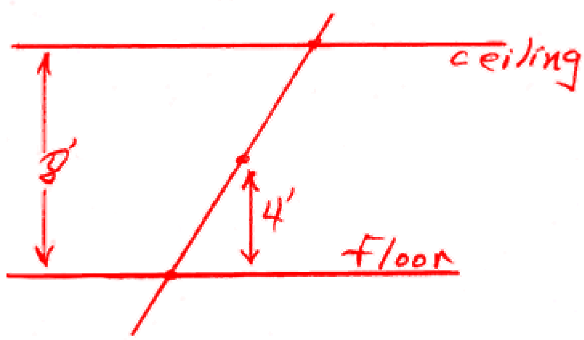
$DB = 44c - 2$
$= 44(2) - 2$
$= 88 - 2 = \boxed{86}$

$BF = 2c + 82$
$= 2 \cdot 2 + 82$
$= 4 + 82$
$= \boxed{86}$

11. Using the results in problem 10, what is the length of $\overline{DF}$ in problem 9?

$DF = DB + BF$
$= 86 + 86$
$= \boxed{172}$

*12. Consider a slanted line coming from above that penetrates your classroom. Now consider the line segment created by the point where this line intersects the plane of the ceiling and the point where the line intersects the plane of the floor. Draw a "side view" of this description. How far above the floor would the midpoint of this line segment be located if your room has 8 ft ceilings?



$\boxed{4 \text{ ft}}$

Unit 02: Review

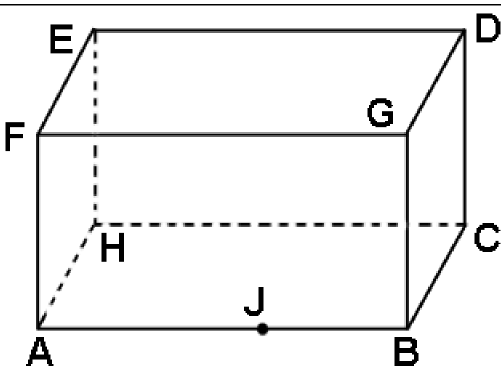
1. Would the place where the 30 yard-line and the sideline meet on a football field be considered a point, line, ray, or plane?

point

2. Would the surface of a football field be considered a point, line, ray, or plane?

plane

Use this drawing to answer questions 3-6.



3. Name the intersection of planes EFA and AJG.

Line $\overleftrightarrow{FA}$

4. Are points A, G, and D coplanar?

Yes, any three points are always coplanar.

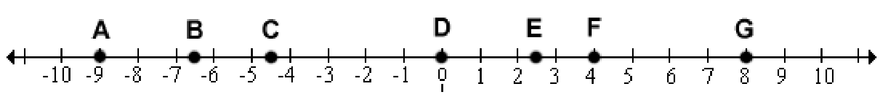
5. Are points A, G, and D collinear?

No

6. Does $\overrightarrow{GF}$ point left, right, up, down, front or back?

Left

Use this drawing for problems 7 & 8.



7. BD = ?

$BD = |-6.5 - 0|$
$= |-6.5|$
$= \boxed{6.5}$

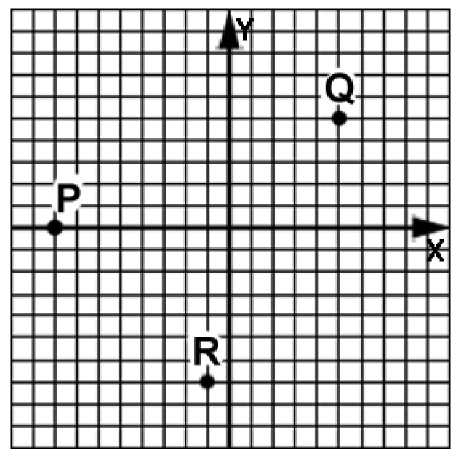
8. FB = ?

$FB = |4 - (-6.5)|$
$= |4 + 6.5|$
$= |10.5| = \boxed{10.5}$

9. Find AB when A is located at (11, -19) and B is at the origin.

$(11, -19) \quad (0, 0)$
$(x_1, y_1) \, (x_2, y_2)$
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$AB = \sqrt{(0 - 11)^2 + (0 - (-19))^2}$
$AB = \sqrt{(-11)^2 + (19)^2}$
$AB = \sqrt{121 + 361} = \sqrt{482}$
$= \boxed{21.95449}$

Use these points to work problems 10 and 11.



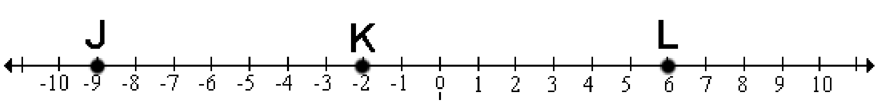
10. What is the length of $\overline{PR}$?

$P \rightarrow (-8, 0) \quad R \rightarrow (-1, -7)$
$PR = \sqrt{(-8 - (-1))^2 + (0 - (-7))^2}$
$= \sqrt{(-8 + 1)^2 + (7)^2}$
$= \sqrt{(-7)^2 + (7)^2} = \sqrt{49 + 49}$
$= \sqrt{98} = \boxed{9.89949}$

11. What is the length of $\overleftrightarrow{RQ}$?

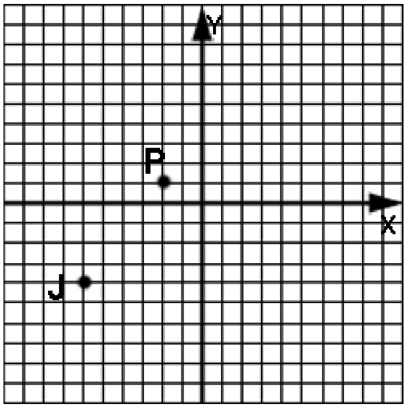
Infinitely long
(it's a line)

12. Find the midpoint of KL.



midpoint $= \frac{-2 + 6}{2} = \frac{4}{2} = \boxed{2}$

13. What are the coordinates of K if J is the midpoint of $\overline{PK}$?



P(−2, 1)

J(−6, −4)

$x_m = \frac{x_2 + x_1}{2}$ | $y_m = \frac{y_2 + y_1}{2}$

$\frac{-6}{1} = \frac{x_K + (-2)}{2}$ | $\frac{-4}{1} = \frac{y_K + 1}{2}$

$-12 = x_K - 2$ | $-8 = y_K + 1$

$-12 + 2 = x_K$ | $-8 - 1 = y_K$

$-10 = x_K$ | $-9 = y_K$

$(x_K, y_K) = \boxed{(-10, -9)}$

14. Use the points given in problem 13 to find the midpoint of line segment $\overline{PJ}$.

$x_m = \frac{-6-2}{2} = \frac{-8}{2} = -4$

$y_m = \frac{-4+1}{2} = \frac{-3}{2}$

$(x_m, y_m) = \boxed{\left(-4, -\frac{3}{2}\right)}$

15. Draw line segment $\overline{PV}$ and point M as its midpoint. If PM = 5f + 2 and VM = 12– f, what is the value of f?

$PM = VM$

$5F + 2 = 12 - F$

$5F + F = 12 - 2$

$6F = 10$

$F = \frac{10}{6} = \boxed{\frac{5}{3}}$

16. Use the information and results of problem 15 to determine PV.

$PV = PM + VM$

$PV = 5F + 2 + 12 - F$

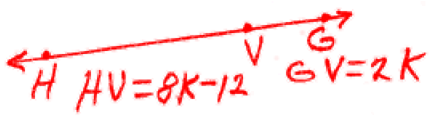
$PV = 5 \cdot \frac{5}{3} + 2 + 12 - \frac{5}{3}$

$= \frac{25}{3} + 14 - \frac{5}{3}$

$= \frac{20}{3} + \frac{14}{1} \cdot \frac{3}{3} = \frac{20}{3} + \frac{42}{3}$

$= \boxed{\frac{62}{3}}$

17. Consider line segment $\overline{HG}$ of length 8 with point V being somewhere between H and G but not necessarily halfway in between. Draw $\overline{HG}$ and V and then determine if V is the midpoint of $\overline{HG}$ if HV = 8k – 12 and GV = 2k.



$HV + GV = HG$

$8K - 12 + 2K = 8$

$10K = 8 + 12$

$10K = 20$

$K = 2$

$HV = 8K - 12 = 8 \cdot 2 - 12$

$= 4$ same

$GV = 2k = 2 \cdot 2 = 4$

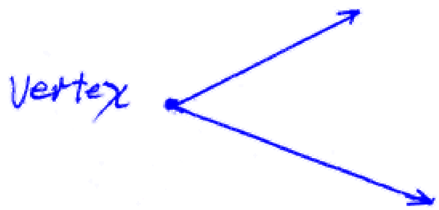
Since $HV = GV$, $\boxed{\text{V is the midpoint of } \overline{HG}}$

Geometry, Unit 3

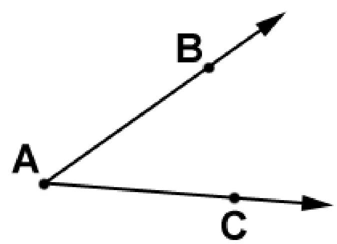
Angles
Angle Relationships

Unit 3: Lesson 01 Angle fundamentals

An **angle** is an object formed by two rays with a common endpoint. The common endpoint is called the **vertex** of the angle.



Angle naming conventions:



The **sides** of the angle are $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

The **vertex** is at A.

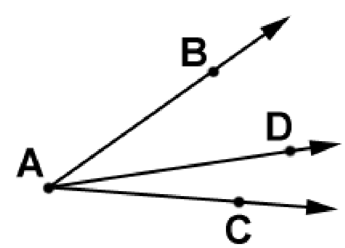
The two acceptable names of the angle using three letters are:

∠BAC and ∠CAB (Notice that the vertex is the center letter.)

The “One-letter” way of naming this angle is:

∠A

Example 1: Name all the angles shown here using the “three-letter” convention.

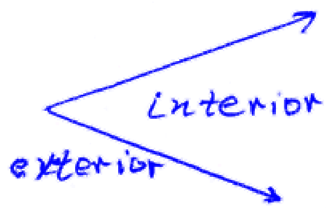


∠ BAC, ∠DAC, and ∠BAD

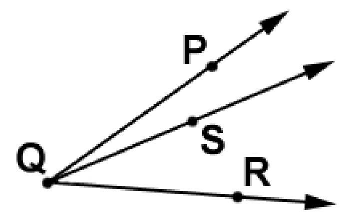
Why is it not possible to use the “single-letter” convention to name these angles?

The letter A serves as the vertex for all three angles and when giving ∠A it would not be clear which angle it specifies.

An angle lies in a plane and creates three separate parts (interior, exterior, and the points along the two rays forming the angle).



Example 2: Name a labeled point that is interior to ∠PQR.



S

Name a labeled point that is exterior to ∠PQS.

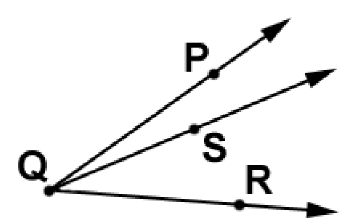
R

Name a labeled point that is exterior to ∠RQS.

P

The measure of angle ∠A in degrees is given symbolically by **m∠A.**

Angle addition postulate: If point S is in the interior of ∠PQR as shown here, then "the two small angles add up to the big one."



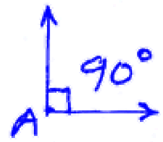
$$m\ \angle PQS + m\ \angle SQR = m\ \angle PQR$$

Example 3: In the drawing just above, find m∠PQR if m∠PQS = 17° and m∠SQR = 32°.

$$m\angle PQR = m\angle PQS + m\angle SQR$$
$$= 17^\circ + 32^\circ = \boxed{49^\circ}$$

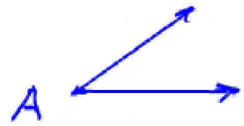
Angles (for example ∠A) are classified **according to size**:

If m∠A = 90°, then it's a **right angle**.

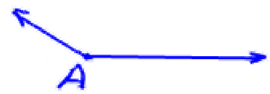


If m∠A = 180°, then it's a **straight angle**.

If m∠A < 90°, then it's an **acute angle**.



If m∠A > 90°, then it's an **obtuse angle**.



Congruent angles: Angles that have the same measure are said to be congruent.

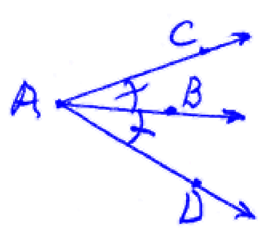
They could fit right on top of each other and match perfectly.

If m∠A = m∠B then

∠A ≅ ∠B where ≅ is the symbol for congruence.

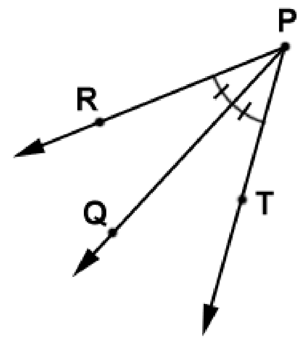
Angle Bisectors: If $\overrightarrow{AB}$ is the bisector of ∠CAD, then ∠CAD is divided into two congruent angles.

m ∠CAB = m ∠BAD



Notice the tic marks on the two angles indicating they are equal.

Example 4: If ∠RPT is bisected by $\overrightarrow{PQ}$, find *x* when m∠RPQ = 2x – 7 and m∠QPT = x + 9. Use *x* to find m∠QPT.



$m\angle RPQ = m\angle QPT$

$2x - 7 = x + 9$

$2x - x = 9 + 7$

$x = \boxed{16}$

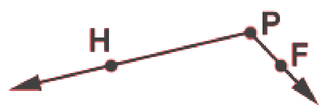
$m\angle QPT$

$= x + 9$

$= 16 + 9 = \boxed{25}$

Assignment:

1. Give two names for this angle using the “three-letter” convention.



$\angle HPF$ and $\angle FPH$

2. Use the drawing in problem 1 to name the angle using the “single-letter” convention.

$\angle P$

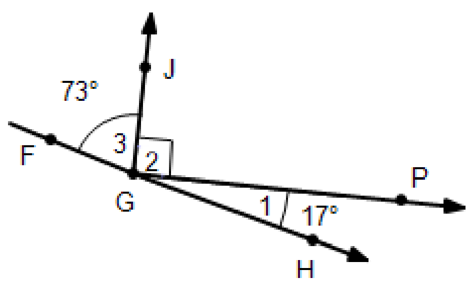
3. Use the drawing in problem 1 to name the two rays that form the angle.

$\overrightarrow{PH}$ and $\overrightarrow{PF}$

4. What is the vertex of the angle in problem 1?

P

Use this drawing for problems 5 - 15.



5. Classify $\angle 1$.

Acute angle

6. Classify $\angle 2$.

Right angle

7. Classify $\angle 3$.

Acute angle

8. Classify $\angle FGH$.

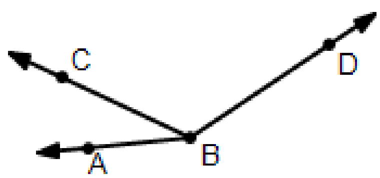
Straight angle

9. Classify $\angle HGJ$.

Obtuse angle

10. What are the two rays that make up ∠PGJ? *$\overrightarrow{GP}$ and $\overrightarrow{GJ}$*	11. What is m∠JGF? *73°*
12. Name ∠2. *∠JGP*	13. Name ∠1 *∠HGP*
14. m∠HGF = ? *180°*	15. m∠2 = ? *90°*

16. If m∠ABC = $(4x - 1)^{\circ}$, m∠CBD = $(6x + 5)^{\circ}$, and m∠ABD = 134°, find the value of *x* and then use it to find m∠ABC.



$m\angle ABC + m\angle CBD = m\angle ABD$

$4x - 1 + 6x + 5 = 134$

$10x + 4 = 134$

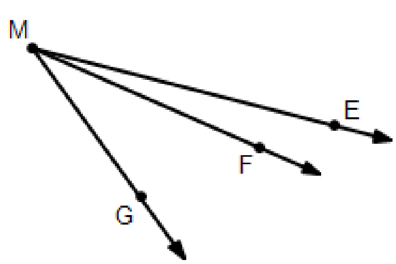
$10x = 134 - 4 = 130$

$x = 130/10 = \boxed{13}$

$m\angle ABC = 4x - 1$

$= 4 \cdot 13 - 1 = 52 - 1 = \boxed{51^{\circ}}$

17. m∠FME = $(8x + 5)^{\circ}$, m∠EMG = $(11x - 1)^{\circ}$, and m∠FMG = x°. Find the value of *x* and then use it to find m∠FME.



$m\angle FME + m\angle FMG = m\angle EMG$

$8x + 5 + x = 11x - 1$

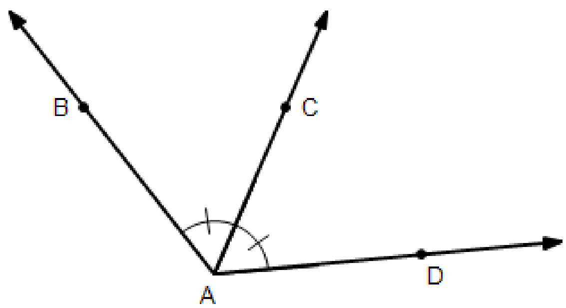
$9x + 5 = 11x - 1$

$6 = 11x - 9x$

$6 = 2x; \ x = \boxed{3}$

$m\angle FME = 8x + 5 = 8 \cdot 3 + 5 = \boxed{29^{\circ}}$

18. If m∠BAC = (8x – 3)° and m∠BAD = (10x + 30)°, find the value of *x* and then use it to find m∠CAD.



$m\angle BAC = m\angle CAD$

$m\angle BAC + m\angle CAD = m\angle BAD$

$8x-3 + 8x-3 = 10x+30$

$16x-6 = 10x+30$

$16x-10x = 30+6$

$6x = 36;\ x = 36/6 = \boxed{6}$

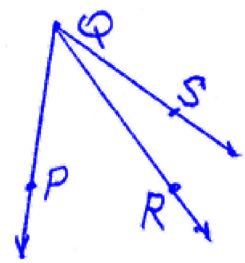
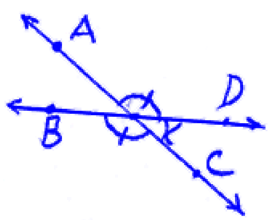
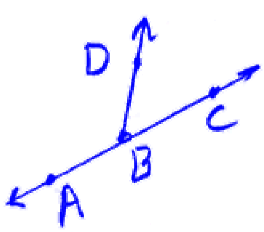
$m\angle CAD = 8x-3$

$= 8\cdot 6-3 = 48-3 = \boxed{45^\circ}$

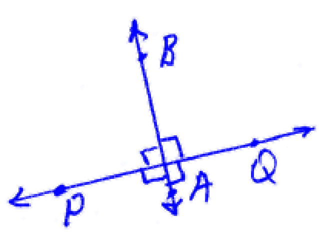
Unit 3: Lesson 02

Special angle pairs, perpendicular lines
Supplementary & complementary angles

Consider special angle pairs formed by the intersection of two lines or rays.

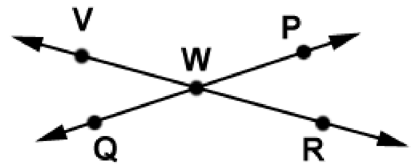
Drawing	Name and description	Examples
	**Adjacent angles:** Angles that have a common vertex and a common side but no common interior points.	*∠PQR and ∠RQS are adjacent angles.*
	**Vertical angles:** Non-adjacent angles formed by an intersecting pair of lines. Vertical angles are congruent.	*∠AXD and ∠BXC are vertical angles.*
	**Linear pair of angles:** Adjacent angles, the sum of whose measures is 180°.	*∠ABD and ∠DBC form a linear pair.*

Perpendicular lines intersect to form 4 right angles. The symbol used to show that lines are perpendicular is ⊥.



$\overline{AB} \perp \overline{PQ}$

Example 1: Use this drawing to find *x* when $m\angle VWQ = 8x - 4$ and $m\angle PWR = 4x + 20$. Then use x to find $m\angle VWQ$. Assume that V, W, & R are collinear and that Q, W, and P are collinear.



$m\angle VWQ = m\angle PWR$
$8x - 4 = 4x + 20$
$8x - 4x = 20 + 4$
$4x = 24$
$x = \boxed{6}$

$m\angle VWQ = 8x - 4$
$= 8 \cdot 6 - 4$
$= 48 - 4 = \boxed{44}$

Example 2: Using the drawing in Example 1, find *x* when $m\angle VWQ = 4x - 20$ and $m\angle VWP = 8x - 4$. Assume that V, W, & R are collinear and that Q, W, and P are collinear.

$m\angle VWP + m\angle VWQ = 180$
$8x - 4 + 4x - 20 = 180$
$12x - 24 = 180$
$12x = 180 + 24 = 204$
$x = 204/12 = \boxed{17}$

If the sum of the measures of two angles is 180°, the angles are said to be **supplementary**.

If the sum of the measures of two angles is 90°, the angles are said to be **complementary**.

To be either supplementary or complementary, the two angles do not necessarily have to be adjacent.

Example 3: If $\angle 3$ and $\angle 7$ are complementary with $m\angle 3 = 4z - 11$ and $m\angle 7 = z - 9$, find z and then use it to find the measure of $\angle 3$.

$m\angle 3 + m\angle 7 = 90$

$4z - 11 + z - 9 = 90$

$5z - 20 = 90$

$5z = 90 + 20 = 110$

$z = 110/5 = \boxed{22}$

$m\angle 3 = 4z - 11$

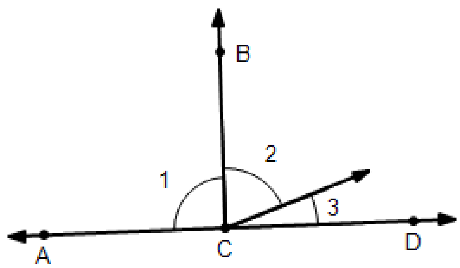
$= 4(22) - 11$

$= 88 - 11 = \boxed{77^\circ}$

See **Theorem Proof A** for a proof of vertical angles being equal.

Assignment:

1. $\overleftrightarrow{AD} \perp \overleftrightarrow{CB}$, m∠3 = (6x – 1)° and m∠2 = (8x + 7)°. Find the value of *x* and use it to find m∠2.



$m\angle 1 + m\angle 3 + m\angle 2 = 180$

$90 + 6x - 1 + 8x + 7 = 180$

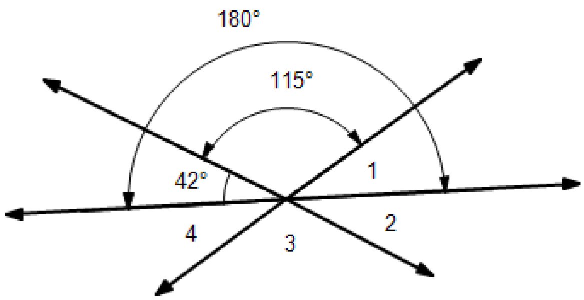
$14x + 96 = 180$

$14x = 180 - 96 = 84$

$x = 84/14 = \boxed{6}$

$m\angle 2 = 8x + 7 = 8 \cdot 6 + 7 = 48 + 7 = \boxed{55}$

2. Find m∠1.



$\angle 1 + 115 + 42 = 180$

$\angle 1 = 180 - 115 - 42 = \boxed{23^\circ}$

3. Using the drawing in problem 2, find m∠2.

$\angle 2 = \boxed{42^\circ}$

vertical angles

4. Using the drawing in problem 2, find m∠3.

$m\angle 3 = \boxed{115^\circ}$

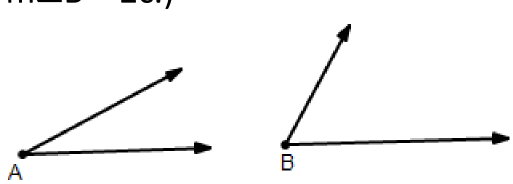
vertical angles

5. Using the drawing in problem 2, find m∠4.

$m\angle 4 = m\angle 1 = \boxed{23^\circ}$

vertical angles

6. If ∠A and ∠B are complementary, find the measure of each. (m∠A = c and m∠B = 2c.)



$m\angle A + m\angle B = 90$

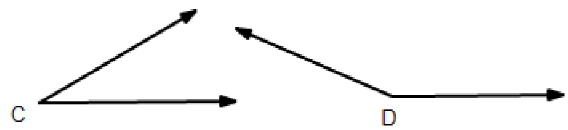
$c + 2c = 90$

$3c = 90$

$c = 90/3 = 30$

$m\angle A = c = \boxed{30^\circ}$ $m\angle B = 2c = \boxed{60^\circ}$

7. If ∠C and ∠D are supplementary, find the measure of each. (m∠C = 4g + 2 and m∠D = 6g – 12.)



$m\angle C + m\angle D = 180$

$4g + 2 + 6g - 12 = 180$

$10g - 10 = 180$

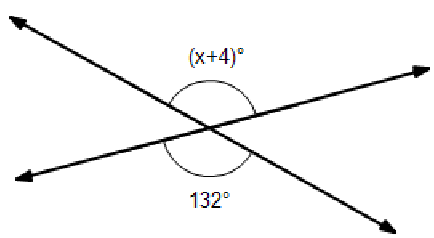
$10g = 180 + 10 = 190$

$g = 190/10 = 19$

$m\angle C = 4g + 2 = 4 \cdot 19 + 2 = \boxed{78}$

$m\angle D = 6g - 12 = 6 \cdot 19 - 12 = \boxed{102}$

8. Find *x*.

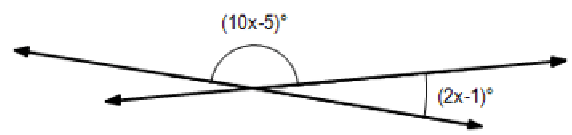


$x + 4 = 132$

$x = 132 - 4$

$x = \boxed{128°}$

9. Find *x*.



$10x - 5 + 2x - 1 = 180$

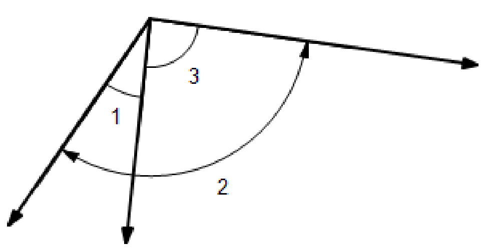
$12x - 6 = 180$

$12x = 180 + 6$

$12x = 186$

$x = 186/12 = \boxed{15.5}$

10. Classify ∠1 as acute, obtuse, right, or straight.



acute

11. Using the drawing in problem 10, classify ∠2 as acute, obtuse, right, or straight.

obtuse

12. Using the drawing in problem 1, classify $\angle 1$ as acute, obtuse, right, or straight.

right

13. Using the drawing in problem 1, classify $\angle ACD$ as acute, obtuse, right, or straight.

straight

14. If $m\angle A = (120 + x)^o$ and $m\angle B = (6 + x)^o$, what is the measure of angle A if $\angle A$ and $\angle B$ are known to be supplementary?

$m\angle A + m\angle B = 180^\circ$

$120 + x + 6 + x = 180$

$126 + 2x = 180$

$2x = 180 - 126$

$2x = 54$

$x = 27$

$m\angle A = 120 + x = 120 + 27$

$= \boxed{147^\circ}$

15. Find the measure of two complementary angles, $\angle C$ and $\angle D$, if $m\angle C = (6x + 4)^o$ and $m\angle D = (4x + 6)^o$.

$m\angle C + m\angle D = 90$

$6x + 4 + 4x + 6 = 90$

$10x + 10 = 90$

$10x = 90 - 10 = 80$

$x = 80/10 = 8$

$m\angle C = 6x + 4 = 6 \cdot 8 + 4 = \boxed{52}$

$m\angle D = 4x + 6 = 4 \cdot 8 + 6 = \boxed{38}$

Unit 3: Lesson 03 Angle word problems

Example 1: Find the measure of an angle if its measure is 50° more than the measure of its supplement.

x = the measure of an angle
180 – x = the measure of its supplement

$x = 180 - x + 50$
$x + x = 230$
$2x = 230;\ x = 230/2 = \boxed{115^\circ}$

Example 2: Find the measure of an angle if its measure is 19° less than its complement.

x = the measure of an angle
90 – x = the measure of its complement

$x = 90 - x - 19$
$x + x = 90 - 19$
$2x = 71$
$x = 71/2 = \boxed{35.5^\circ}$

Example 3: Find the measure of an angle if its measure is 40° less than twice its supplement.

x = the measure of an angle
180 – x = the measure of its supplement

$x = 2(180 - x) - 40$
$x = 360 - 2x - 40$
$3x = 320$
$x = 320/3 = \boxed{106.\overline{6}^\circ}$

***Example 4:** Is it possible to have an angle whose supplement is 20^o more than twice its complement?

x = the measure of an angle
90 – x = the measure of its complement
180 – x = the measure of its supplement

$$180 - x = 2(90 - x) + 20$$
$$\cancel{180} - x = \cancel{180} - 2x + 20$$
$$2x - x = 20$$
$$x = \boxed{20}$$

Yes, x is the measure of the angle and it's positive. If it had been negative or greater than 90° (that would make 90 – x negative), the answer would have been "No".

Assignment:

1. Find the measure of an angle if its measure is 42° more than the measure of its supplement.

x = the measure of an angle
$180 - x$ = the measure of its supplement

$x = 180 - x + 42$
$x + x = 222$
$2x = 222$; $x = 222/2 = \boxed{111^{\circ}}$

---

2. Find the measure of an angle if its measure is 50° less than its complement.

x = the measure of an angle
$90 - x$ = the measure of its complement

$x = 90 - x - 50$
$x + x = 40$
$2x = 40$
$x = \boxed{20^{\circ}}$

---

3. Find the measure of an angle if its measure is 10° more than three times its supplement.

x = the measure of an angle
$180 - x$ = the measure of its supplement

$x = 3(180 - x) + 10$
$x = 540 - 3x + 10$
$4x = 550$
$x = 550/4 = \boxed{137.5^{\circ}}$

4. Find the measure of an angle if its measure is 20° less than three times its complement.

x = the measure of an angle
$90 - x$ = the measure of its complement

$x = 3(90 - x) - 20$
$x = 270 - 3x - 20$
$x = 250 - 3x$
$3x + x = 250$; $4x = 250$; $x = 250/4 = \boxed{62.5^{\circ}}$

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*5. Is it possible to have an angle whose supplement is 20° less than twice its complement?

x = the measure of an angle
$90 - x$ = the measure of its complement
$180 - x$ = the measure of its supplement

$180 - x = 2(90 - x) - 20$
$\cancel{180} - x = \cancel{180} - 2x - 20$
$2x - x = -20$; $x = \boxed{-20}$

No, x is the measure of the angle and it can't be negative.

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6. Find the measure of an angle if its measure is half that of its complement.

x = the measure of an angle
$90 - x$ = the measure of its complement

$x = \frac{1}{2}(90 - x)$
$x = 45 - \frac{1}{2}x$
$x + \frac{1}{2}x = 45$
$\frac{3}{2}x = 45$; $x = \frac{45}{1}\cdot\frac{2}{3} = \boxed{30^{\circ}}$

7. Find the measure of an angle if its measure is double the measure of its supplement.

x = the measure of an angle
180 – x = the measure of its supplement

$x = 2(180 - x)$
$x = 360 - 2x$
$2x + x = 360$
$3x = 360$
$x = 360/3 = \boxed{120°}$

---

8. Is it possible for the complement of an angle to be equal to its supplement?

x = the measure of an angle
90 – x = the measure of its complement
180 – x = the measure of its supplement

$90 - x = 180 - x$
$-x + x = 180 - 90$
$0 \neq 90$

No solution
Not possible

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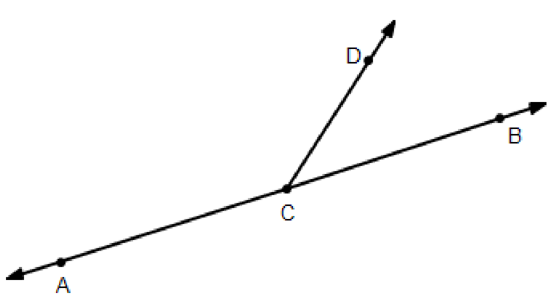
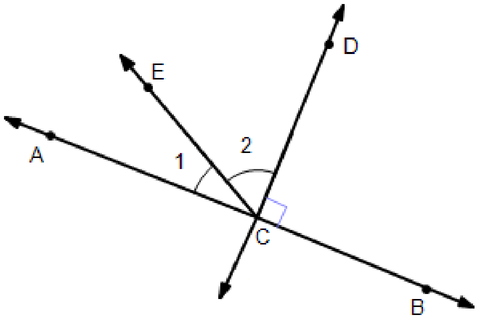
9. If points A, C, and B are collinear and m$\angle ACD$ is three times the measure of its supplement, what is the measure of $\angle BCD$?



$x = 3(180 - x)$
$x = 540 - 3x$
$3x + x = 540$
$4x = 540$
$x = 540/4 = 135°$
$m\angle BCD = 180 - x = 180 - 135$
$= \boxed{45°}$

x = m ∠ACD
180 – x = (supplement of m ∠ACD) = m ∠DCB

10. $\overleftrightarrow{CD}$ is perpendicular to $\overline{AB}$. If m∠ECB = 140°, what is the sum of the measure of the supplement of ∠ECB and ∠2?



Since the supplement of ∠ECB is ∠1, (suppl. of ∠ECB) + ∠2 is equivalent to ∠1 + ∠2.

$\angle 1 + \angle 2 = \boxed{90^\circ}$

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11. Using the drawing and information in problem 10, what is the measure of ∠2?

$$m\angle ECB + m\angle 1 = 180$$
$$140 + m\angle 1 = 180$$
$$m\angle 1 = 180 - 140 = 40^\circ$$
$$m\angle 1 + m\angle 2 = 90^\circ$$
$$40 + m\angle 2 = 90$$
$$m\angle 2 = 90 - 40 = \boxed{50^\circ}$$

Unit 3: Lesson 04 — Construction fundamentals

Copying segments & angles; bisecting segments & angles

In this lesson we will learn how to use a **straight edge** (ruler) and a **compass** to

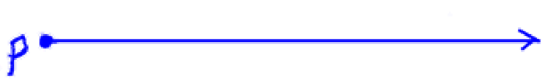
- copy a line segment,
- construct a perpendicular bisector of a line segment,
- copy an angle, and
- bisect an angle.

Copying a line segment:

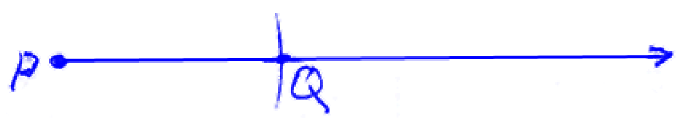
Begin with a given line segment $\overline{AB}$.

Place the point of the compass at point A and adjust the compass so that the pencil is at point B.

Use a straight edge to draw a line segment and mark point P at one end.



Without readjusting the compass, place the point of the compass a point P and strike an arc at point Q.



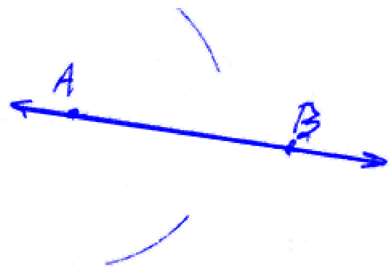
The line segment $\overline{AB}$ is now congruent to $\overline{PQ}$.

$\overline{AB} \cong \overline{PQ}$ *The line segments are congruent because the compass span that corresponds to the distance from A to B is the same span that establishes the distance from P to Q.*

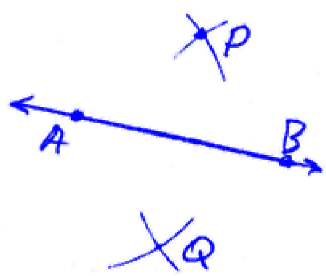
Bisecting a line segment (perpendicular bisector):

Begin with a given line segment $\overline{AB}$.

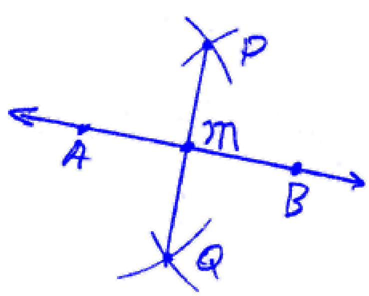
Place the point of the compass at point A, adjust its span greater than half AB, and then strike an arc above and below $\overline{AB}$.



With the same setting on the compass, place its point at B and strike arcs above and below $\overline{AB}$ that intersect the previous arcs. Call the points where the arcs meet points P and Q.



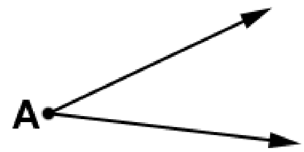
Use a straight edge to draw $\overline{PQ}$. Designate the point where $\overline{PQ}$ intersects $\overline{AB}$ as M.



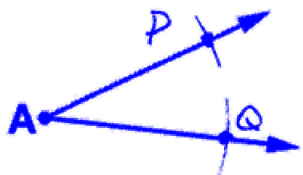
M is the midpoint of $\overline{AB}$ and PQ is a perpendicular bisector of $\overline{AB}$.

Copy an angle:

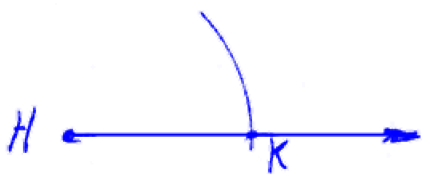
Begin with angle $\angle A$:



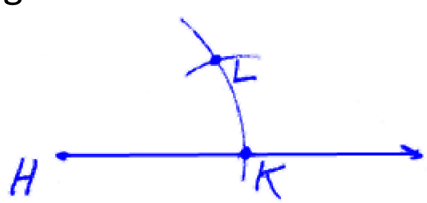
Place the point of the compass at A and strike arcs on the two rays. Label the points where the arcs intersect the rays at P and Q.



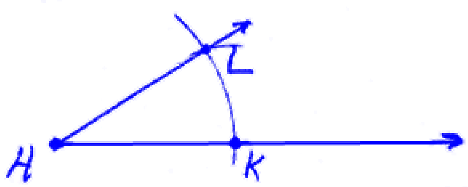
Draw a ray and label the end point as H. With the same setting on the compass as before and with the point of the compass at H, strike a large arc as shown. Label the point where the arc intersects the ray as K.



Go back to the previous drawing, place the point of the compass at P and adjust the span of the compass to reach point Q. Now place the point of the compass at K and strike an arc intersecting the previous large arc as shown. Call this point of intersection L.



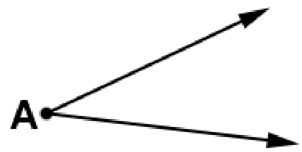
Draw the ray $\overrightarrow{HL}$.



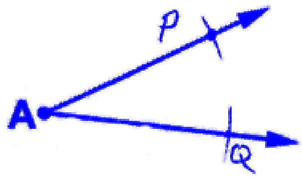
$\angle$**KHL** ***is a copy of*** $\angle$**A.** $\angle$**KHL** $\cong$ $\angle$**A**

Bisecting an angle:

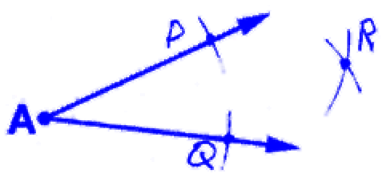
Begin with angle $\angle A$:



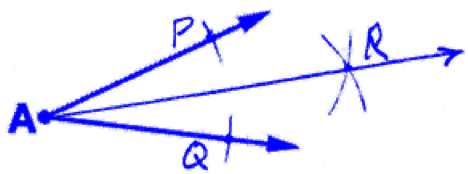
With the point of the compass at A, strike two arcs that intersect both rays. Call these points of intersection P and Q.



With a span set on the compass that is greater than the distance between P and Q, and with the point of the compass at point P, strike an arc as shown. Similarly, with the point of the compass at Q, strike another arc intersecting the first. Call the point of intersection of the arcs, R.



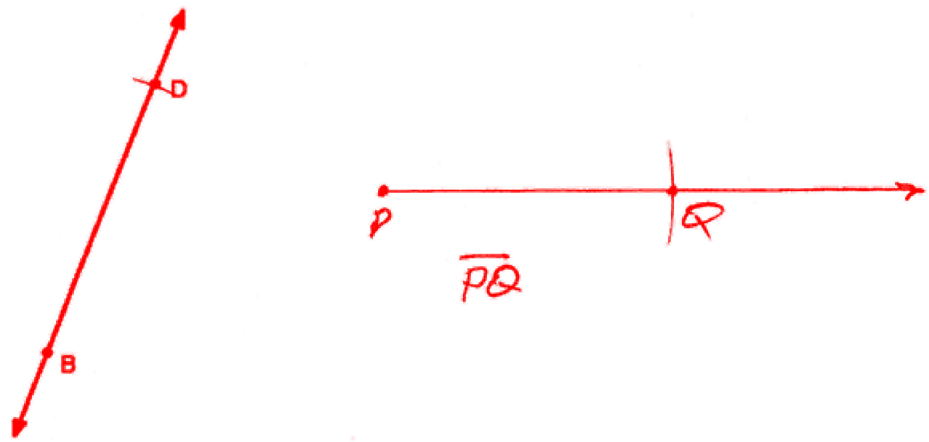
With a straight edge, draw ray $\overrightarrow{AR}$.



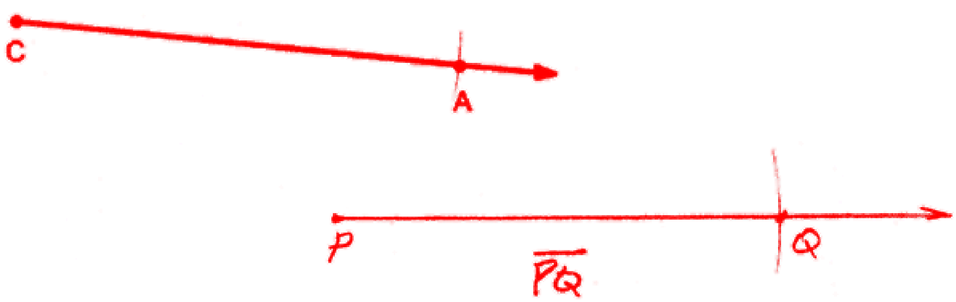
$\overrightarrow{\mathbf{AR}}$ ***bisects the original angle*** $\angle \mathbf{A}$. $\angle \mathbf{PAR} \cong \angle \mathbf{RAQ}$

Assignment: Use construction techniques for these problems.

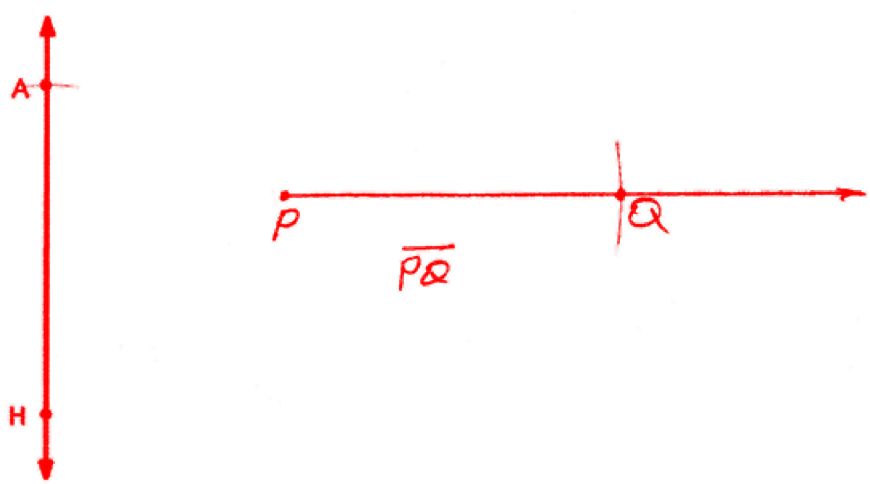
1. Make a copy ($\overline{PQ}$) of line segment $\overline{BD}$.



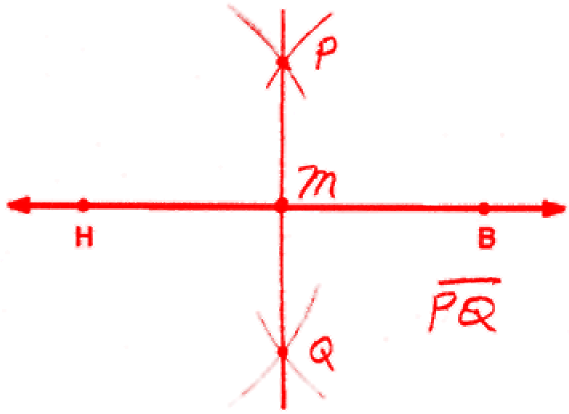
2. Make a copy ($\overline{PQ}$) of line segment $\overline{CA}$.



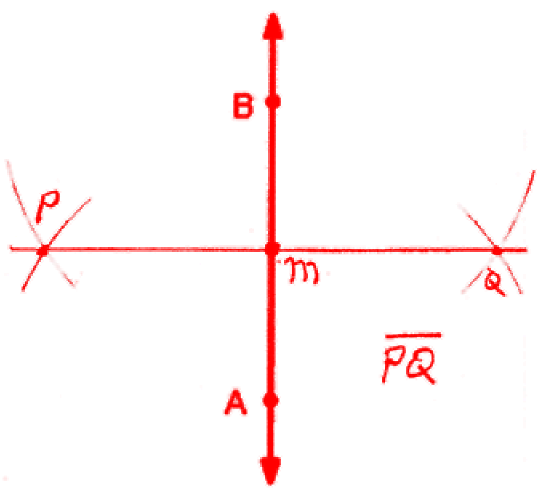
3. Make a copy ($\overline{PQ}$) of line segment $\overline{HA}$.



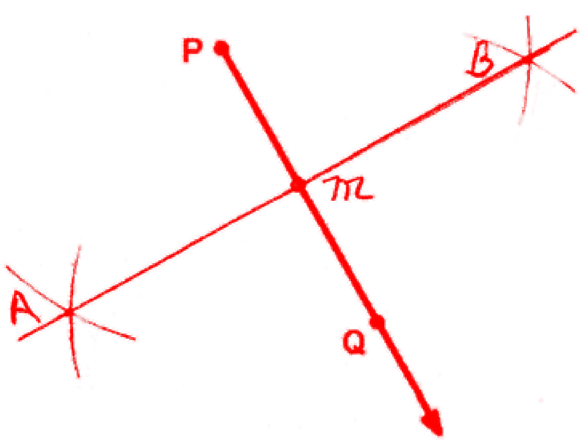
4. Construct the perpendicular bisector ($\overline{PQ}$) of $\overline{HB}$.



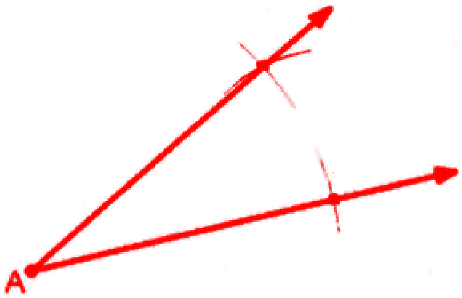
5. Construct the perpendicular bisector ($\overline{PQ}$) of $\overline{AB}$.

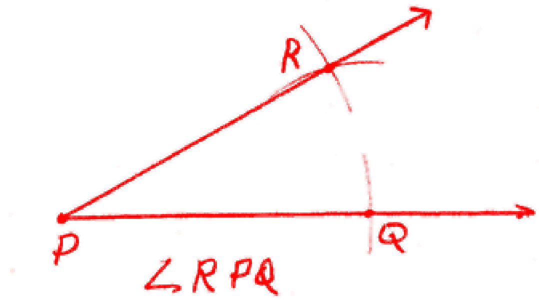


6. Construct the perpendicular bisector ($\overline{AB}$) of $\overline{PQ}$.

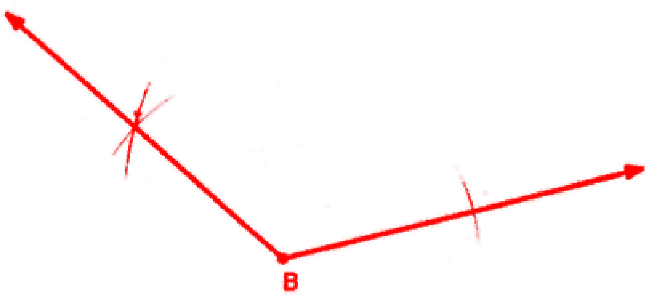


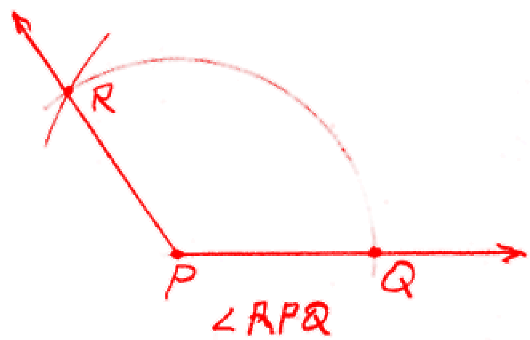
7. Make a copy of angle $\angle A$.



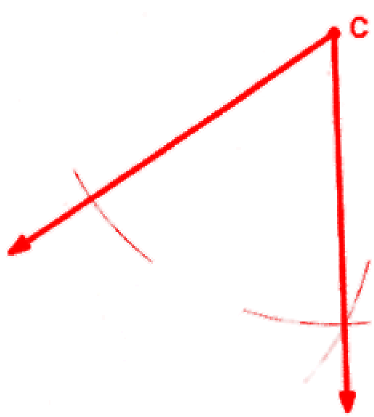


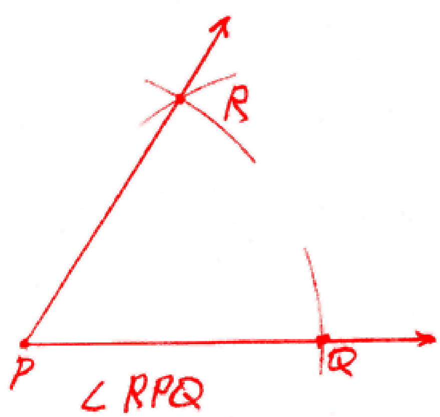
8. Make a copy of angle $\angle B$.



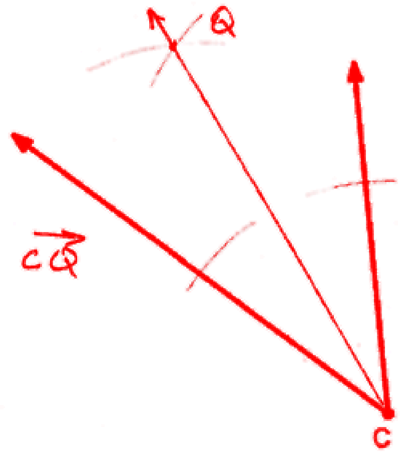


9. Make a copy of angle $\angle C$.

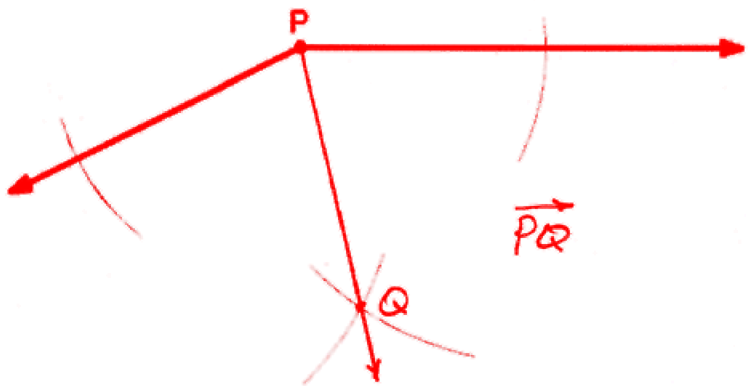




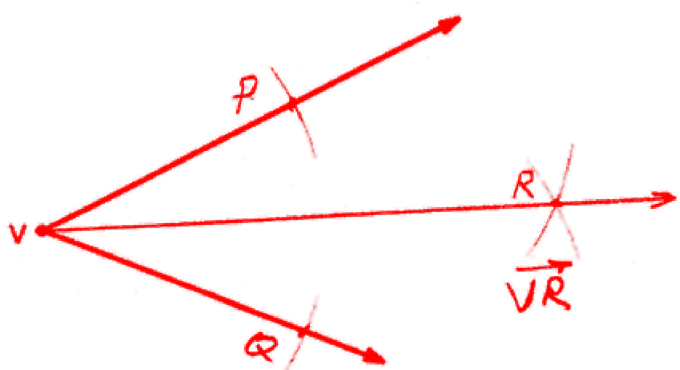
10. Create a ray ($\overrightarrow{CQ}$) that biscects $\angle C$.



11. Create a ray ($\overrightarrow{PQ}$) that bisects $\angle P$.



12. Create a ray ($\overrightarrow{VR}$) that biscects $\angle V$.



Unit 3: Cumulative Review

1. Factor $4x^2 + 19x + 21$.

	x	3
$4x$	$4x^2$	$12x$
7	$7x$	21

Product: $84x^2$

Sum: $19x$

$4x^2+19x+21 = \boxed{(x+3)(4x+7)}$

2. Factor $h^2 - 16$.

$a^2 - b^2 = (a-b)(a+b)$
$h^2 - 16 = \boxed{(h-4)(h+4)}$

3. Factor $p^2 + 14p + 49$.

$a^2 + 2ab + b^2 = (a+b)^2$
$p^2 + 2\cdot 7p + 7^2 = \boxed{(p+7)^2}$

4. Multiply $(2x - 8)(2x + 8)$.

$(a-b)(a+b) = a^2 - b^2$
$(2x-8)(2x+8) = \boxed{4x^2 - 64}$

5. Solve $5(x + 3) = 11x - 3$.

$5(x+3) = 11x - 3$
$5x + 15 = 11x - 3$
$5x - 11x = -3 - 15$
$-6x = -18$
$x = -18/(-6) = \boxed{3}$

6. Solve $x^2 + x - 56 = 0$ by factoring.

$x^2 + x - 56 = 0$
$(x+8)(x-7) = 0$
$x + 8 = 0 \quad x - 7 = 0$
$x = \boxed{-8} \quad x = \boxed{7}$

7. Write the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

8. Solve $2x^2 - 3x + 1 = 0$ using the quadratic formula.

$a = 2 \quad b = -3 \quad c = 1$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(2)1}}{2 \cdot 2}$

$x = \dfrac{3 \pm \sqrt{9-8}}{4}$

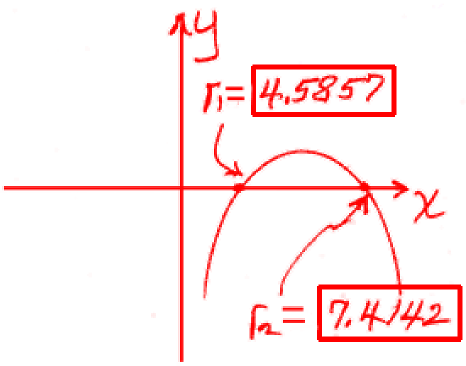
$x = \dfrac{3 \pm \sqrt{1}}{4}$

$x = \dfrac{3 \pm 1}{4}$

$x = \dfrac{3+1}{4}$ $\qquad$ $x = \dfrac{3-1}{4}$

$x = \dfrac{4}{4} = \boxed{1}$ $\qquad$ $x = \dfrac{2}{4} = \boxed{\dfrac{1}{2}}$

9. Solve $-x^2 + 12x - 34 = 0$ using a graphing calculator. Make a sketch of the calculator display.



10. Would the location on a map where two streets intersect be described as a point, line, line segment, ray, or plane?

point

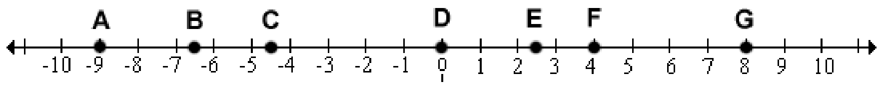
11. Suppose one end of a rubber band is attached to a wall and the other end stretched forever (assume it doesn't break). What best describes this? (A point, line, line segment, ray, or plane)

ray

12. A building sits beside a parking lot. Would the place where the parking lot first meets the building be described as a point, line, line segment, ray, or plane?

line segment

Use this number line for problems 13 & 14.



13. What is FB?

$$FB = |4 - (-6.5)|$$
$$= |4 + 6.5|$$
$$= |10.5|$$
$$= \boxed{10.5}$$

14. What is the midpoint of $\overline{FB}$?

$$m = \frac{-6.5 + 4}{2}$$
$$= \frac{-2.5}{2} = \boxed{-1.25}$$

15. What is the distance between (-8, 2) and (16, -4)?

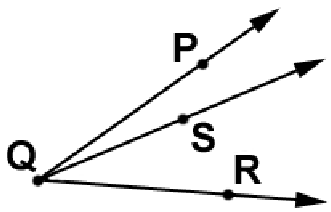
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(16 - (-8))^2 + (-4 - 2)^2}$$
$$d = \sqrt{(16 + 8)^2 + (-6)^2}$$
$$d = \sqrt{24^2 + 36}$$
$$= \sqrt{612} = \boxed{24.7386}$$

16. What are the coordinates of the point midway between (-8, 2) and (16, -4)?

$$x_m = \frac{16 + (-8)}{2}$$
$$= \frac{8}{2} = 4$$
$$y_m = \frac{-4 + 2}{2}$$
$$= \frac{-2}{2} = -1$$
$$(x_m, y_m) = \boxed{(4, -1)}$$

Unit 3: Review

1. Find m∠PQR if m∠PQS = 17° and m∠SQR = 32°.



$m\angle PQS + m\angle SQR = m\angle PQR$

$17 + 32 = m\angle PQR$

$\boxed{49^\circ} = m\angle PQR$

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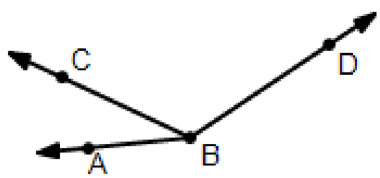
2. If m∠ABC = (2x – 1)°, m∠CBD = (6x + 3)°, and m∠ABD = 130°, find the value of *x* and then use it to find m∠DBC.



$m\angle ABC + m\angle CBD = m\angle ABD$

$2x - 1 + 6x + 3 = 130$

$8x + 2 = 130$

$8x = 130 - 2 = 128$

$x = 128/8 = \boxed{16}$

$m\angle DBC = 6x + 3$

$= 6 \cdot 16 + 3 = \boxed{99^\circ}$

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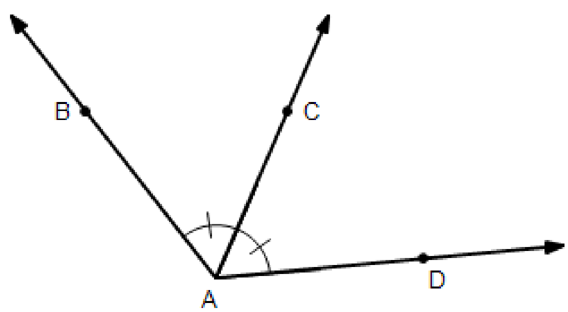
3. If m∠BAC = (4x – 2)° and m∠BAD = 126°, what is m∠CAD?



$m\angle BAC = m\angle CAD = 4x - 2$

$2\, m\angle BAC = m\angle BAD$

$2(4x - 2) = 126$

$8x - 4 = 126$

$8x = 126 + 4 = 130$

$x = 130/8 = 16.25$

$m\angle CAD = 4x - 2 = 4(16.25) - 2 = \boxed{63^\circ}$

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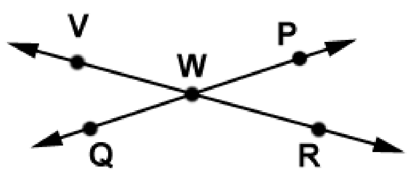
4. Name two different pairs of vertical angles. What is the relationship of vertical angles?

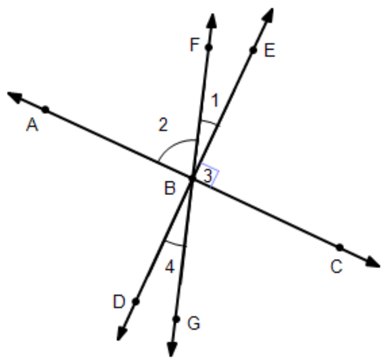


∠VWQ & ∠PWR

∠QWR & ∠VWP

Vertical angles are congruent.

5. $\overleftrightarrow{AC} \perp \overleftrightarrow{DE}$. Name the angle that is the supplement to $\angle 3$.



$\angle ABE$

6. Using the drawing in problem 5, name the angle that is the complement to $\angle 1$.

$\angle 2$ or $\angle GBC$ (Compl angles do not necessarily have to be adjacent.)

7. Using the drawing in problem 5, if $m\angle 2 = 70^o$, what is $m\angle 1$?

$m\angle 2 + m\angle 1 = 90$

$70 + m\angle 1 = 90$

$m\angle 1 = 90 - 70$

$m\angle 1 = \boxed{20^\circ}$

8. Using the drawing in problem 5, if $m\angle 2 = 70^o$, what is $m\angle 4$?

From #7, $m\angle 1 = 20^o$.
$\angle 1$ & $\angle 4$ are vertical angles.

$m\angle 1 = m\angle 4 = \boxed{20^\circ}$

9. Using the drawing in problem 5, what is $m\angle ABF + m\angle FBE + m\angle EBC$?

180^o. The sum of these angles is a straight angle.

10. Using the drawing in problem 5, if $\overrightarrow{BF}$ bisects $\angle ABE$, what would be the measure of $\angle 2$?

45^o. $\angle ABE$ is a right angle (90^o), and when bisected, each half is 45^o.

11. Find the measure of an angle if its measure is 30^o less than its complement.

x = the measure of the angle
$90 - x$ = the measure of its complement

$x = 90 - x - 30$

$x + x = 60$

$2x = 60$

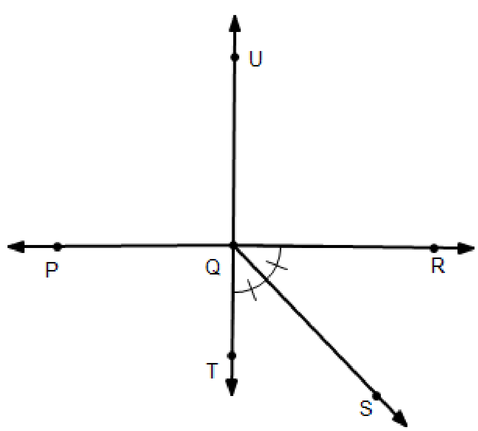
$x = 60/2 = \boxed{30^\circ}$

12. Find the measure of an angle if its measure is 20° more than one-third its supplement.

x = the measure of the angle
180 – x = the measure of its supplement

$$x = \frac{1}{3}(180 - x) + 20$$
$$x = 60 - \frac{1}{3}x + 20$$
$$\frac{3}{3}x + \frac{1}{3}x = 80$$
$$\frac{4}{3}x = 80$$
$$\cancel{\left(\frac{3}{4}\right)}\cancel{\frac{4}{3}}x = \cancel{80}^{20}\left(\frac{3}{\cancel{4}}\right)$$
$$x = \boxed{60^\circ}$$

13. $\overleftrightarrow{PR} \perp \overleftrightarrow{UT}$. Name three other angles that have the same measure as ∠TQR.



∠PQU, ∠PQT, & ∠UQR, all 90°.

14. Using the drawing in problem 13, what is the sum of the angles ∠RQS and ∠SQT?

90°

15. Using the drawing in problem 13, what is the sum of the angles ∠PQT and ∠UQR?

180°

16. Using the drawing in problem 13, classify ∠RQS as acute, obtuse, right, or straight.

acute

17. Using the drawing in problem 13, classify ∠PQS as acute, obtuse, right, or straight.

obtuse

18. Using the drawing in problem 13, classify ∠PQR as acute, obtuse, right, or straight.

straight

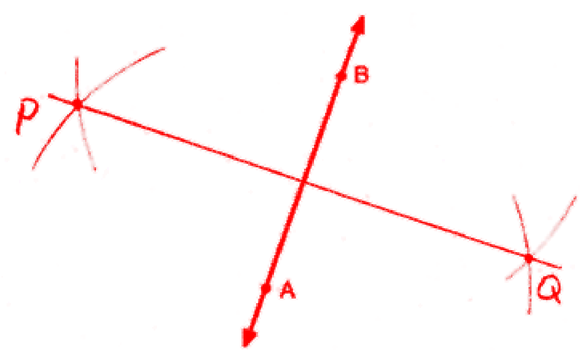
19. Using the drawing in problem 13, classify ∠RQT as acute, obtuse, right, or straight.

right

20. Using the drawing in problem 13, ∠UQR and what other angle form a pair of vertical angles?

∠PQT

21. Construct a perpendicular bisector of $\overleftrightarrow{AB}$.



22. Construct a bisector of ∠P.