Blue Pelican

15 Minute MathLabs
Teacher Manual

Preface

The concept of <u>15 Minute MathLabs</u> came about as a result of a few colleagues having a discussion about how to motivate math students: what teacher hasn't had that conversation? As the argument raged back and forth, the three main ideas emerged that were seemingly at odds with each other:

> *"Math projects (labs) are needed to show students the really* ***practical side of math****. And, besides, labs provide a little* ***spice and variety*** *to what is otherwise considered by many students as dull and irrelevant."*
>
> *"We must constantly hammer on the fundamentals while continuing to* ***move forward*** *through the curriculum.* ***Math labs take too much time****. They often require an entire period and many times the outcome is not what was predicted. Not only was instruction time lost, the whole thing was counterproductive."*
>
> *"It's such a* ***hassle trying to round up all of the items*** *and equipment for a lab."*

In the grip of this impasse someone wearily suggested, "Wouldn't it be nice if we could have occasional 15 minute labs that really worked and all the equipment was right there." …Thus the idea was born along with the name.

The 30 labs and videos presented here have been reviewed by a number of people and occasionally prompted comments like, "*Looks like there is as much science in there as math.*" To this rather oblique "criticism" our comment is:

> Yes! Thank you. The practical side of math **is** to a large extent science and engineering. After all, our math students are someone else's science students. Since science **needs** math and we **need** to show a practical side, labs that are a **mixture of both** would seem to be the perfect marriage.

Our entire philosophy concerning math labs can be summarized as:

- Provide labs that **actually work** and with **predictable results**.
- Emphasize **specific mathematics principles** in each lab.
- Relate labs to **other subject areas** such as science.
- Provide for quick and easy **teachers preparation** for the labs (videos and manual).
- Provide supplemental **in depth questions** and **calculator activities.**
- Provide a **kit of all the necessary items** for all labs.
- Confine labs to **15 minutes**… and then **right back to the regular lesson**.

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MathLab 19 The "Focus" on Alternative Energy ML 19-1

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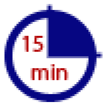


MathLab 09 – Some More Beans, Please

Purpose: Use physical data from an experiment using two different types of beans to write two equations in two unknowns and then solve for the number of each type of bean.

Math Principles: Solution of two simultaneous equations for two unknowns. Interpolate reading on a graduated cylinder.

Use two students to demo this lab:

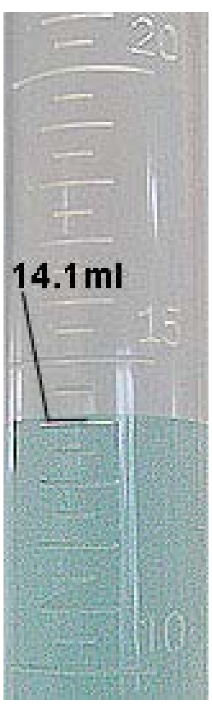
1. Student 1 will record data on the board.
2. Student 2 will read the levels on a 25ml graduated cylinder.

Items needed: Water, <u>25 ml graduated cylinder</u>, <u>large lima beans</u>, <u>kidney beans</u>. (Underlined items are supplied in the accompanying kit.)

What it's all about:
Knowing the **total volume** occupied by a combination of two different kinds of beans and knowing the **total bean count**, we are to determine the number of each type of bean. Solve by creating and then solving two linear equations.

In MathLab 07 the average volume for the two different types of beans (kidney and lima beans) was determined. Use those values (or your own) here:

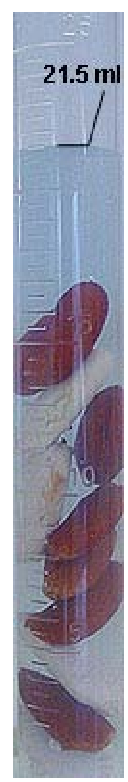
Average volume of a single kidney bean = .5 ml
Average volume of a single lima bean = 1 ml



Add some water:
Have student # 2 fill the 25 ml graduated cylinder about half full of water and then read the level. Student # 1 will record the level on the board. In the picture to the left it is **14.1 ml**.

Keep it a secret:
Shield the lower part of the cylinder from the class and secretly drop in **7 kidney beans and 4 of the large lima beans**. These two numbers represent the answer to the problem for which the students will solve.

Read the new level:
With the "bean portion" of the graduated cylinder still shielded, have student #2 read the new level of water in the cylinder. Have Student # 2 record this new level. In the picture to the right it is **21.5 ml**.



Let them dry:
Remove the beans, blot with a paper towel and let them air dry out of site of the students. (Leaving the beans in the water will eventually make them swell and make them unsuitable for later use.)

Make a guess:
Before working the problem, let the students guess how many of each type of bean there are.

Two pieces of information:
The teacher will now give students the **necessary information** so they will be able to **set up to equations** and thus be able to say **how many** of each type of bean was placed in the cylinder.

The necessary information that we now have is:

1. The **total bean count** is 11.

2. The **total volume** occupied by all 11 beans is the difference in the two readings taken above: 21.5 ml - 14.1 ml = 7.4 ml

   We must also use information mentioned above from MathLab 07 concerning the average volume of a single bean of each type:
   kidney bean volume = .5 ml, lima bean volume = 1 ml.

The two equations:
First we must define our variables:

L = number of lima beams
K = number of kidney beans

The first equation is for the **total bean count**:
L + K = 11

The second equation expresses the total volume of the combination as the **sum of the volumes** of the two different bean types:

1L + .5K = 7.4

Have your students solve these two equations for the variables L and K. The following shows how to do this using the elimination method:

Multiply by -1 → -1(L + K) = (-1)11 → -1L – 1K = -11
1L + .5K = 7.4 → 1L + .5K = 7.4
-.5K = -3.6
K = -3.6/(-.5) = **7.2 kidneys**

Substituting K= 7.2 into L + K = 11 we get:
L= 3.8 limas

Since we can only have a whole number of beans, round these answers to the nearest whole number of beans and get:

L= 4 lima beans **K = 7 kidney beans**

Enrichment activities:

Calculator activity

From the “Enrichment” portion of the DVD menu, see the video on how to solve the two equations graphically.

In depth questions:

1. Why might we not always get integer answers for L and K?

   If we used different beans in Lab 7 (where the volume was determined) than here, then because of the variation in the size of beans the volume readings may be in error. Also, all of the volume readings are just estimates.

2. How could the accuracy of the lab results be improved?

   Choose beans that are more uniform in size. Use a cylinder with finer graduations for more accurate volumes.

3. For a small diameter cylinder the meniscus formed by the water becomes more apparent and affects the readings. What is a meniscus? (Research required)

   It is the curved shape of the top of the liquid as it hugs the walls of the cylinder.

4. What would be the result of leaving the beans in the water for an extended period of time and then trying to use them for a lab in a class later in the day?

   The beans would swell and their volumes would be different from that determined in MathLab 07 thus making that data invalid for use in a later lab.

MathLab 18 – The Blacksmith and the Broken Wheel

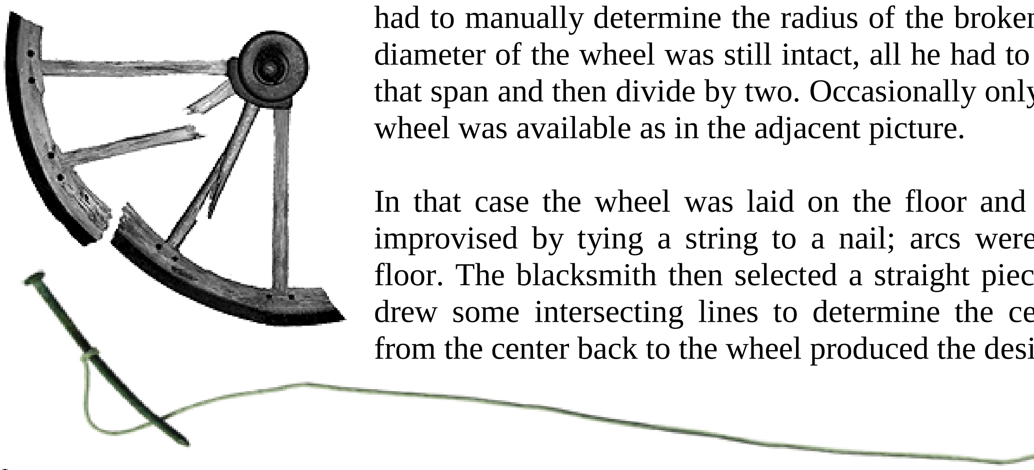
Purpose: Use construction techniques to show how to take a portion of a circle and find the center and radius.

Math Principles: Geometry theorems, construction techniques, perpendicular bisector, chord, radius, center.

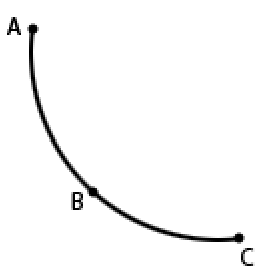
Use two students to demo this lab:
One student will create arcs with a compass and the other will draw lines with a straightedge.

Items needed: <u>White board compass</u>, straightedge (Underlined item is supplied in the accompanying kit.)

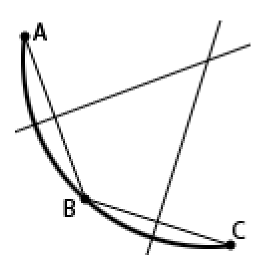
What it's all about:
In the old days it was common for a blacksmith to be given a piece of a broken wheel and then be asked to make a replacement wheel. There was very little standardization of wheel sizes so he had to manually determine the radius of the broken wheel. If a full diameter of the wheel was still intact, all he had to do was measure that span and then divide by two. Occasionally only a fragment of a wheel was available as in the adjacent picture.



In that case the wheel was laid on the floor and with a compass improvised by tying a string to a nail; arcs were marked on the floor. The blacksmith then selected a straight piece of lumber and drew some intersecting lines to determine the center. Measuring from the center back to the wheel produced the desired radius.

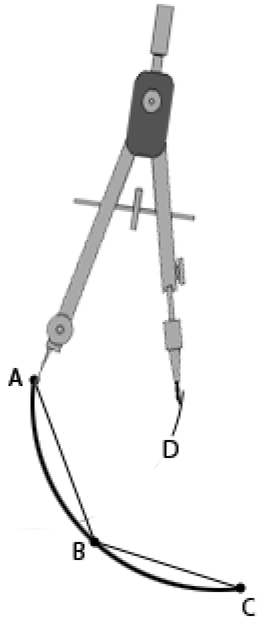


Begin with a piece of a circle. Mark three points:
The partial circle here represents our broken wheel. Mark three points on this arc. For greatest accuracy, place a point at each end of the arc. The third point can be placed somewhere near the center of the arc. (A student worksheet is provided on the last page of this lab so students can work along with the demonstration… or as an assignment sheet.)



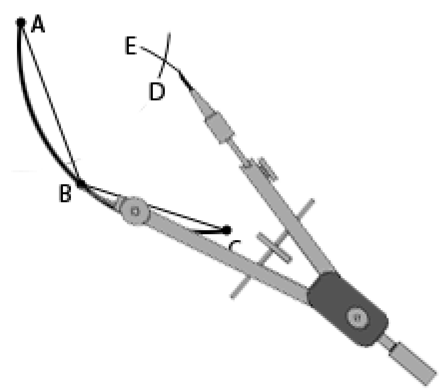
Here's the plan:
First, draw the two chords AB and BC. Then draw the perpendicular bisector of each. Extend these bisectors until they meet. That point will be the center of our circle. All we would have to do at this point is measure from that center to any point on the arc to get the radius.

Easier said than done:
Ok, but how do we do it? Instead of scratching a nail mark on the floor and drawing a line with a piece of lumber, we will use a compass and ruler as shown in the diagrams that follow.

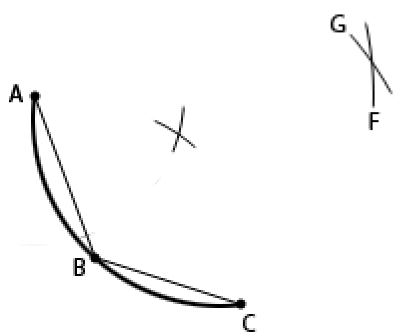


Strike some arcs:
Just bear in mind that the first three diagrams below produce two points that allow us to draw the perpendicular bisector of chord AB.

Begin by setting a spread on the compass a little larger than the length of AB. As shown in the picture to the left, put the point of the compass at A and strike arc D.



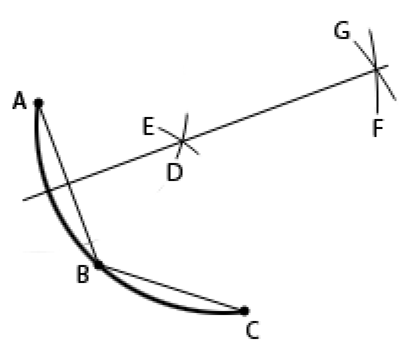
With this **same setting** on the compass, center the compass at point B and strike arc E as shown to the right. The intersection of arcs D and E establish one point on the perpendicular bisector that we will eventually draw.



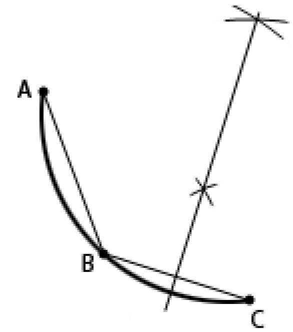
In a similar way with a **larger setting** on the compass, strike arcs F and G as shown in the picture to the left. The intersection of these two arcs defines the second point for our perpendicular bisector.

Draw the perpendicular bisector:
With a straightedge aligned to the points defined by the intersecting arcs just produced, draw the perpendicular bisector of chord AB. (Recall the theorem from geometry that says this bisector goes through the center of the circle.)

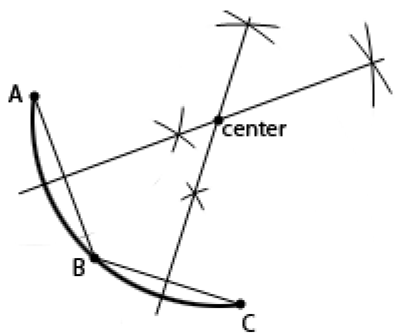
We still don’t know where the center of the circle is, but at least we do know **it’s somewhere** on the line just drawn.



Do it again:
In exactly the same way that the perpendicular bisector of chord AB was produced, use a compass and a straightedge to similarly produce the perpendicular bisector of chord BC.



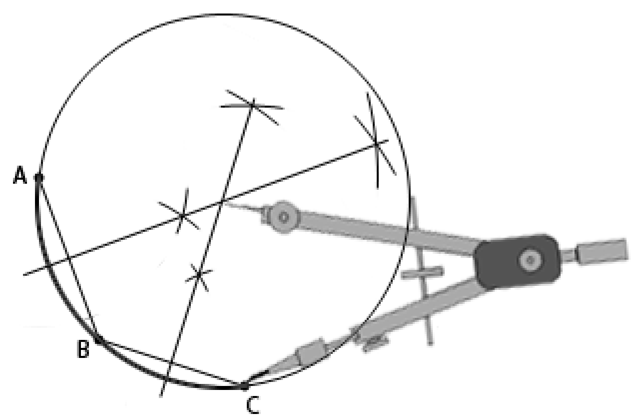
Locate the center:
The center of the circle is at the intersection of the two perpendiculars we have drawn.



Why? Because the center of a circle is guaranteed to be on the perpendicular bisector of a chord of that circle. The intersection point of our two lines is the **only point** on **both** perpendicular bisectors, so it **must be the center of the circle**.

At this point the blacksmith would measure the distance from the center to **any point** on the circle to find the radius of the wheel.

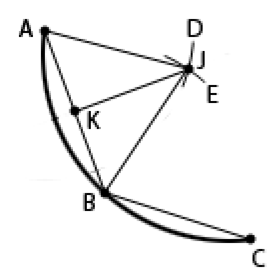
How can we be sure it worked?
Still not convinced? Center the compass at the center point and open up the compass so it just touches point C. Then draw a full circle. Observe that it also **goes through points A and B** thus proving that we had found the true center.



Enrichment activities:

Geometry proof:

Some students may have lingering doubts or curiosity about why these techniques work. Why does the line joining the intersecting arcs produce a perpendicular bisector of the chord?



The secret is to show triangles to be congruent: consider triangles AJK and BJK. Line segment AJ is equal in length to segment JB because they are both radii of the same circle. Both triangles share a common side. With these few hints, a well motivated student should be able to extend the reasoning and see that the lines connecting the intersecting arcs are, indeed, perpendicular bisectors of the chords.

Calculator Activity # 1:

Show how to graph a circle. Two equations will be needed since solving for y in the equation for a circle centered at the origin, $x^2 + y^2 = r^2$, produces a plus or minus… ($y = \pm\sqrt{r^2 - x^2}$)

Calculator Activity # 2:

Consider a more general equation of a circle that is **not** centered at the origin:

$$(x-h)^2 + (y-k)^2 = r^2, \qquad y = \pm\sqrt{r^2 - (x-h)^2} + k$$

Show how to graph this general circle using two different functions (one for the plus and one for the minus).

In depth questions:

1. When we measured from the center to any of points A, B, or C to obtain the radius, were there other point to which we could have measured?

2. Why would it be important for blacksmith to use a non stretchable string?

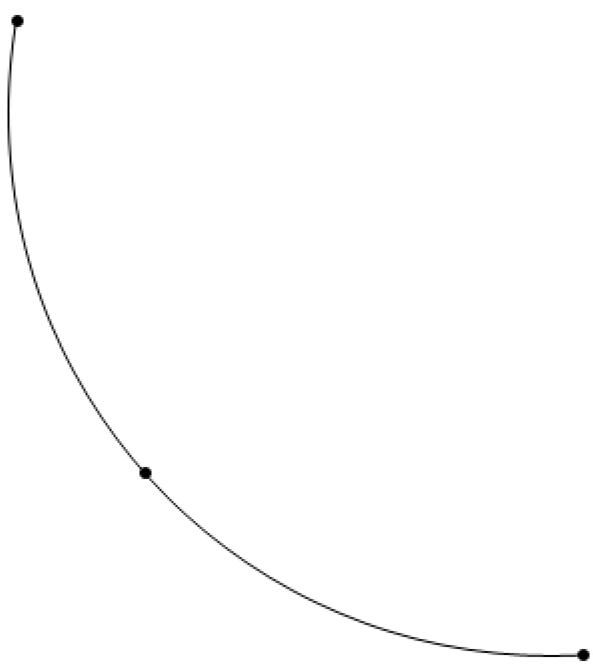
3. Would the perpendicular bisector of a fourth chord have passed through the intersection point of the first two?

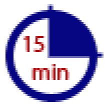
4. In choosing the three points on the arc, why is it important not to choose points that are near each other?

5. Is it always possible to find a circle that will pass through three noncollinear points?

6. Is it always possible to find a circle that will pass through four noncollinear points?

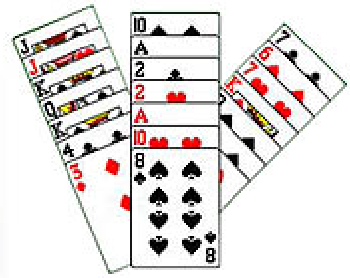
Student work sheet for MathLab 18





MathLab 21 – An Unusual Approach to Probability

Purpose: Explore the idea that the sum of all probabilities is one, and then use it to solve a problem.



Math principles/concepts: Probability, sum of all probabilities, theoretical probability, experimental probability, solving a linear equation.

Use three students to demo this lab:
Two students will operate the two spinners while the third records the results on the board.

Items needed: <u>Four-segment spinner</u>, <u>five-segment spinner</u> (Underlined items supplied in the accompanying kit.)

What it's all about:
Consider the two probability spinners shown here. We are interested in when the spinners stop on the red sections. The probability of getting a red with the four-segment spinner is **1/4** since there are four different colored segments but only one that is red. Similarly, the five-segment spinner's probability of coming up red is **1/5**.

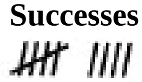
The problem we are to solve here is to find the probability of getting **at least one red** when considering **both spinners simultaneously**.

Experimental determination:
With two students operating the spinners, have them spin both devices while the third records a tally on the board of **both** the total number of trials **and** the number of successes. Emphasize that a **success could either be just one red or both red**.

Do at least 20 trials and more if time permits. Make sure the students understand that the greater the number of trials done, the closer will be the agreement of this experimental probability (**successes/trials**) with the theoretical probability.

Trials

Successes



Theoretical determination:
Begin with a discussion and emphasis on the fact that the sum of the probabilities of all possibilities is one. This may be a new concept to students, so make sure they understand since it is at the very heart of this lab.

> An example, students might relate to a weather prediction of 75 % rain. Of course, this means a 25% chance of no rain. Point out that these two probabilities add to 100% which is equivalent to a probability of one.

With regard to getting a red with the two spinners, all possible probabilities are:

P_0 = probability of no reds
P_1 = probability of exactly one red
P_2 = probability of two reds

Expressing the sum of these probabilities, we get the fundamental equation with which we will solve our problem:

$$P_0 + P_1 + P_2 = 1$$

At this point it is important for the students to realize that the probability we want (the probability of getting at least one red) is the **probability of getting exactly one red plus the probability of getting two reds**. This can be expressed as:

$$P_1 + P_2$$

Rewrite the "sum" equation, isolate these two terms in parenthesis, and then subtract P_0 from both sides:

$$P_0 + (P_1 + P_2) = 1$$
$$(P_1 + P_2) = \mathbf{1 - P_0}$$

Instead of finding two probabilities (P_1 & P_2) and adding them, we have the easier task of finding P_0 and subtracting it from 1:

$$P_0 = (\text{prob of no red on spinner}_5) \text{ X } (\text{prob of no red on spinner}_4)$$

$$P_0 = \frac{\#\ of\ non\ reds}{5\ spaces} X \frac{\#\ of\ non\ reds}{4\ spaces}$$

$$P_0 = \quad \frac{4}{5} \quad X \quad \frac{3}{4} = \frac{12}{20}$$

Finally, the solution is:

$$\text{P1 + P2} = 1 - P_0 = 1 - \frac{12}{20} = \frac{8}{20} = \frac{2}{5} = .4$$

Enrichment activities:

Calculator Activity # 1:

In the theoretical analysis above, we looked at P_1, the probability of getting exactly one red. Actually, there are **two** ways to get a single red: "$spinner_4$ is red and $spinner_5$ is not" or "$spinner_5$ is red and $spinner_4$ is not". In other words, with two items from which to choose, there are two different ways to choose a single item.

This gets into the realm of "combinations" upon which so much probability theory is so heavily dependent. For example, the meaning of ${}_5C_3$ is the number of ways 3 items can be selected from among 5: ${}_5C_3 = \frac{5!}{(5-3)!3!} = 10$

This calculator activity demonstrates how to evaluate ${}_nC_r$ on a graphing calculator.

Calculator Activity # 2:

In Calculator activity #1 above, mention was made of factorials. For example, 5! is read, "five factorial" and simply means $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 20$. In this calculator activity, factorials are demonstrated on the graphing calculator.

In depth questions:

1. Since probabilities can't exceed 1, how can the probability of rain be 25% since 25 is greater than 1.

2. Suppose in addition to the 4 and 5 color spinners we additionally have a 6 color spinner that also has just one red section. Spinning all three simultaneously, what would be the probability of at least one red?

3. For the 4-color spinner, would it be possible to spin a million times and never get a red?

4. For the 4-color spinner, would we be more likely to get 30,000 reds or 300,000 reds when spinning a million times? Why?

5. What is the probability for an event that is absolutely certain to happen?

6. What is the probability for an event that is absolutely certain not to happen?

7. Is it possible for a probability to be negative?

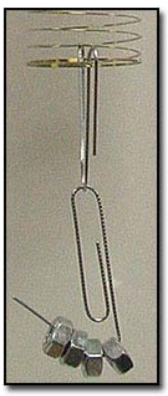
MathLab 23 – Slinky Secrets Revealed

Purpose: Demonstrate that the force used to stretch a spring is **proportional** to the amount it stretches. Find the constant of proportionality.

Math principles/concepts: Direct variation, scatter-plot, slope, measuring with a ruler, linear regression.

Use three students to demo this lab:
One student will hang weights on and stabilize the slinky, another will mark positions of the stretched spring, and the third will record data on the board.

Items needed: Miniature slinky(spring), nuts(1/4 – 20), centimeter rule. (Underlined items supplied in the accompanying kit.)



What it's all about:
The amount of force (F) needed to stretch a spring varies directly as the amount of stretch (x). The equation that describes this is:

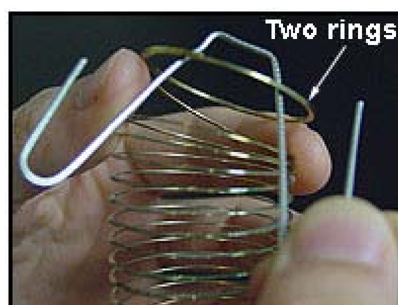
$F = (const)x.$

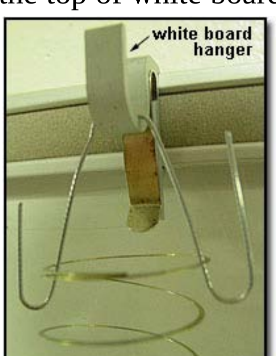
In this lab the stretching force is provided by the weight of nuts hung on the end of a spring (slinky). Since this weight is proportional to the number of nuts, the equation will be modified to the following where N is the number of nuts:

$N = kx$

Marks will be made on the background (chalk board or white board) as several nuts are successively placed on the hanger. Distances between the marks are measured for later analysis in which the constant of proportionality will be determined.

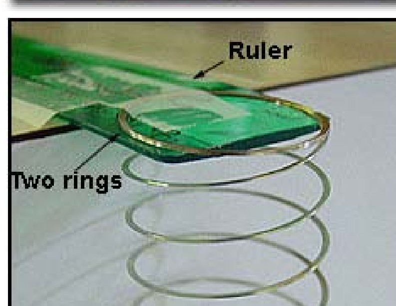
Insert the top hanger:
Use the provided square-top hanger so as to provide a means of hanging the slinky from a hook such as those typically found at the top of white boards. With the fingers, separate **two rings** of the slinky from the others and insert the square-top hanger as shown to the right.



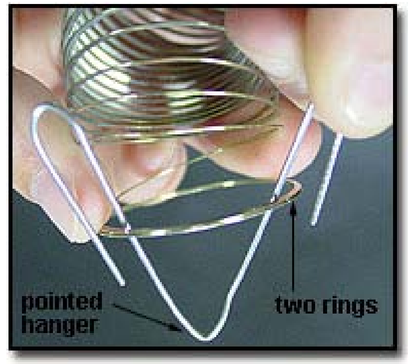


Hang:
Hang from a hook that will allow the slinky to hang down in front of a chalk board or white board.

…or use a ruler:
If such a hanger is not available, hang a ruler over the edge of a table as shown in the picture to the right and secure with tape. Again, only use **two rings** of the slinky on top of the ruler. By hanging from only two rings, your students should get data similar to that shown on the following pages.

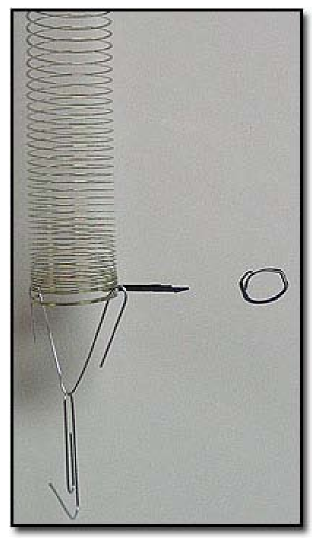


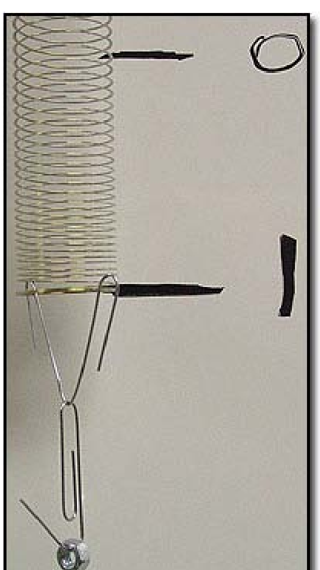
Insert the bottom hanger:
Use the provided pointed hanger so as to provide a means of hanging weights (nuts) on the slinky. With the fingers, separate **two rings** of the slinky from the others and insert the pointed hanger as shown to the right.



Attach the nut hanger:
Finally, attach the nut hanger (yes, it's a paperclip) as shown to the left. The nuts will be hung on the protruding segment.

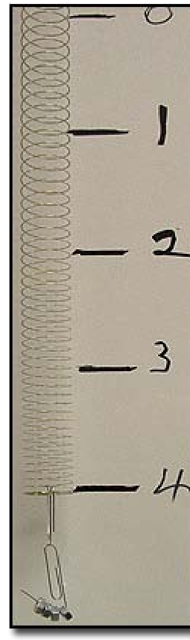
Mark the starting point:
With the spring hung as shown to the right and **with no added weights**, stabilize it (stop it from bouncing and shaking) and mark the bottom of the spring. **All measurements will be referenced from this point**. Make sure the student marking this and following positions has his **eyes level** with the bottom of the spring so as to accurately mark the true position.

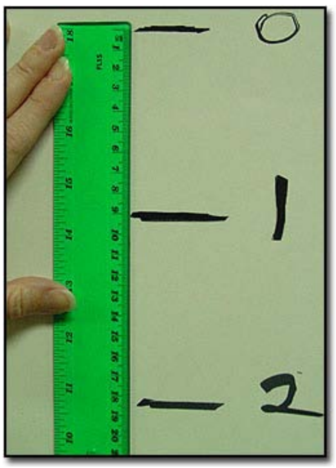




Place one nut on the hanger:
Gently hang one nut on the hanger, stabilize the spring (stop it from bouncing and shaking), mark the position, and label with "1". Again, make sure eye level is at the bottom of the spring so as to place an accurate mark.

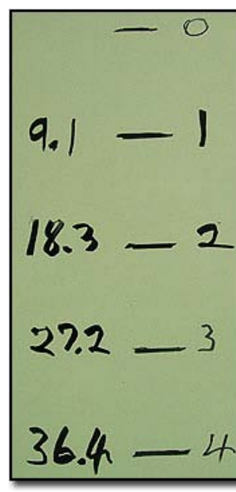
Continue adding weight:
One at a time, add nuts (up to a total of four) to the hanger. Each time mark the position of the bottom of the spring and record the number of nuts beside each mark. The picture to the right shows the marks and labeling after all four nuts have been attached.

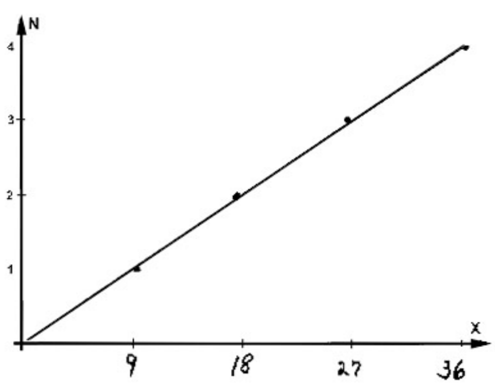




Measure the distances:
Using a centimeter ruler, measure the distances of each position of the spring (1, 2, 3, & 4) **from the zero mark** as shown to the left.

Completing the measurements:
After all four measurements are taken they should be close to those shown to the right only if the bottom and top hangers for the slinky were each hung on **two rings** (see pages 1 & 2). Different readings will also be the result of a distorted slinky (caused by excessive stretching). If any of these things happen in your case, **you will still get valid results** for your lab: they will just be different from those results shown here. Please note that all these readings are in centimeters.

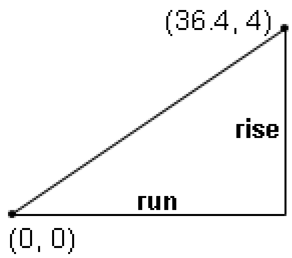




Putting it all together:
Using the graph paper provided as the last page of this lab, tape one piece to the board and have a student make a scatter-plot of the points with a large black marker. Be sure to make enough copies of the graph paper so that all the students in the class can participate. The results should look something like the adjacent drawing.

Notice that the line that best fits the data is drawn. See Calculator Activity #1 and #2 for a demonstration of how to display these scatter-plot points along with a best-fit line (linear regression). Also, notice that this graph is consistent with the equation *N* = *kx* since both have a y-intercept (actually an *N* intercept) of zero.

Finding the constant of proportionality:
Determine the constant of proportionality (*k*, also the slope of the line) for this line as follows:



Since the line goes through the origin, use (0, 0) as one point and (36.4, 4) as the other. The slope (and therefore *k*) is:

$$k = \frac{rise}{run} = \frac{N_2 - N_1}{x_1 - x_1} = \frac{4-0}{36.4-0} = \frac{4}{36.4} = .10989$$

Finally, the complete linear function is:

F = .10989x

Enrichment activities:

Calculator Activity # 1:

Use "scatter-plot" on the calculator to display the data points (9.1, 1), (18.3, 2), (27.2, 3), and (36.4, 4).

Calculator Activity # 2:

Using the points entered in Calculator Activity # 1, perform a linear regression and graph the resulting best-fit line simultaneously with the scatter plot.

In depth questions:

1. Using the equation for the number of nuts ($N = .10989x$), what would be the number of nuts if the amount of stretch is 54.6 cm?

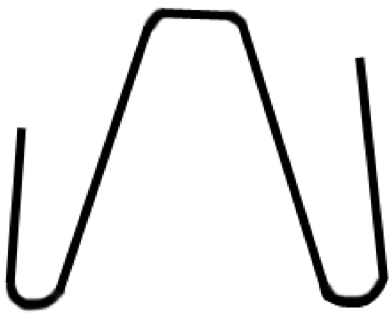
2. From physics what is the name of the law that expresses $F = kx$ where F is the force, k is the spring constant, and x is the amount of stretch. (Research needed)

3. Rewrite the function $N(x) = kx$ so that instead of N being a function of x, x is a function of N.

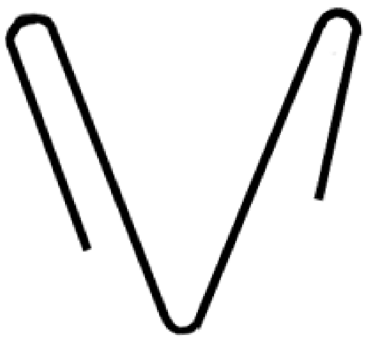
4. If we had nuts that weighed 1.5 times as much as those used in this lab, how much would 3 nuts stretch the spring?

5. When a linear regression is done on the calculator the N-intercept is -.00539721 whereas it's theoretical value is 0. Why the difference?

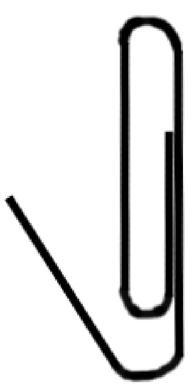
Paperclip templates used with the miniature slinky spring.



Square top hanger

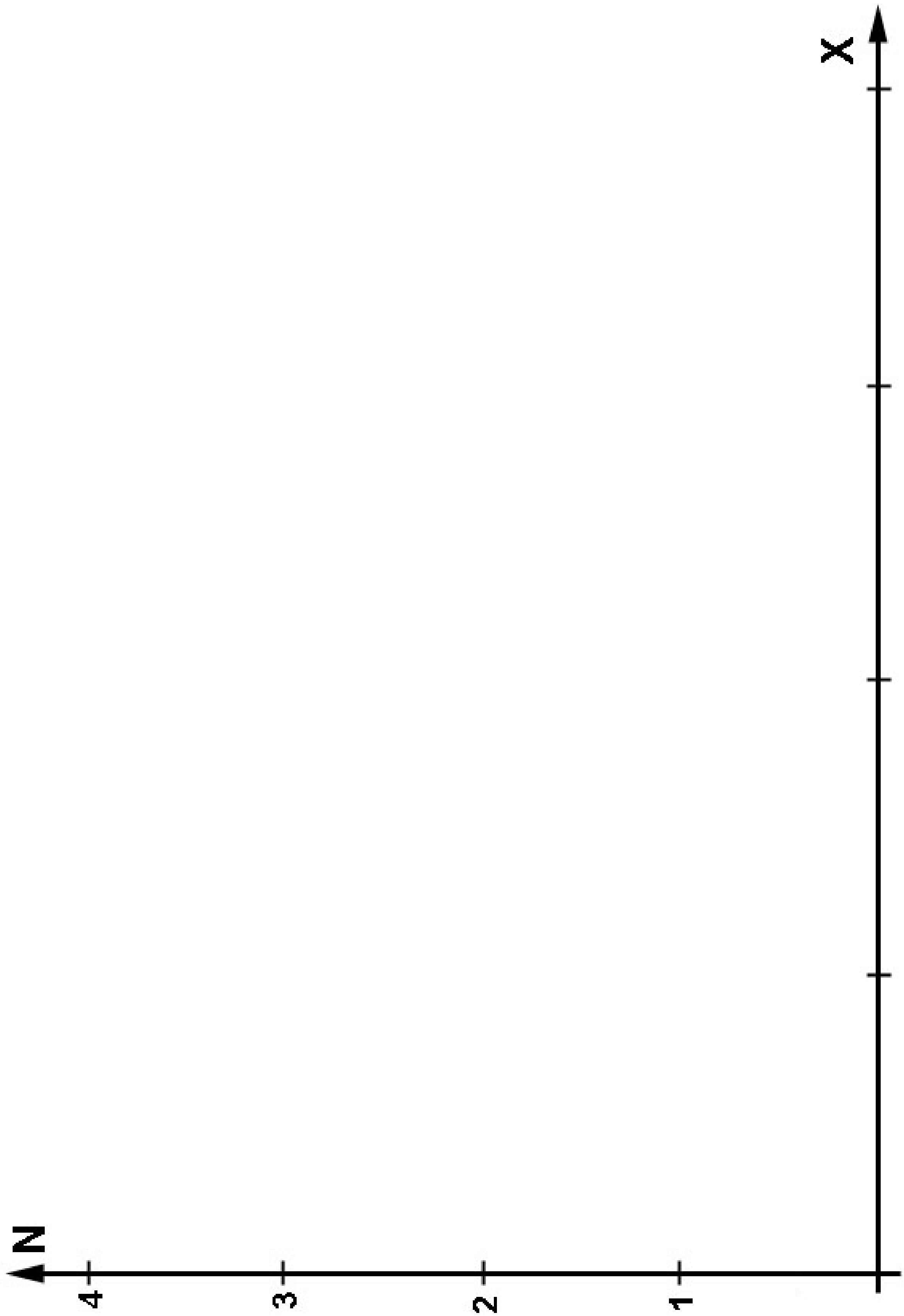


Pointed bottom hanger



Nut hanger

Graph Paper for MathLab 23



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