Blue Pelican Pre-Calculus

First Semester

Teacher Version 1.01

Pre Calculus Syllabus (First Semester)

Unit 1: Algebra review

Lesson 01: Review: multiplying and factoring polynomials

Lesson 02: Review: rational expressions, complex fractions

Lesson 03: Review: solving equations

Lesson 04: Review: equations of linear functions (lines)
Linear regression review

Lesson 05: Review: solutions of linear systems

Unit 1 review
Test: Unit 1 test

Unit 2: Basic trigonometry

Lesson 01: Angle conventions; definitions of the six trig functions

Lesson 02: Angle units; degrees(minutes & seconds), radians

Lesson 03: Given one trig ratio, find the others

Lesson 04: Special angles (0^o, 30^o, 60^o, 45^o, 90^o, 180^o, 270^o, 360^o)

Lesson 05: Evaluating trig function on the graphing calculator

Unit 2 review
Test: Unit 2 test

Unit 3: Triangle solutions

Lesson 01: Abstract solutions of right triangles

Lesson 02: Right triangle word problems, triangle area

Lesson 03: Vectors

Lesson 04: Sine law, more triangle area formulas

Lesson 05: Ambiguous case of the sine law

Lesson 06: Cosine law

Cumulative review, unit 6
Unit 6 review
Unit 6 test

Unit 7: Quadratic functions (parabolas)

Lesson 1: Transformations of quadratic functions

Lesson 2: Three forms of quadratic functions

Lesson 3: Quadratic calculator applications

Lesson 4: Quadratic area applications

Cumulative review, unit 7
Unit 7 review
Unit 7 test

Unit 8: Special functions

Lesson 1: Square root and semicircle functions

Lesson 2: Absolute value functions (|f(x)| and f(|x|) reflections)

Lesson 3: Piecewise functions, continuity

Lesson 4: Rate of change, piecewise word problems

Lesson 5: Greatest integer function, power functions

Cumulative review, unit 8
Unit 8 review
Unit 8 test

Unit 9: Polynomial functions

Lesson 1: Polynomial fundamentals, roots, end behavior

Lesson 2. Creating polynomial functions

Lesson 3: Long division (factoring, finding roots)

Lesson 4: Analyzing polynomials with a graphing calculator

Lesson 5: Applications (maximizing volume)

Cumulative review, unit 9
Unit 9 review
Unit 9 test

Semester summary

Semester review
Semester test

Enrichment Topics

Topic A: Analysis of absolute value inequalities

Topic B: Linear Programming

Topic C: Point-slope and intercept forms of a line

Topic D: The summation operator, Σ

Topic E: An unusual look at probability

Topic F: Rotations

Topic G: Absolute value parent functions

Topic H: Dimension changes affecting perimeter, area, and volume

Topic J: Algebraic solution to quadratic systems of equations.

Topic K: Derivation of the sine law

Topic L: Derivation of the cosine law

Topic M: Tangent composite function derivations

Topic N: Locating the vertex of a standard-form parabola

Topic O: Algebraic manipulation of inverse trig functions

Topic P: Logarithm theorem derivations

Topic Q: Arithmetic and geometric sum formulas

Topic R: Converting general form conics into standard form (completing-the-square)

Topic S: Conic section applications

Pre Calculus, Unit 1

Algebra Review

Unit 1: Lesson 01 Review: multiplying and factoring polynomials

Basic formulas that should be memorized:

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$
$$(a-b)(a+b) = a^2 - b^2$$
$$A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$$

Example 1: Multiply $(3x - 2y)(3x + 2y)$

$$(a-b)(a+b) = a^2 - b^2$$
$$(3x-2y)(3x+2y)$$
$$= \boxed{9x^2 - 4y^2}$$

Example 2: Factor $x^2 - 9$

$$a^2 - b^2 = (a-b)(a+b)$$
$$x^2 - 9 = \boxed{(x-3)(x+3)}$$

Example 3: Square $(x - 6z)^2$

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(x-6z)^2 = \boxed{x^2 - 12xz + 36z^2}$$

Example 4: Factor $x^2 + 8x + 16$

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$x^2 + 8x + 16 = \boxed{(x+4)^2}$$

Example 5: Multiply
$(x - 5y)(x^2 + 5xy + 25y^2)$

$$(a-b)(a^2+ab+b^2) = a^3 - b^3$$
$$(x-5y)(x^2+5xy+25y^2)$$
$$= x^3 - (5y)^3$$
$$= \boxed{x^3 - 125y^3}$$

Example 6: Factor $27x^3 - z^3$

$$(a)^3 - (b)^3$$
$$27x^3 - z^3 = (3x)^3 - (z)^3$$
$$= \boxed{(3x-z)(9x^2+3xz+z^2)}$$
$$(a - b)(a^2 + ab + b^2)$$

Example 7: Multiply using FOIL (first, outside, inside, last). (2x – 5) (x +3)

$$(2x-5)(x+3) = \overset{F}{2x^2} + \overset{O}{6x} \overset{I}{-5x} \overset{L}{-15}$$

$$= \boxed{2x^2 + x - 15}$$

When factoring, always look for a greatest common factor (GCF).

Example 8: Factor $5x^2 + 5x - 30$

GCF = 5

$$5x^2 + 5x - 30 = 5(x^2 + x - 6)$$

$$= \boxed{5(x+3)(x-2)}$$

Assignment: In problems 1-8, perform the indicated multiplications.

1. $(y - x)(y + 11x)$

FOIL

$= y^2 + 11xy - 1xy - 11x^2$

$= \boxed{y^2 + 10xy - 11x^2}$

2. $3(x - 4)(x + 4)$

$(a - b)(a + b)$

$= 3(x^2 - 16) = \boxed{3x^2 - 48}$

3. $(A - 3B)^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$[A - (3B)]^2 = \boxed{A^2 - 6AB + 9B^2}$

4. $(4mb + 9a)^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$[(4mb) + (9a)]^2$

$= \boxed{16m^2b^2 + 72mab + 81a^2}$

5. $(j + k)(j^2 - jk + k^2)$

$(a + b)(a^2 - ab + b^2) = a^3 + b^3$

$(j + k)(j^2 - jk + k^2)$

$= \boxed{j^3 + k^3}$

6. $(5x + 20d)(5x - 20d)$

$(a + b)(a - b)$

$(5x + 20d)(5x - 20d)$

$= \boxed{25x^2 - 400d^2}$

7. (2m + 8n)(m + n)

FOIL

$= \overset{F}{2m^2} + \overset{O}{2mn} + \overset{I}{8mn} + \overset{L}{8n^2}$

$= \boxed{2m^2 + 10mn + 8n^2}$

8. $(p^2 + 4h)^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(p^2 + 4h)^2 = \boxed{p^4 + 8p^2h + 16h^2}$

Factor the given polynomial in the following problems:

9. $x^2 - 6x + 9$

$a^2 - 2ab + b^2 = (a - b)^2$

$x^2 - 6x + 9 = \boxed{(x - 3)^2}$

10. $2x^2 - 4x - 48$

GCF=2

$= 2(x^2 - 2x - 24)$

$= \boxed{2(x - 6)(x + 4)}$

11. $3x^2 - 48$

GCF= 3

$= 3(\underbrace{x^2 - 16}_{a^2 - b^2}) = \boxed{3(x - 4)(x + 4)}$

12. $y^2 - 18y + 81$

$a^2 - 2ab + b^2 = (a - b)^2$

$y^2 - 18y + 81 = \boxed{(y - 9)^2}$

13. $v^2 + 12v + 36$

$a^2+2ab+b^2=(a+b)^2$

$\boxed{v^2+12v+36=(v+6)^2}$

14. $9f^2 - 100v^2$

$(a)^2-(b)^2=(a-b)(a+b)$

$(3f)^2-(10v)^2=\boxed{(3f-10v)(3f+10v)}$

15. $d^3 - c^3$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$d^3-c^3=\boxed{(d-c)(d^2+dc+c^2)}$

16. $54x^3 - 2y^3$

$=2(27x^3-y^3)$ GCF = 2

$(A)^3-(B)^3$

$=2[(3x)^3-(y)^3]$

$=\boxed{2(3x-y)(9x^2+3xy+y^2)}$

17. $x^2 + 12x + 35$

$=\boxed{(x+7)(x+5)}$

18. $x^2 - 10x - 24$

$=\boxed{(x-12)(x+2)}$

Unit 1: Lesson 02 **Review: rational expressions, complex fractions**

Rational expressions are of the form $\frac{\mathrm{P(x)}}{\mathrm{Q(x)}}$ where P(x) and Q(x) are polynomials. Rational expressions can be added, subtracted, multiplied, and divided.

Example 1: Find a common denominator and add or subtract as appropriate:

$$\frac{5}{x^2-9} - \frac{x-1}{x^2+x-12}$$

$$= \frac{5}{(x-3)(x+3)} - \frac{x-1}{(x+4)(x-3)}$$

$$= \frac{5}{(x-3)(x+3)} \cdot \frac{x+4}{x+4} - \frac{x-1}{(x+4)(x-3)} \cdot \frac{x+3}{x+3}$$

$$= \frac{5x+20-x^2-2x+3}{(x-3)(x+3)(x+4)} = \boxed{\frac{-x^2+3x+23}{(x-3)(x+3)(x+4)}}$$

Example 2: Simplify $\frac{x^2-16}{x+5} \quad \frac{x^2+10x+25}{x^2+x-12}$

$$= \frac{(x-4)\cancel{(x+4)}}{\cancel{x+5}} \cdot \frac{(x+5)^{\cancel{2}}}{\cancel{(x+4)}(x-3)} = \boxed{\frac{(x-4)(x+5)}{x-3}}$$

Example 3: Simplify $\frac{\mathrm{x}^3-1}{\mathrm{x}-5} \div \frac{\mathrm{x}^2-1}{\mathrm{x}^2-8\mathrm{x}+15}$

$$= \frac{\cancel{(x-1)}(x^2+1x+1)}{\cancel{(x-5)}} \cdot \frac{(x-3)\cancel{(x-5)}}{\cancel{(x-1)}(x+1)}$$

$$= \boxed{\frac{(x^2+x+1)(x-3)}{x+1}}$$

Example 4: Simplify this complex fraction. $\dfrac{\dfrac{2}{z^2} - \dfrac{5}{mz} - \dfrac{3}{m^2}}{\dfrac{2}{z^2} + \dfrac{7}{mz} + \dfrac{3}{m^2}}$

$$\frac{\frac{2}{z^2} - \frac{5}{mz} - \frac{3}{m^2}}{\frac{2}{z^2} + \frac{7}{mz} + \frac{3}{m^2}} \cdot \frac{m^2z^2}{m^2z^2} = \frac{\frac{2m^2z^2}{z^2} - \frac{5m^2z^2}{mz} - \frac{3m^2z^2}{m^2}}{\frac{2m^2z^2}{z^2} + \frac{7m^2z^2}{mz} + \frac{3m^2z^2}{m^2}}$$

$$= \frac{2m^2 - 5mz - 3z^2}{2m^2 + 7mz + 3z^2}$$

$$= \frac{\cancel{(2m+z)}(m-3z)}{\cancel{(2m+z)}(m+3z)} = \boxed{\frac{m-3z}{m+3z}}$$

Assignment: In problems 1-4, simplify by finding a common denominator and combining into one fraction.

1. $\frac{3}{x} + \frac{2}{y}$

$= \frac{3}{x}\cdot\frac{y}{y} + \frac{2}{y}\cdot\frac{x}{x}$

$= \boxed{\frac{3y+2x}{xy}}$

2. $\frac{4}{8x^2} - \frac{3}{4xy}$

$= \frac{4}{8x^2}\cdot\frac{y}{y} - \frac{3}{4xy}\cdot\frac{2x}{2x}$

$= \boxed{\frac{4y-6x}{8x^2y}}$

3. $\frac{3y}{y^2-5y+6} + \frac{4y}{y^2-4}$

$= \frac{3y}{(y-3)(y-2)} + \frac{4y}{(y-2)(y+2)}$

$= \frac{3y}{(y-3)(y-2)}\cdot\frac{y+2}{y+2} + \frac{4y}{(y-2)(y+2)}\cdot\frac{y-3}{y-3}$

$= \frac{3y^2+6y+4y^2-12y}{(y-3)(y-2)(y+2)} = \boxed{\frac{7y^2-6y}{(y-3)(y-2)(y+2)}}$

4. $\frac{3}{x^2+3x+2} + \frac{x-3}{x^2-2x-3}$

$= \frac{3}{(x+2)(x+1)} + \frac{x-3}{(x-3)(x+1)}$

$= \frac{3}{(x+2)(x+1)}\cdot\frac{(x-3)}{(x-3)} + \frac{x-3}{(x-3)(x+1)}\cdot\frac{x+2}{x+2}$

$= \frac{3x-9+x^2-x-6}{(x+2)(x+1)(x-3)} = \frac{x^2+2x-15}{(x+2)(x+1)(x-3)}$

$= \frac{(x+5)\cancel{(x-3)}}{(x+2)(x+1)\cancel{(x-3)}} = \boxed{\frac{x+5}{(x+2)(x+1)}}$

5. Simplify $\frac{x^3-8}{x^2-4}\,\frac{x+3}{x^2-x-12}$

$$= \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}\,\frac{\cancel{x+3}}{(x-4)\cancel{(x+3)}}$$

$$= \boxed{\frac{x^2+2x+4}{(x+2)(x-4)}}$$

6. Simplify $\frac{2x^2-3x-2}{4x^2-1} \div \frac{x^2-4}{2x^2-5x+2}$

$$= \frac{\cancel{(2x+1)}(x-2)}{\cancel{(2x-1)}\cancel{(2x+1)}}\,\frac{\cancel{(2x-1)}\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$$

$$= \boxed{\frac{x-2}{x+2}}$$

7. Simplify $\dfrac{\frac{2}{x-3}+\frac{-1}{x^2-9}}{\frac{5}{x^3-27}}$

$$= \dfrac{\frac{2}{x-3}+\frac{-1}{(x-3)(x+3)}}{\frac{5}{(x-3)(x^2+3x+9)}}$$

$$= \dfrac{\frac{2}{x-3}+\frac{-1}{(x-3)(x+3)}}{\frac{5}{(x-3)(x^2+3x+9)}}\;\frac{(x-3)(x+3)(x^2+3x+9)}{(x-3)(x+3)(x^2+3x+9)}$$

$$= \boxed{\frac{2(x+3)(x^2+3x+9)-1(x^2+3x+9)}{5(x+3)}}$$

8. Simplify $\dfrac{\frac{1}{x-3}}{\frac{x-5}{2x^2-4x-6}+3}$ $= \dfrac{\frac{1}{x-3}}{\frac{x-5}{2(x^2-2x-3)}+\frac{3}{1}}$

$$= \frac{\frac{1}{x-3}}{\frac{x-5}{2(x-3)(x+1)}+\frac{3}{1}} \quad \frac{2(x-3)(x+1)}{2(x-3)(x+1)}$$

$$= \frac{\frac{2\cancel{(x-3)}(x+1)}{\cancel{x-3}}}{\frac{(x-5)\cancel{2}\cancel{(x-3)}\cancel{(x+1)}}{\cancel{2}\cancel{(x-3)}\cancel{(x+1)}}+6(x-3)(x+1)}$$

$$= \frac{2x+2}{x-5+6(x^2-2x-3)} = \frac{2x+2}{x-5+6x^2-12x-18}$$

$$= \boxed{\frac{2x+2}{6x^2-11x-23}}$$

Unit 1: Lesson 03 Review: solving equations

Solve these problems for the given variables:

Example 1: $\frac{2x}{3} - \frac{1}{5}(x - 4) = 5$

$\frac{2x}{\cancel{3}}\cancel{15}^{5} - \frac{1}{\cancel{5}}\cancel{15}^{3}(x-4) = 5(15)$

$10x - 3(x-4) = 75$

$10x - 3x + 12 = 75$

$7x = 75 - 12 = 63$

$x = \frac{63}{7} = \boxed{9}$

Example 2: $3x^2 - 75 = 0$

$3(x^2 - 25) = 0$

$3(x-5)(x+5) = 0$

$x - 5 = 0 \quad x + 5 = 0$

$x = \boxed{5} \quad x = \boxed{-5}$

Example 3: $x^2 - x - 42 = 0$

$(x-7)(x+6) = 0$

$x - 7 = 0 \quad x + 6 = 0$

$x = \boxed{7} \quad x = \boxed{-6}$

Example 4: $\frac{1}{x-2} = \frac{x}{3}$

$\frac{1}{x-2} = \frac{x}{3}$ cross multiply

$x(x-2) = 3$

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \quad x + 1 = 0$

$x = \boxed{3} \quad x = \boxed{-1}$

Example 5: $3x^3 - x^2 - 12x + 4 = 0$

$[x^2(3x-1) - 4(3x-1)] = 0$

$(3x-1)[x^2 - 4] = 0$

$(3x-1)(x-2)(x+2) = 0$

$3x - 1 = 0 \quad x - 2 = 0 \quad x + 2 = 0$

$3x = 1 \quad x = \boxed{2} \quad x = \boxed{-2}$

$x = \boxed{\frac{1}{3}}$

Example 6: $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$

$$\frac{3}{\cancel{x+2}} \cdot \frac{5x(\cancel{x+2})}{1} - \frac{1}{\cancel{x}} \cdot \frac{5\cancel{x}(x+2)}{1} = \frac{1}{\cancel{5x}} \cdot \frac{\cancel{5x}(x+2)}{1}$$

$$15x - 5(x+2) = x+2$$

$$15x - 5x - 10 = x + 2$$

$$10x - x = 2 + 10$$

$$9x = 12$$

$$x = \frac{12}{9} = \boxed{\frac{4}{3}}$$

Example 7: $\sqrt{x+4} = 7$

$$\left(\sqrt{x+4}\right)^2 = (7)^2$$

$$x + 4 = 49$$

$$x = 49 - 4$$

$$x = \boxed{45}$$

check: (Because we raised to an even power)

$$\sqrt{45+4} = 7$$

$$\sqrt{49} = 7$$

$$7 \overset{\checkmark}{=} 7$$

Assignment: Solve for the variables.

1. $4 + \frac{x}{4} = \frac{2(x-2)}{5}$

$4 \cdot 20 + \frac{x}{\cancel{4}} \cancel{20}^{5} = \frac{2(x-2)}{\cancel{5}} \cancel{20}^{4}$

$80 + 5x = 8x - 16$

$5x - 8x = -16 - 80$

$-3x = -96$

$x = \boxed{32}$

2. 5x – 3 = 7x + 1

$5x - 7x = 3 + 1$

$-2x = 4$

$x = \boxed{-2}$

3. $x^2 + 9x + 18 = 0$

$(x+6)(x+3) = 0$

$x + 6 = 0 \quad x + 3 = 0$

$x = \boxed{-6} \quad x = \boxed{-3}$

4. $x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x - 3 = 0 \quad x + 3 = 0$

$x = \boxed{3} \quad x = \boxed{-3}$

5. $2x^2 + 10x + 3x + 15 = 0$

$[2x(x+5) + 3(x+5)] = 0$

$(x+5)[2x+3] = 0$

$x + 5 = 0 \quad 2x + 3 = 0$

$x = \boxed{-5} \quad 2x = -3$

$x = \boxed{-\frac{3}{2}}$

6. $5x^2 - 5x + x - 1 = 0$

$[5x(x-1) + (x-1)] = 0$

$(x-1)[5x+1] = 0$

$x - 1 = 0 \quad 5x + 1 = 0$

$x = \boxed{1} \quad 5x = -1$

$x = \boxed{-\frac{1}{5}}$

7. $\frac{56}{x-2} = x - 3$

$\frac{56}{\cancel{x-2}}\cancel{(x-2)} = (x-3)(x-2)$

$56 = x^2 - 5x + 6$

$0 = x^2 - 5x + 6 - 56$

$0 = x^2 - 5x - 50$

$0 = (x-10)(x+5)$

$x - 10 = 0 \qquad x + 5 = 0$

$x = \boxed{10} \qquad x = \boxed{-5}$

8. (3/2)x + .5x + 7 = 0

$\frac{3}{\cancel{2}}x\cdot\cancel{2} + \frac{1}{2}x\cdot 2 + 7\cdot 2 = 0\cdot 2$

$3x + x + 14 = 0$

$4x = -14$

$x = -\frac{14}{4}$

$x = \boxed{-\frac{7}{2}}$

9. $x^2 = 81$

$x^2 - 81 = 0$

$(x-9)(x+9) = 0$

$x - 9 = 0 \qquad x + 9 = 0$

$x = \boxed{9} \qquad x = \boxed{-9}$

10. $2x^3 - 5x^2 - 12x = 0$

$x(2x^2 - 5x - 12) = 0$

$x(2x+3)(x-4) = 0$

$x = \boxed{0} \qquad 2x + 3 = 0 \qquad x - 4 = 0$

$2x = -3 \qquad x = \boxed{4}$

$x = \boxed{-\frac{3}{2}}$

11. $\sqrt{x-2} = 6$

$(\sqrt{x-2})^2 = 6^2$

$x - 2 = 36$

$x = 36 + 2 = \boxed{38}$

check:

$\sqrt{38-2} = 6$

$6 \overset{\checkmark}{=} 6$

12. $\frac{3}{2x} + \frac{11}{5x} = 1$

$\frac{3}{\cancel{2x}}\cancel{10x}^{5} + \frac{11}{\cancel{5x}}\cancel{10x}^{2} = 10x$

$15 + 22 = 10x$

$37 = 10x$

$\boxed{\frac{37}{10}} = x$

Unit 1: Lesson 04

Review: equations of linear functions (lines)
Linear regression review

There are four forms of the equation of a line (a linear function):

- Ax + By + C = 0, **General** form (some textbooks put C on the right side of the equation)
- y = mx + b, **Slope-intercept** form
- $y - y_1 = m(x - x_1)$, **Point-slope** form
- $\frac{x}{a} + \frac{y}{b} = 1$, **Intercept** form

In the above equations *m* is the slope, (x_1, y_1) and (x_2, y_2) are points on the line, *a* is the x-intercept, and *b* is the y-intercept.

Notice in all forms both **x and y are to the one power** (degree 1).

Special cases:

- Equations of **vertical lines** are always of the form x = a where *a* is the place on the x-axis through which the line passes.
- Equations of **horizontal lines** are always of the form y = b where *b* is the place on the y-axis through which the line passes.

Example 1: Find the equation of the line (in slope-intercept form) passing through the points (4, 3) and (1, –6).

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-3}{1-4} = 3$

$y = mx + b$; $y = 3x + b$

$3 = 3(4) + b \leftarrow$ sub in (4,3)

$3 - 12 = b$

$-9 = b$

$y = mx + b$

$\boxed{y = 3x - 9}$

Example 2: Find the equation of the line (in slope-intercept form) parallel to the line given by 3x + 2y = –1 and having y-intercept 5.

$3x + 2y = -1$

$2y = -3x - 1$

$y = -\frac{3}{2}x - \frac{1}{2}$; $m = -3/2$

y-intc = 5 $\rightarrow$ $b = 5$

$y = mx + b$

$\boxed{y = -\frac{3}{2}x + 5}$

Example 3: Express y = (3/4)x – 7 in general form.

$y=\frac{3}{4}x-7$

$4y=\frac{3}{4}x(4)-7\cdot 4$

$4y=3x-28$

$\boxed{-3x+4y+28=0}$

Example 4: Express y = (3/4)x – 7 in intercept form.

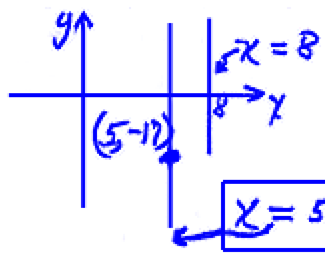
$-\frac{3}{4}x+y=-7$ ← need 1 here

$-\frac{3x}{4(-7)}+\frac{y}{-7}=\frac{-7}{-7}$ Divide by -7

$\frac{3x}{28}+\frac{y}{-7}=1$

$\boxed{\frac{x}{28/3}+\frac{y}{-7}=1}$

Example 5: Find the equation of the line parallel to the line x = 8 and passing through the point (5, –17).



$\boxed{x=5}$

Example 6: Find the equation of the line (in point-slope form) passing through (–1, 6) and perpendicular to the line given by 3x + 7y = –9.

$3x+7y=-9$

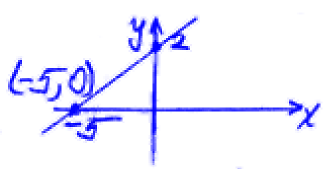
$7y=-3x-9$

$m=-\frac{3}{7}$ $\quad y=-\frac{3}{7}x-9/7$

$m_{\perp}=\frac{7}{3}$ $\quad y-y_1=m(x-x_1)$

$\boxed{y-6=\frac{7}{3}(x+1)}$

Example 7: Find the equation of the line (in intercept form) that passes through (–5, 0) and has y-intercept 2.



$a=-5$; $b=2$

$\frac{x}{a}+\frac{y}{b}=1$

$\boxed{\frac{x}{-5}+\frac{y}{2}=1}$

Example 8: Find the equation of the line (in slope-intercept form) that has x-intercept –8 and y-intercept 9.

y-intc → $b=9$

x-intc → $(-8,0)$

$y=mx+b$; $y=mx+9$

$0=m(-8)+9$; sub in (-8,0)

$8m=9$; $m=9/8$

$y=mx+b$

$\boxed{y=\frac{9}{8}x+9}$

Recall from Alg II, that scattered points (a scatter plot) can often be approximately fit with a straight line. The process of finding the line of best fit is known as **linear regression**.

For a review on this process, see **Calculator Appendix M** (scatter plots) and **Calculator Appendix N** (regression). Note that with regression it is also possible for other than linear functions (exponential, logarithm, etc.) to be made to fit scatter plots.

Take special note of the **correlation factor, *r*,** given by a regression which is basically a score of how good the fit is. The value of *r* is restricted to $-1 \leq r \leq 1$. The closer $|r|$ is to 1, the better the fit.

When the best fit line has a negative slope, the correlation is said to be negative. Likewise, when the slope is positive, the correlation is said to be positive.

Assignment:

1. Find the equation of the line (in intercept form) that has x-intercept 2 and y-intercept –7.

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\boxed{\frac{x}{2}+\frac{y}{-7}=1}$$

2. Find the equation of the line (in slope-intercept form) that passes through (5,6) and (1, –4).

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-6}{1-5}=\frac{5}{2}$$

$$y=mx+b \rightarrow y=\frac{5}{2}x+b$$

sub in $(1,-4) \rightarrow -4=\frac{5}{2}(1)+b$

$$-4\frac{2}{2}-\frac{5}{2}=b;\ b=\frac{-13}{2}$$

$$y=mx+b$$

$$\boxed{y=\frac{5}{2}x-\frac{13}{2}}$$

3. Express 3x + 7y = 9 in intercept form.

$3x+7y=9 \leftarrow$ need 1 here

$$\frac{3x}{9}+\frac{7y}{9}=\frac{9}{9}$$

$$\frac{x}{3}+\frac{7y}{9}=1$$

$$\boxed{\frac{x}{3}+\frac{y}{9/7}=1}$$

4. Express 3x + 7y = 9 in slope-intercept form.

$3x+7y=9$; solve for y

$$7y=-3x+9$$

$$\boxed{y=\frac{-3}{7}x+\frac{9}{7}}$$

5. Write the equation of the line that is perpendicular to the x-axis and passing through the point (5, 1).

$$\boxed{x=5}$$

6. Write the equation of the line (in slope-intercept form) passing through (4, 6) and intersecting the y-axis at 17.

$$b=17$$

$$y=mx+b \rightarrow y=mx+17$$

sub in $(4,6) \rightarrow 6=m4+17$

$$6-17=4m \rightarrow m=\frac{-11}{4}$$

$$y=mx+b$$

$$\boxed{y=\frac{-11}{4}x+17}$$

7. Write the equation of the line (in general form) that is perpendicular to the line given by x = 2y – 6 and passing through (101, 79).

$x+6=2y$
$\frac{1}{2}x+3=y$
$m=\frac{1}{2}$
$m_{\perp}=-2$

$y=mx+b$
$y=-2x+b$
$79=-2(101)+b$
$79+202=b$
$281=b$

$y=mx+b$
$y=-2x+281$
$\boxed{2x+y-281=0}$

8. Write the equation of the line (in point-slope form) having slope 5 and passing through (– 6, 2).

$y-y_1=m(x-x_1)$
$y-2=5(x-(-6))$
$\boxed{y-2=5(x+6)}$

$m=5$
$(x_1,y_1)=(-6,2)$

9. Write the equation of the line (in slope-intercept form) having slope 5 and passing through (– 6, 2).

$y=5x+b \leftarrow m=5$
$+2=5(-6)+b \leftarrow$ sub in $(-6,2)$
$32=b$
$y=mx+b \rightarrow \boxed{y=5x+32}$

10. Write the equation of a line having slope 2 and y-intercept 19.

$m=2 \quad b=19$
$y=mx+b$
$\boxed{y=2x+19}$

11. What is the slope of a line perpendicular to x/4 – y/7 = 1?

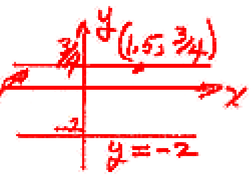
solve for y
$\frac{x}{4}-\frac{y}{7}=1$
$-\frac{y}{7}=-\frac{x}{4}+1$

$y=-\frac{x}{4}(-7)+1(-7)$
$y=\frac{7}{4}x-7$
$m=\frac{7}{4} \rightarrow m_{\perp}=\boxed{-\frac{4}{7}}$

12. What is the equation of the line parallel to the line given by y + 2 = 0 and passing through (1.5, 3/4)?

$y+2=0$
$y=-2$

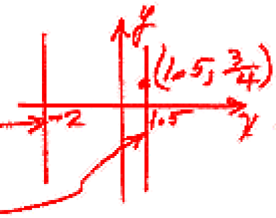
$\boxed{y=\frac{3}{4}}$



13. What is the equation of the line parallel to the line given by x + 2 = 0 and passing through (1.5, 3/4)?

$x+2=0$
$x=-2$

$\boxed{x=1.5}$



14. Find the equation of the line passing through (2, 5) and (– 2, 1) in point slope form.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-5}{-2-2}=\frac{-4}{-4}=1$$

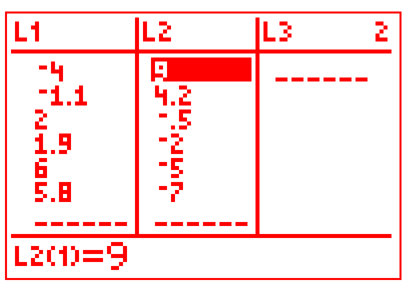
$$y-y_2=m(x-x_2)$$
$$y-1=1(x-(-2))$$
$$\boxed{y-1=1(x+2)}$$

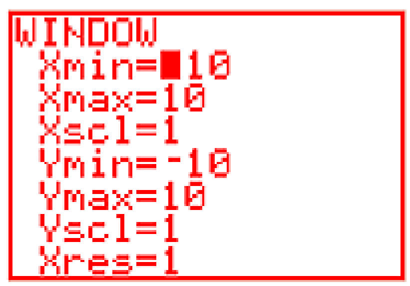
or

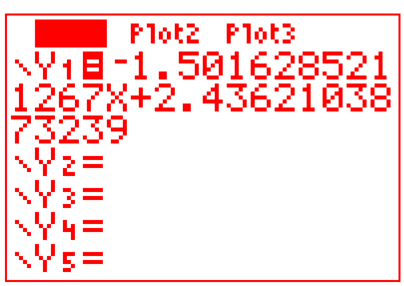
$$y-y_1=m(x-x_1)$$
$$\boxed{y-5=1(x-2)}$$

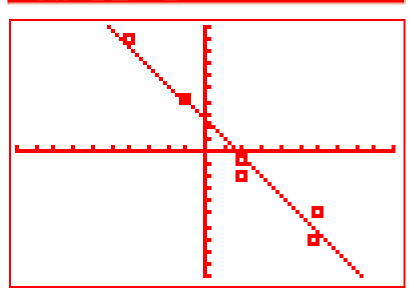
15. Perform a linear regression on this data and show the equation of the best-fit line along with a sketch of the line and scatter-plot. Give the correlation coefficient.

x	y
–4.0	9.0
–1.1	4.2
2.0	–0.5
1.9	–2.0
6.0	–5.0
5.8	–7.0







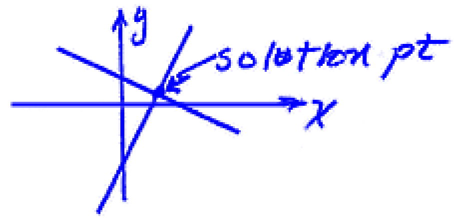


$r = -.98333$

Unit 1: Lesson 05 — Review: solving linear systems

Solving the following "system" of linear equations is equivalent to finding the (x, y) **intersection point** of the two lines:

$y = 3x - 6$
$y = -x + 4$



The following example demonstrates the solution of a linear system of equations using the method of **elimination**.

Example 1: $x + y = -1$
$3x - 2y = 11$

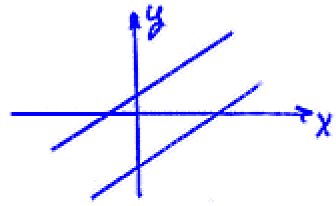
$$2(x + y) = -1(2) \longrightarrow 2x + \cancel{2y} = -2$$
$$3x - 2y = 11 \longrightarrow 3x - \cancel{2y} = 11$$
$$5x = 9$$
$$x = \boxed{\tfrac{9}{5}}$$
$$x + y = -1$$
$$\tfrac{9}{5} + y = -1$$
$$y = -1 - \tfrac{9}{5} = -\tfrac{5}{5} - \tfrac{9}{5} = \boxed{-\tfrac{14}{5}}$$

The following example demonstrates the solution of a linear system of equations using the method of **substitution**.

Example 2: $x + y = -1$
$3x - 2y = 11$

$$y = -x - 1$$
$$3x - 2(-x - 1) = 11$$
$$3x + 2x + 2 = 11$$
$$5x = 9$$
$$x = \boxed{\tfrac{9}{5}}$$
$$x + y = -1$$
$$\tfrac{9}{5} + y = -1$$
$$y = -\tfrac{9}{5} - 1$$
$$y = \boxed{-\tfrac{14}{5}}$$

When algebraically solving a linear system, equations are sometimes produced that are obviously not true. This means the equations of the system are **inconsistent** and **independent** and there is no solution. This happens when the two **lines are parallel and separated**, and as a result there is no intersection point.



No solution

Notice in the following example that a nonsense equation is produced, thus indicating no solution.

Example 3: $4x - y = -2$
$12x - 3y = 5$

$-3(4x - y) = -2(-3) \rightarrow -\cancel{12x} + \cancel{3y} = +6$

$12x - 3y = 5 \rightarrow \cancel{12x} - \cancel{3y} = 5$

$0 \neq 11$

$\boxed{\text{No solution}}$

Occasionally during the solution of a linear system a true equation is produced that does not lead to a solution. In this case the equations are **consistent** and **dependent**. The two lines are **right on top of each other** producing an infinite number of intersection points along the lines.

This is illustrated in the following example.

Example 4: $y = 2x - 5$ and $-6x + 3y = -15$

$y = 2x - 5 \quad -6x + 3(2x - 5) = -15$

$-\cancel{6x} + \cancel{6x} - 15 = -15$

$-15 = -15$

$\boxed{\text{Infinite \# of solution pts along the line } y = 2x - 5}$

See **Enrichment Topic I** for how to solve three equations for three variables.

See **Enrichment Topic J** for how to solve quadratic systems of equations.

Assignment:

In the following problems, solve for the intersection point(s) of the systems of linear equations using the **substitution** method:

1. $y = 8x - 11 \quad and \quad x - 2y = 1$

$x - 2(8x - 11) = 1$
$x - 16x + 22 = 1$
$-15x + 22 = 1$
$-15x = 1 - 22$
$x = \frac{-21}{-15} = \boxed{\frac{7}{5}}$
$y = 8\left(\frac{7}{5}\right) - 11$
$y = \frac{56}{5} - \frac{11}{1}\left(\frac{5}{5}\right) = \frac{56}{5} - \frac{55}{5}$
$y = \boxed{\frac{1}{5}}$

2. $x = -(1/3)y + 4$ and $y = -3x + 1$

$y = -3\left(-\frac{1}{3}y + 4\right) + 1$
$\cancel{y} = \cancel{y} - 12 + 1$
$0 \neq -11$

$\boxed{\text{No solution, Parallel, separated lines,}}$

3. $x + y = 6 \quad \text{and} \quad 2y + 2x = 0$

$x = 6 - y$
$2y + 2(6 - y) = 0$
$2y + 12 - 2y = 0$
$0y + 12 = 0$
$12 \neq 0$

$\boxed{\text{No Solution}}$

4. $\left(\frac{1}{2}\right)y = \left(\frac{1}{3}\right)x + 2$ and $x - y - 1 = 0$

$6\left(\frac{1}{2}\right)y = 6\left(\frac{1}{3}\right)x + 6 \cdot 2$
$3y = 2x + 12$
$x = y + 1$
$3y = 2(y + 1) + 12$
$3y = 2y + 2 + 12$
$3y = 2y + 14$
$3y - 2y = 14$
$\boxed{y = 14}$
$x = 14 + 1$
$\boxed{x = 15}$

5. $x = 1 \quad \text{and} \quad y = x + 2$

$\boxed{x = 1}$
$y = 1 + 2$
$\boxed{y = 3}$

6. $18x - .5y = 7 \quad and \quad y = 8$

$18x - .5(8) = 7$
$18x - 4 = 7$
$18x = 7 + 4$
$18x = 11$
$\boxed{x = \frac{11}{18}}$ $\boxed{y = 8}$

In the following problems, solve for the intersection point(s) of the systems of linear equations using the **elimination** method:

7. $x + y = 5$ and $-x + 11y = 0$

$$x+y=5$$
$$-x+11y=0$$
$$12y=5$$
$$\boxed{y=\frac{5}{12}}$$

$$-x+11\left(\frac{5}{12}\right)=0$$
$$-x+\frac{55}{12}=0$$
$$-x=-\frac{55}{12}$$
$$\boxed{x=\frac{55}{12}}$$

8. $4 = x + 2y$ and $-2y + x = 1$

$$-4=-(x+2y)$$
$$-4=-x-2y$$
$$2y+x=4$$

$$-\cancel{2y}+x=1$$
$$\cancel{2y}+x=4$$
$$2x=5$$
$$\boxed{x=\frac{5}{2}}$$

$$2y+\frac{5}{2}=4$$
$$2y=4-\frac{5}{2}$$
$$y=2-\frac{5}{4}=\frac{8}{4}-\frac{5}{4}$$
$$\boxed{y=\frac{3}{4}}$$

9. $-3x + y = 1$ and $18x - 6y = 1$

$$6(-3x+y)=1\cdot 6 \rightarrow -\cancel{18x}+\cancel{6y}=6$$
$$18x-6y=1 \rightarrow \cancel{18x}-\cancel{6y}=1$$
$$0 \neq 7$$

$\boxed{\text{No solution. Lines are parallel + separate}}$

10. $x + y + 1 = 0$ and $2x - 9y = 3$

$$-2(x+y)=-1(2) \rightarrow -\cancel{2x}-2y=+2$$
$$\cancel{2x}-9y=3$$
$$-11y=5$$
$$y=\boxed{-\frac{5}{11}}$$

$$x+y=-1$$
$$x-\frac{5}{11}=-1$$
$$x=\frac{5}{11}-1$$
$$=\frac{5}{11}-\frac{11}{11}=\boxed{-\frac{6}{11}}$$

11. $x = 2$ and $x - 5y = 2$

$$x-5y=2$$
$$-x \quad = -2$$
$$-5y=0$$

$\boxed{x=2}$ $\boxed{y=0}$

12. $y = 5x - 6$ and $-15x + 3y = -18$

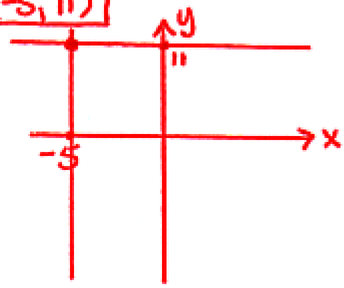
$$3(5x-y)=6(3) \rightarrow \cancel{15x}-\cancel{3y}=\cancel{18}$$
$$-15x+3y=-18 \rightarrow -\cancel{15x}+\cancel{3y}=-\cancel{18}$$
$$0=0$$

$\boxed{\text{Infinite number of solution points along the line } y=5x-6}$

In the following problems, use any technique. Hint: Make a sketch of the two lines in which case you might be able to "see" the answer. This is called solving by "inspection."

13. $x = -5 \quad and \quad y = 11$

$\boxed{(-5, 11)}$



14. $x + 2 = 0 \quad and \quad 2(8 - y) = 0$

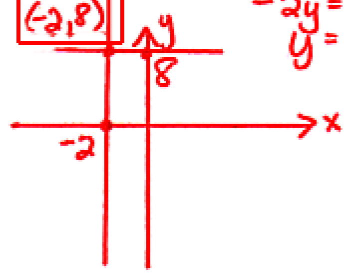
$x = -2$

$16 - 2y = 0$

$-2y = -16$

$y = 8$

$\boxed{(-2, 8)}$



Unit 01: Review

1. Multiply (3x – 9)(5x + 2)

F O I L

$= 15x^2 + 6x - 45x - 18$

$= \boxed{15x^2 - 39x - 18}$

2. Factor $5x^2 - 45$

$= 5(x^2 - 9)$

$= \boxed{5(x-3)(x+3)}$

3. Factor $x^2 + 4x - 21$

$= \boxed{(x+7)(x-3)}$

4. Factor $8x^3 - 27$

$(A)^3 - (B)^3$

$= (2x)^3 - (3)^3$

$= \boxed{(2x-3)(4x^2+6x+9)}$

$(A-B)(A^2+AB+B^2)$

5. Simplify $\frac{x^2-16}{x-3}\frac{2x-6}{x^2+10x+24}$

$= \frac{(x-4)\cancel{(x+4)}\,2\cancel{(x-3)}}{\cancel{(x-3)}(x+6)\cancel{(x+4)}}$

$= \boxed{\frac{2(x-4)}{x+6}}$

6. Combine into a single fraction and simplify: $\frac{x^3-1}{x^2+x+1}+\frac{16}{2x^2+18x-20}$

$$=\frac{(x-1)\cancel{(x^2+x+1)}}{\cancel{x^2+x+1}}+\frac{16}{2(x^2+9x-10)}$$

$$=\frac{x-1}{1}+\frac{16}{2(x+10)(x-1)}=$$

$$=\frac{(x-1)\,2(x+10)(x-1)}{2(x+10)(x-1)}+\frac{\cancel{16}\;8}{2(x+10)(x-1)}$$

$$=\boxed{\frac{(x+10)(x-1)^2+8}{(x+10)(x-1)}}$$

7. Simplify $\dfrac{\frac{5}{x-5}}{\frac{7}{2x}+\frac{x+1}{x^2-25}}$

$$=\frac{\frac{5}{x-5}}{\frac{7}{2x}+\frac{x+1}{(x-5)(x+5)}}\cdot\frac{2x(x-5)(x+5)}{2x(x-5)(x+5)}$$

$$=\frac{\frac{5(2x)\cancel{(x-5)}(x+5)}{\cancel{x-5}}}{\frac{7\cdot\cancel{2x}(x-5)(x+5)}{\cancel{2x}}+\frac{(x+1)2x\cancel{(x-5)}\cancel{(x+5)}}{\cancel{(x-5)}\cancel{(x+5)}}}$$

$$=\frac{10x(x+5)}{7(x^2-25)+2x^2+2x}=\boxed{\frac{10x(x+5)}{9x^2+2x-175}}$$

8. Expand $(2x-8)^2$

$$(a-b)^2=a^2-2ab+b^2$$

$$(2x-8)^2=\boxed{4x^2-32x+64}$$

9. Expand $2(x+9y)^2$

$$2(a+b)^2=2(a^2+2ab+b^2)$$

$$=2(x+9y)^2=2(x^2+18xy+81y^2)$$

$$=\boxed{2x^2+36xy+162y^2}$$

10. Solve $\frac{2}{x-6} + 3 = 11$

$\frac{2}{\cancel{x-6}} \cdot \frac{\cancel{x-6}}{1} + 3(x-6) = 11(x-6)$

$2 + 3x - 18 = 11x - 66$

$3x - 11x = -2 + 18 - 66$

$-8x = -50$

$\boxed{x = \frac{50}{8} = \frac{25}{4}}$

11. Solve $2x^2 + 12x + 10 = 0$

$2(x^2 + 6x + 5) = 0$

$2(x+5)(x+1) = 0$

$x+5=0 \quad x+1=0$

$\boxed{x=-5 \quad x=-1}$

12. Solve $3x^2 - 7x + 6x - 14 = 0$

$[x(3x-7) + 2(3x-7)] = 0$

$(3x-7)[x+2] = 0$

$3x-7=0$

$3x = 7$

$x = \boxed{\frac{7}{3}}$

$x+2=0$

$x = \boxed{-2}$

13. Solve $t^3 - 36t = 0$

$t(t^2-36)=0$

$t(t-6)(t+6)=0$

$t=\boxed{0} \quad t-6=0 \quad t+6=0$

$t=\boxed{6} \quad t=\boxed{-6}$

14. Solve $\sqrt{x+3} = -4$

$(\sqrt{x+3})^2 = (-4)^2$

$x+3 = 16$

$x = 16-3 = 13$

check:

$\sqrt{13+3} = -4$

$\sqrt{16} = -4$

$4 \neq -4$ $\boxed{\text{No solution}}$

15. Find the equation (in slope-intercept form) of the line passing through (5, 1) and (–6, 2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-6-5} = \frac{1}{-11}$$

$y = mx + b$

$y = -\frac{1}{11}x + b$

sub in (5,1) → $1 = -\frac{1}{11}(5) + b$

$1 + \frac{5}{11} = b$

$\frac{16}{11} = b$

$y = mx + b$

$\boxed{y = -\frac{1}{11}x + \frac{16}{11}}$

---

16. Find the equation (in point-slope form) of the line perpendicular to the line given by x + 5y = 19 and passing through (11, 8).

$x + 5y = 19$

$5y = -x + 19$

$y = -\frac{1}{5}x + \frac{19}{5}$

$m = -\frac{1}{5};\ m_{\perp} = 5$

$(x_1, y_1) = (11, 8)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 8 = +5(x - 11)}$

---

17. Find the equation (in intercept form) of the line passing through (0, 8) and having slope –3.

$y - y_1 = m(x - x_1)$

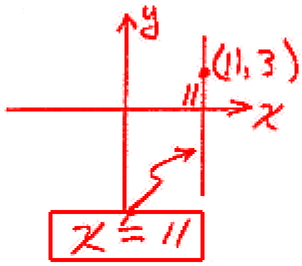
$y - 8 = -3(x - 0)$

$y - 8 = -3x$

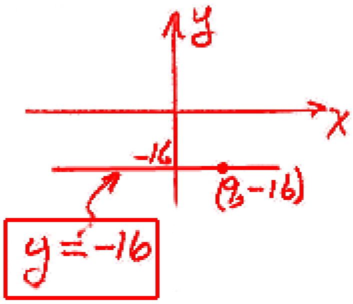
$3x + y = 8$ we need 1 here, so ÷ by 8

$\frac{3x}{8} + \frac{y}{8} = 1 \longrightarrow \boxed{\frac{x}{8/3} + \frac{y}{8} = 1}$

18. What is the equation of the line perpendicular to the x-axis and passing through (11, 3)?



19. What is the equation of the horizontal line passing through (9, –16)?



20. Find the intersection point of these two lines using substitution:
x + y = 18 and 4x – y = 12

$y = -x + 18$

$y = -6 + 18$

$y = \boxed{12}$

$4x - (y) = 12$

$4x - (-x + 18) = 12$

$4x + x - 18 = 12$

$5x = 18 + 12$

$5x = 30$

$x = \boxed{6}$

21. Find the intersection point of these two lines using elimination:
x + y = 18 and 4x – y = 11

$x + \cancel{y} = 18$

$4x - \cancel{y} = 11$

$5x = 29$

$x = \boxed{\frac{29}{5}}$

$x + y = 18$

$\frac{29}{5} + y = 18$

$y = 18 - \frac{29}{5}$

$y = \frac{18}{1}\frac{5}{5} - \frac{29}{5} = \boxed{\frac{61}{5}}$

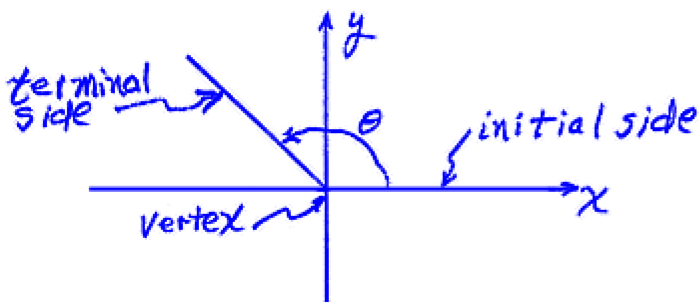
Pre Calculus, Unit 2

Basic Trigonometry

Unit 2: Lesson 01 Angle conventions, definitions of the six trig functions

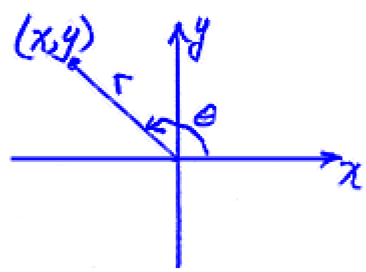
The standard position of an angle throughout all mathematics (not just trig) has its **vertex at the origin** and **initial side on the positive x-axis**. The other side is called the **terminal side**.

Note that **positive angles** rotate **counter clock-wise**.



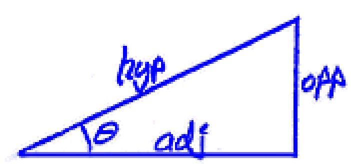
Three definitions of the sine (abbreviated sin), cosine (abbreviated cos), and tangent (abbreviated tan) of an angle (we will call our angle by the Greek letter theta, θ):

First definition (x, y, & r):
Begin with an angle in standard position and with the terminal side in the 2nd quadrant (it could be in any quadrant).



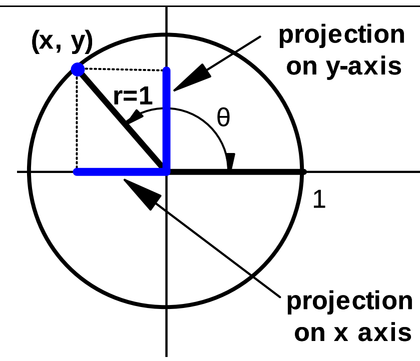
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Second definition (opp, adj, hyp):
(opposite, adjacent, hypotenuse)
Let θ be one of the acute angles of a right triangle.



$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Third definition (projection): Draw an angle in standard position with a circle centered at the origin with radius 1 (called a unit circle).



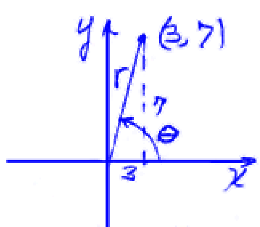
$\sin\theta =$ projection on y axis
$\cos\theta =$ projection on x-axis
$\tan\theta \rightarrow$ No proj definition

Defining the other three trig functions:

The basic fact to remember here is that cosecant (csc), secant (sec), and cotangent (cot) are **reciprocals** respectively of sin, cos, and tan.

$\sec\theta = \frac{r}{x} = \frac{hyp}{adj} \longrightarrow$ reciprocal of cos
$\csc\theta = \frac{r}{y} = \frac{hyp}{opp} \longrightarrow$ " " sin
$\cot\theta = \frac{x}{y} = \frac{adj}{opp} \longrightarrow$ " " tan

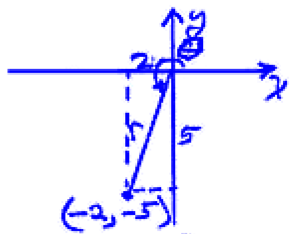
Example 1: Draw the angle, θ, in standard position where (3, 7) lies on the terminal side. Find the values of all six trig functions.



$r^2 = 3^2 + 7^2$
$r = \sqrt{9+49} = \sqrt{58}$

$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{58}}$
$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{58}}$
$\tan\theta = \frac{y}{x} = \frac{7}{3}$
$\cot\theta = \frac{x}{y} = \frac{3}{7}$
$\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$
$\csc\theta = \frac{r}{y} = \frac{\sqrt{58}}{7}$

Example 2: Draw the angle, θ, in standard position where (–2, –5) lies on the terminal side. Find the values of all six trig functions.



$r^2 = 2^2 + 5^2$

$r = \sqrt{4+25} = \sqrt{29}$

$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}}$

$\tan\theta = \frac{y}{x} = \frac{-5}{-2} = \frac{5}{2}$

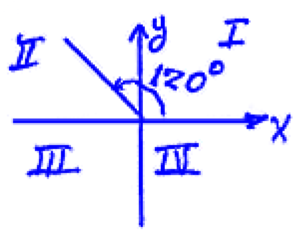
$\cot\theta = \frac{x}{y} = \frac{-2}{-5} = \frac{2}{5}$

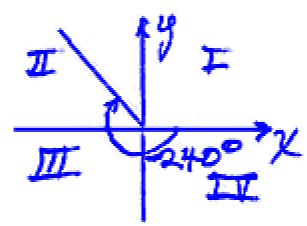
$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-5}$

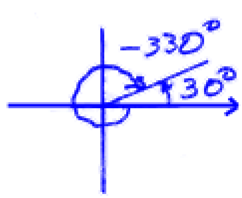
Co-terminal angles have a common terminal side. Naturally, the values of all six trig functions of an angle are the same as those angles with which it is co-terminal.

Example 3: Draw a 120^o angle in standard position. Then draw a negative angle in standard position that is co-terminal with the 120^o angle. Identify the four quadrants.





Example 4: If sin(30^o) = .5 what is csc(–330^o)?



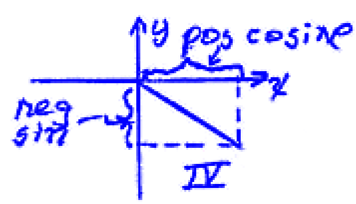
These angles are co-terminal so they have the same sin.

$\sin(-330) = .5$

$\csc(-330) = \frac{1}{.5} = \boxed{2}$

reciprocal

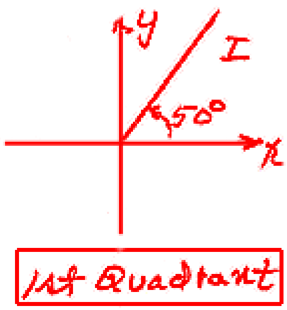
Example 5: If the sine of an angle is negative and the cosine is positive, in what quadrant is the angle?



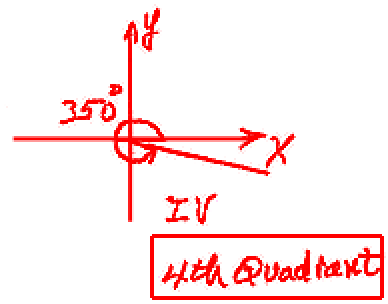
4th quadrant

Assignment: In problems 1-4, draw the angle in standard position and identify the quadrant of the terminal side.

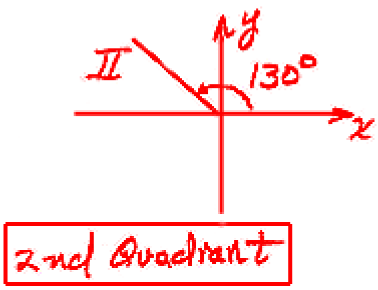
1. 50^{o}



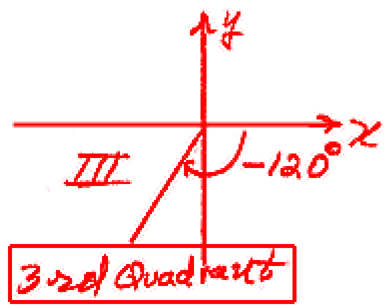
2. 350^{o}



3. 130^{o}



4. -120^{o}



5. Define all six trig functions in terms of x, y, and r.

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{r}{y}$

6. Define all six trig functions in terms of opp, adj, and hyp.

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

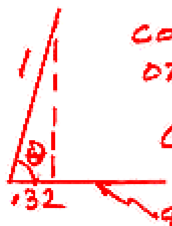
$\tan\theta = \frac{opp}{adj}$

$\cot\theta = \frac{adj}{opp}$

$\sec\theta = \frac{hyp}{adj}$

$\csc\theta = \frac{hyp}{opp}$

7. A straight stick protrudes out of the ground exactly 1 meter. As a result of the sun being directly overhead, the shadow on the ground of the slightly leaning stick is exactly .32 meters. What is the cosine of the angle between the stick and the horizontal ground?

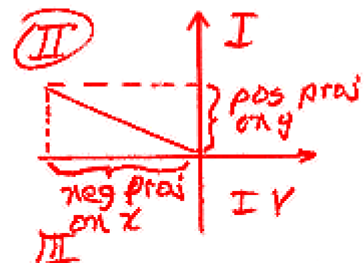


cosine is projection on the horizontal

$\cos\theta = \boxed{.32}$

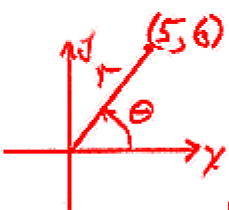
ground

8. Draw an angle in standard position in which the projection of the terminal side is positive on the y-axis and negative on the x-axis. In which quadrant is the terminal side?



$\boxed{\text{2nd Quadrant}}$

9. Draw an angle, θ, in standard position whose terminal side includes the point (5, 6). Find sin θ, cos θ, and tan θ.



$r^2 = 5^2 + 6^2$

$r = \sqrt{25+36}$

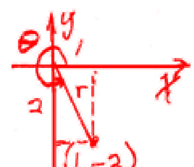
$r = \sqrt{61}$

$\sin\theta = \frac{y}{r} = \frac{6}{\sqrt{61}}$

$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{61}}$

$\tan\theta = \frac{y}{x} = \frac{6}{5}$

10. Draw an angle, θ, in standard position whose terminal side includes the point (1, –2). Find sin θ, cos θ, and tan θ.



$r^2 = 1^2 + 2^2$

$r = \sqrt{1+4}$

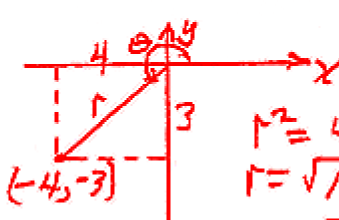
$r = \sqrt{5}$

$\sin\theta = \frac{y}{r} = \boxed{\frac{-2}{\sqrt{5}}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{1}{\sqrt{5}}}$

$\tan\theta = \frac{y}{x} = \frac{-2}{1} = \boxed{-2}$

11. Draw an angle, θ, in standard position whose terminal side includes the point (–4, –3). Find csc θ, sec θ, and cot θ.



$r^2 = 4^2 + 3^2$

$r = \sqrt{16+9}$

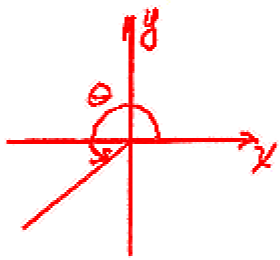
$= 5$

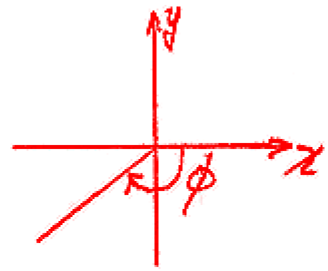
$\csc\theta = \frac{r}{y} = \frac{5}{-3} = \boxed{-\frac{5}{3}}$

$\sec\theta = \frac{r}{x} = \frac{5}{-4} = \boxed{-\frac{5}{4}}$

$\cot\theta = \frac{x}{y} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$

12. Draw two angles in standard position (but each on its own coordinate system) that are co-terminal. The terminal side of both angles should be in the third quadrant.





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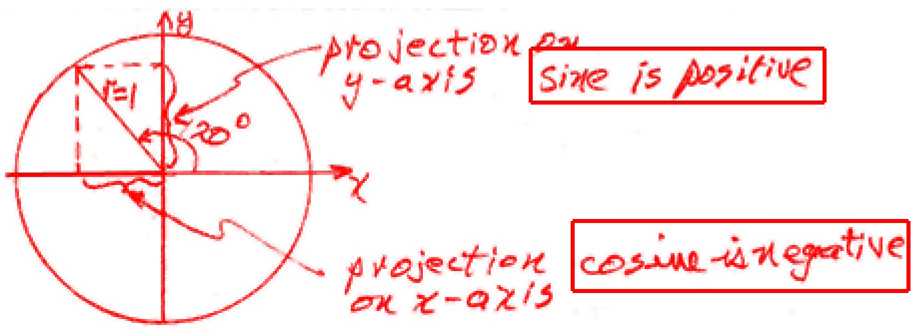
13. Inside a circle of radius 1, draw a 120^{o} angle in standard position. Show the projection of this angle on the two axes. What conclusion can be reached regarding the sign of the sine and cosine of 120^{o}?



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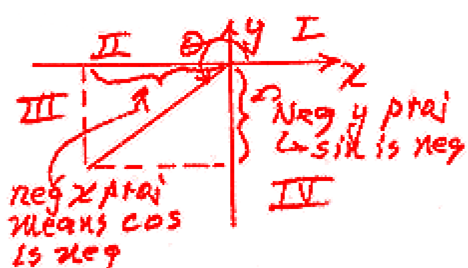
14. If tan θ = 11.2 what would be the value of cot θ?

tanθ + cotθ are recip. of each other

cotθ = $\boxed{\frac{1}{11.2} \approx .0892857}$

15. If the sine and cosine of an angle are both negative, in what quadrant is the angle? Draw the angle.



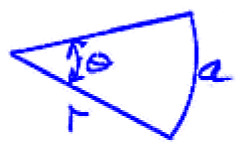
$\boxed{\text{Third Quadrant}}$

Unit 2: Lesson 02 Angle units; degrees(minutes & seconds), radians

Here are two units of measure for angles with which we should be familiar.

- Degrees … 360° in a full circle.
- Radians … 2π(radians) ≈ 6.28 in a full circle

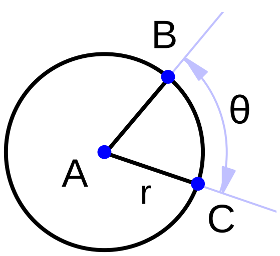
The definition of radians comes from a drawing and an associated formula:



$$\theta = \frac{a}{r}$$

In the drawing above, *r* is the radius, θ is the **angle in radians**, and, *a* is the arc length.

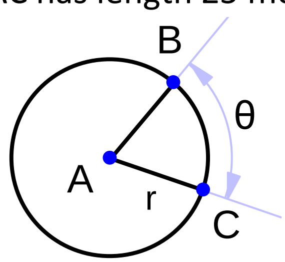
Example 1: In the drawing below the arc length BC is 20 ft and the radius is 18 ft. Find the measure of θ in radians.



$$\theta = \frac{BC}{r}$$

$$\theta = \frac{20}{18} = \boxed{1.\overline{1}\text{ radians}}$$

Example 2: In the drawing below find the length of the arc BC when the angle θ is π/3 radians and the line segment AC has length 25 meters.



$$\theta = \frac{BC}{r}$$

$$\frac{\pi}{3} = \frac{BC}{25}$$

$$3BC = 25\pi$$

$$BC = \frac{25\pi}{3} = \boxed{26.1799\,m}$$

Conversions between degrees and radians (**these must be memorized**):

$90^o = \pi/2$ radians $\qquad$ $180^o = \pi$ radians

$270^o = 3\,\pi/2$ radians $\qquad$ $360^o = 2\pi$ radians

For non-special angles we will use the conversion formula: $\frac{deg}{rad} = \frac{deg}{rad}$

For the left side of the equation we will use the correspondence of 180^o to π radians, so the formula becomes: $\frac{180}{\pi} = \frac{deg}{rad}$

Example 3: Convert 38^o into radians.

$$\frac{180}{\pi} = \frac{38}{\theta}$$

$$180\theta = 38\pi$$

$$\theta = \frac{38\pi}{180} = \boxed{.663225 \text{ rad.}}$$

Example 4: Convert $2\pi/7$ radians to degrees.

$$\frac{180}{\pi} = \frac{\theta}{2\pi/7}$$

$$\cancel{\pi}\theta = 180(2\cancel{\pi}/7)$$

$$\theta = \frac{360}{7} = \boxed{51.42857^o}$$

Each degree can be broken up into **minutes** and **seconds**. These are **angular measures**, **not time**, although the analogy is certainly there.

1 degree = 60 minutes

1 minute = 60 seconds

Example 5: Express an angle of 14 degrees, 57 minutes and 32 seconds in "shorthand" notation.

$14^o\,57'\,32''$

Example 6: Convert $123^o\,5'\,49''$ into decimal form.

$$49'' = \frac{49}{60}{}' = .81\bar{6}'$$

$$5' + .81\bar{6}' = 5.81\bar{6}'$$

$$5.81\bar{6}' = \frac{5.81\bar{6}}{60}{}^o = .0969\bar{4}^o$$

$$123^o + .0969\bar{4}^o$$

$$= \boxed{123.09694^o}$$

Example 7: Convert 238.458° into degrees, minutes, and seconds.

$.458^\circ = .458(60)' = 27.48' = 27' + .48'$

$.48' = .48(60)'' = 28.8''$

238.458°

$= \boxed{238^\circ\ 27'\ 28.8''}$

See **Calculator Appendix S** (and an associated video) for how to convert between degrees, minutes, seconds, and radians on a TI graphing calculator.

Assignment: In all conversions to degrees, do so in decimal form unless otherwise specifically instructed to use degrees, minutes, and seconds.

1. Convert 122° to radians.

$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{122}{\theta}$

$180\theta = 122\pi$

$\theta = \frac{122\pi}{180} = \boxed{2.1293 \text{ radians}}$

2. Convert 7π/12 radians to degrees.

$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{\theta}{7\pi/12}$

$\pi\theta = 180(7\pi/12)$

$\theta = \boxed{105^\circ}$

3. Convert – π/6 radians to degrees.

$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{\theta}{-\pi/6}$

$\pi\theta = -\pi 180/6$

$\theta = \boxed{-30^\circ}$

4. Convert 72° to radians.

$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{72}{\theta}$

$180\theta = 72\pi$

$\theta = \frac{72\pi}{180} = \boxed{1.2566 \text{ radians}}$

5. Convert 41 seconds into minutes.

$41'' = \frac{41}{60} = \boxed{.68\overline{3}'}$

6. Convert 19 minutes into degrees.

$19' = \frac{19}{60} = \boxed{.31\overline{6}^\circ}$

7. Convert π/2 into degrees.

Could use $\frac{deg}{rad} = \frac{deg}{rad}$

Memory work

$\pi/2 \text{ rad} = \boxed{90^\circ}$

8. Convert 270° into radians.

Memorize

$270^\circ = \boxed{\frac{3\pi}{2} \text{ rad}}$

9. Approximately how many degrees is one radian?

$$\frac{deg}{rad} = \frac{deg}{rad}$$

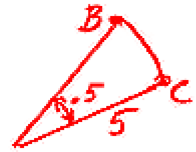
$$\frac{180}{\pi} = \frac{\theta}{1}$$

$$\pi\theta = 180$$

$$\theta = \frac{180}{\pi} = 57.29577^\circ$$

$$\approx \boxed{57^\circ}$$

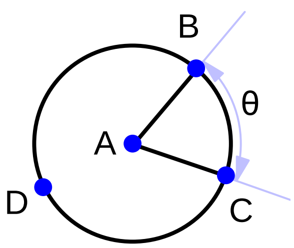
10. Draw a central angle of .5 radians with a radius of 5 out to the arc BC. What is the length of arc BC?



$$\theta = \frac{a}{r} \qquad BC = 5(.5)$$

$$.5 = \frac{BC}{5} \qquad BC = \boxed{2.5}$$

11. In the drawing below find the length of arc BDC when θ = 60° and AC = 22.



$$360 - 60 = 300^\circ$$

$$\frac{180}{\pi} = \frac{300}{\varnothing}$$

$$\varnothing = \boxed{5.23598 \text{ rad.}}$$

$$\varnothing = \frac{BDC}{22}$$

$$5.23598 = \frac{BDC}{22}$$

$$BDC = 22(5.23598)$$

$$BDC = \boxed{115.19156}$$

12. Convert 187.926° into degrees, minutes, and seconds.

$$.926^\circ = .926(60) = 55.56'$$

$$.56' = .56(60) = 33.6''$$

$$187.926^\circ = \boxed{187^\circ 55' 33.6''}$$

13. Convert 15° 11' 46'' into decimal degrees.

$$46'' = \frac{46}{60} = .7\bar{6}'$$

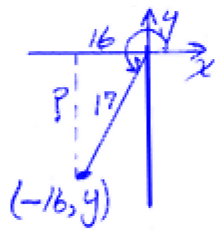
$$11' + .7\bar{6}' = 11.7\bar{6}' = 11.7\bar{6}/60 = .196\bar{1}^\circ$$

$$15^\circ + .196\bar{1}^\circ = \boxed{15.196\bar{1}^\circ}$$

Unit 2: Lesson 03 Given one trig ratio, find the others

In the following examples, point P (x, y) is on the terminal side of an angle in standard position. The distance from the vertex to P is *r*. Find the one of *x*, *y*, and *r* that is missing.

Example 1: x = –16, r = 17, P(–16, y) is in the third quadrant.



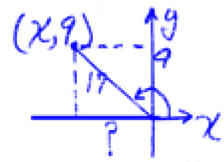
$r^2 = x^2 + y^2$

$y^2 = r^2 - x^2$

$y = \pm\sqrt{17^2 - (16)^2} = \boxed{-5.74456}$

choose − because of 3rd quad.

Example 2: y = 9, r = 17, P(x, 9) is in the second quadrant.



$x^2 + y^2 = r^2$

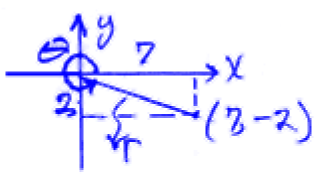
$x^2 = r^2 - y^2$

$x = \pm\sqrt{17^2 - 9^2}$

$x = \boxed{-14.4222}$

choose − sign because of 2nd quad

Example 3: x = 7, y = –2



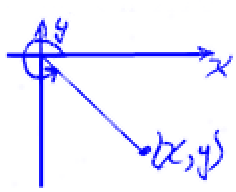
$r^2 = x^2 + y^2$

$r = \sqrt{7^2 + (-2)^2}$

$r = \sqrt{53} = \boxed{7.280109}$

In the following examples, draw an angle in standard position that satisfies the given conditions and then find the other five trig functions.

Example 4: cos θ = 5/13, θ in the fourth quadrant



$\cos\theta = \frac{5}{13} = \frac{x}{r}$

$x = 5 \quad r = 13$

$y = \pm\sqrt{r^2 - x^2}$

$y = \pm\sqrt{13^2 - 5^2}$

$y = -12$

$\sin\theta = \frac{y}{r} = \frac{-12}{13}$

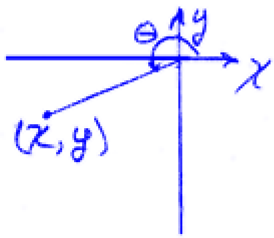
$\tan\theta = \frac{y}{x} = \frac{-12}{5}$

$\cot\theta = \frac{x}{y} = \frac{-5}{12}$

$\sec\theta = \frac{r}{x} = \frac{13}{5}$

$\csc\theta = \frac{r}{y} = \frac{-13}{12}$

Example 5: tan θ = 8/15, θ in the third quadrant



$\tan\theta = \frac{8}{15} = \frac{-8}{-15} = \frac{y}{x}$

$x = -15 \quad y = -8$

$r^2 = x^2 + y^2$

$r = \sqrt{(-15)^2 + (-8)^2}$

$r = 17$

$\sin\theta = \frac{y}{r} = \frac{-8}{17}$

$\cos\theta = \frac{x}{r} = \frac{-15}{17}$

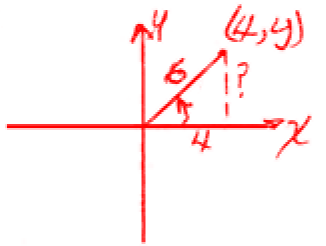
$\cot\theta = \frac{x}{y} = \frac{-15}{-8} = \frac{15}{8}$

$\sec\theta = \frac{r}{x} = \frac{17}{-15}$

$\csc\theta = \frac{r}{y} = \frac{17}{-8}$

Assignment: In problems 1-4, point P (x, y) is on the terminal side of an angle in standard position. The distance from the vertex to P is *r*. Find the one of *x*, *y*, and *r* that is missing.

1. x = 4, r = 6, P(4, y) in the first quadrant



$x^2+y^2=r^2$

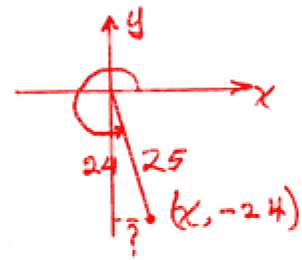
$y^2=r^2-x^2$

$y=\pm\sqrt{6^2-4^2}$

+ because of 1st quad $=+\sqrt{20}$

$=\boxed{4.472135}$

2. y = –24, r = 25, P(x, –24) in the fourth quadrant



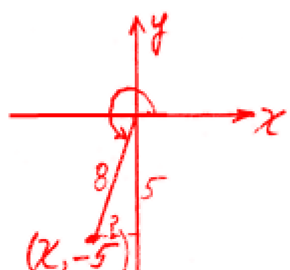
$x^2+y^2=r^2$

$x=\pm\sqrt{r^2-y^2}$

$x=\pm\sqrt{25^2-(-24)^2}$

$x=\boxed{+7}$ + because of 4th quad

3. y = –5, r = 8, P(x, –5) in the third quadrant



$x^2+y^2=r^2$

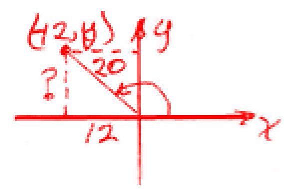
$x=\pm\sqrt{r^2-y^2}$

$x=\pm\sqrt{8^2-(-5)^2}$

$x=\boxed{-6.244997}$

Neg because of 3rd quad

4. x = –12, r = 20, P(–12, y) in the second quadrant



$x^2+y^2=r^2$

$y=\pm\sqrt{r^2-x^2}$

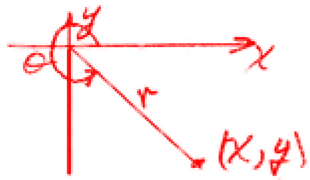
$y=+\sqrt{20^2-12^2}$

$y=\boxed{+16}$

+ because of 2nd quad.

In the following problems, draw an angle in standard position that satisfies the given conditions and then find the remaining unknown trig functions. When a point P is given it is assumed to be on the terminal side of the angle.

5. tan θ = –3/4, θ in the fourth quadrant



$\tan\theta = -\frac{3}{4} = \frac{-3}{4} = \frac{y}{x}$

$x = 4\ ;\ y = -3$

$r = \sqrt{x^2+y^2} = \sqrt{4^2+(-3)^2} = 5$

$\sin\theta = \frac{y}{r} = \boxed{\frac{-3}{5}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{4}{5}}$

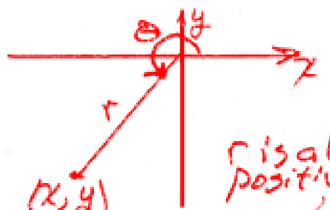
$\cot\theta = \frac{x}{y} = \boxed{-\frac{4}{3}}$

$\sec\theta = \frac{r}{x} = \boxed{\frac{5}{4}}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{5}{-3}}$

---

6. sec θ = –17/8, θ in the third quadrant



r is always positive

$\sec\theta = \frac{r}{x} = \frac{17}{-8}$ $x = -8$, $r = 17$

$y = \pm\sqrt{r^2 - x^2} = \sqrt{17^2-(-8)^2}$

$= -15$ neg because of 3rd quad

$\sin\theta = \frac{y}{r} = \boxed{\frac{-15}{17}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{-8}{17}}$

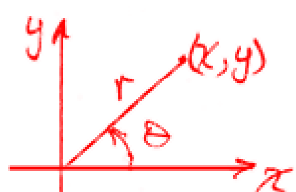
$\tan\theta = \frac{y}{x} = \boxed{\frac{-15}{-8} = \frac{15}{8}}$

$\cot\theta = \frac{x}{y} = \boxed{\frac{-8}{-15} = \frac{8}{15}}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{17}{-15}}$

---

7. cot θ = 5/3, θ in the first quadrant



$\cot\theta = \frac{x}{y} = \frac{5}{3}$

$x = 5\ ;\ y = 3$

$r = \sqrt{x^2+y^2} = \sqrt{25+9} = \sqrt{34}$

$\sin\theta = \frac{y}{r} = \boxed{\frac{3}{\sqrt{34}}}$

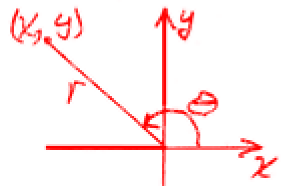
$\cos\theta = \frac{x}{r} = \boxed{\frac{5}{\sqrt{34}}}$

$\tan\theta = \frac{y}{x} = \boxed{\frac{3}{5}}$

$\sec\theta = \frac{r}{x} = \boxed{\frac{\sqrt{34}}{5}}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{\sqrt{34}}{3}}$

8. sin θ = 5/6, θ in the 2nd quadrant



$\sin\theta = \frac{5}{6} = \frac{y}{r}$

$y = 5 \quad r = 6$

$x = \pm\sqrt{r^2 - y^2} = \pm\sqrt{36 - 25}$

− because of 2nd Quad $= -\sqrt{11}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{-\sqrt{11}}{6}}$

$\tan\theta = \frac{y}{x} = \boxed{\frac{5}{-\sqrt{11}}}$

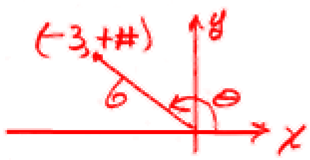
$\cot\theta = \frac{x}{y} = \boxed{\frac{-\sqrt{11}}{5}}$

$\sec\theta = \frac{r}{x} = \boxed{\frac{6}{-\sqrt{11}}}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{6}{5}}$

---

9. P(–3, y), y > 0, radius of 6



$r = 6, \; x = -3$

$x^2 + y^2 = r^2$

$y = \pm\sqrt{r^2 - x^2}$

$y = +\sqrt{36 - (-3)^2}$

$y = \sqrt{27}$ pos because of 2nd Quad

$\sin\theta = \frac{y}{r} = \boxed{\frac{\sqrt{27}}{6}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{-3}{6} = -\frac{1}{2}}$

$\tan\theta = \frac{y}{x} = \boxed{\frac{\sqrt{27}}{-3}}$

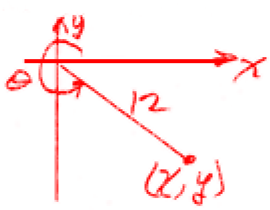
$\cot\theta = \frac{x}{y} = \boxed{\frac{-3}{\sqrt{27}}}$

$\sec\theta = \frac{r}{x} = \boxed{\frac{6}{-3} = -2}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{6}{\sqrt{27}}}$

---

10. P(x, –7), x > 0, radius of 12



$x^2 + y^2 = r^2 \; ; x = \pm\sqrt{r^2 - y^2}$

$x = \pm\sqrt{12^2 - (-7)^2} = \sqrt{95}$

$x = \sqrt{95} \; ; y = -7 \; ; r = 12$

$\sin\theta = \frac{y}{r} = \boxed{\frac{-7}{12}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{\sqrt{95}}{12}}$

$\tan\theta = \frac{y}{x} = \boxed{\frac{-7}{\sqrt{95}}}$

$\cot\theta = \frac{x}{y} = \boxed{\frac{\sqrt{95}}{-7}}$

$\sec\theta = \frac{r}{x} = \boxed{\frac{12}{\sqrt{95}}}$

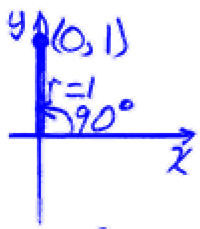
$\csc\theta = \frac{r}{y} = \boxed{\frac{12}{-7}}$

Unit 2: Lesson 04 Special angles (0°, 30°, 60°, 45°, 90°, 180°, 270°, 360°)

The quadrantal angles are those that are integral multiples of 90° (0°, 180°, 270°, 360°). The easiest way to evaluate the trig function values of these angles is by using the *x*, *y*, *r* definitions.

In the following two examples, draw the terminal side of the angle in standard position containing point P with a radius of 1. Label the (x, y) values of this point. Then determine the values of sin, cos, and tan.

Example 1: 90°



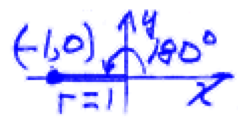
$x = 0$
$y = 1$
$r = 1$

$\sin 90 = \frac{y}{r} = \frac{1}{1} = \boxed{1}$

$\cos 90 = \frac{x}{r} = \frac{0}{1} = \boxed{0}$

$\tan 90 = \frac{y}{x} = \frac{1}{0}$ $\boxed{\text{undefined}}$

Example 2: 180°



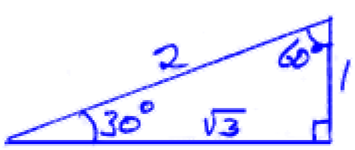
$x = -1$
$y = 0$
$r = 1$

$\sin 180 = \frac{y}{r} = \frac{0}{1} = \boxed{0}$

$\cos 180 = \frac{x}{r} = \frac{-1}{1} = \boxed{-1}$

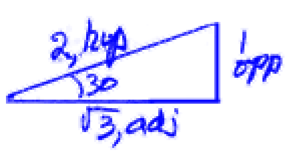
$\tan 180 = \frac{y}{x} = \frac{0}{-1} = \boxed{0}$

The angles 30° and 60° are special angles, both coming from a 30-60-90 triangle. Recall from geometry the following relationship between the sides of this special triangle (**must be memorized**):



prove with
$2^2 = 1^2 + (\sqrt{3})^2$

Example 3: Having previously memorized the six trig functions in terms of opp, adj, & hyp, these functions are easily determined for 30°.



$\sin 30 = \frac{opp}{hyp} = \boxed{\frac{1}{2}}$

$\cos 30 = \frac{adj}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$

$\tan 30 = \frac{opp}{adj} = \boxed{\frac{1}{\sqrt{3}}}$

$\csc 30 = \frac{hyp}{opp} = \boxed{\frac{2}{1} = 2}$

$\sec 30 = \frac{hyp}{adj} = \boxed{\frac{2}{\sqrt{3}}}$

$\cot 30 = \frac{adj}{opp} = \boxed{\frac{\sqrt{3}}{1}}$

Example 4: Similarly the six trig functions of 60^o are determined.

$\sin 60 = \frac{opp}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$

$\cos 60 = \frac{adj}{hyp} = \boxed{\frac{1}{2}}$

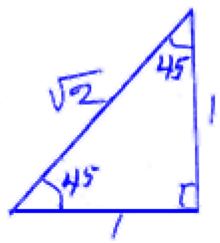
$\tan 60 = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

$\cot 60 = \frac{adj}{opp} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$\sec 60 = \frac{hyp}{adj} = \frac{2}{1} = \boxed{2}$

$\csc 60 = \frac{hyp}{opp} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

The final special triangle we will consider is the 45-45-90 triangle. Recall from geometry the following relationship between the sides of this special triangle (**must be memorized**):



prove with
$(\sqrt{2})^2 = 1^2 + 1^2$

Example 5: Having previously memorized the six trig functions in terms of opp, adj, & hyp, these functions are easily determined for 45^o.

opp & adj are both 1
$hyp = \sqrt{2}$

$\sin 45 = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\cos 45 = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

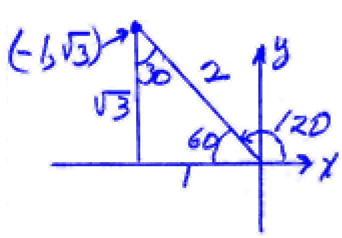
$\tan 45 = \frac{opp}{adj} = \frac{1}{1} = \boxed{1}$

$\cot 45 = \frac{adj}{opp} = \frac{1}{1} = \boxed{1}$

$\sec 45 = \frac{hyp}{adj} = \frac{\sqrt{2}}{1} = \boxed{\sqrt{2}}$

$\csc 45 = \frac{hyp}{opp} = \frac{\sqrt{2}}{1} = \boxed{\sqrt{2}}$

Example 6: Evaluate the sine, cosine, and tangent of 120^o.



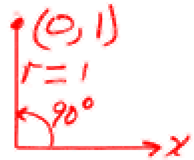
$\sin 120 = \frac{y}{r} = \boxed{\frac{\sqrt{3}}{2}}$

$\cos 120 = \frac{x}{r} = \boxed{\frac{-1}{2}}$

$\tan 120 = \frac{y}{x} = \boxed{\frac{\sqrt{3}}{-1} = -\sqrt{3}}$

Assignment: In problems 1-4, draw the terminal side of the angle in standard position containing point P with a radius of 1. Label the (x, y) values of this point. Then determine the values of sine, cosine, and tangent.

1. 90^{o}



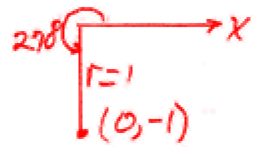
$x = 0$
$y = 1$
$r = 1$

$\sin 90 = \frac{y}{r} = \frac{1}{1} = \boxed{1}$

$\cos 90 = \frac{x}{r} = \frac{0}{1} = \boxed{0}$

$\tan 90 = \frac{y}{x} = \frac{1}{0} \rightarrow \boxed{\text{undefined}}$

2. 270^{o}



$x = 0$
$y = -1$
$r = 1$

$\sin 270 = \frac{y}{r} = \frac{-1}{1} = \boxed{-1}$

$\cos 270 = \frac{x}{r} = \frac{0}{1} = \boxed{0}$

$\tan 270 = \frac{y}{x} = \frac{-1}{0} \rightarrow \boxed{\text{undefined}}$

3. 360^{o}

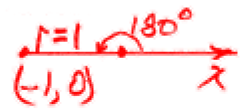
$x = 1$
$y = 0$
$r = 1$

$\sin 360 = \frac{y}{r} = \frac{0}{1} = \boxed{0}$

$\cos 360 = \frac{x}{r} = \frac{1}{1} = \boxed{1}$

$\tan 360 = \frac{y}{x} = \frac{0}{1} = \boxed{0}$

4. 180^{o}



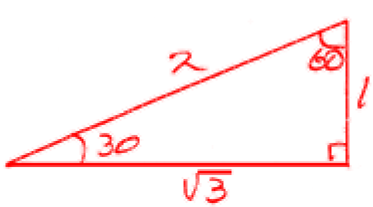
$x = -1$
$y = 0$
$r = 1$

$\sin 180 = \frac{y}{r} = \frac{0}{1} = \boxed{0}$

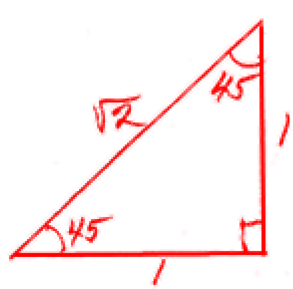
$\cos 180 = \frac{x}{r} = \frac{-1}{1} = \boxed{-1}$

$\tan 180 = \frac{y}{x} = \frac{0}{-1} = \boxed{0}$

5. Draw a 30-60-90 triangle and label the standard lengths of the sides.

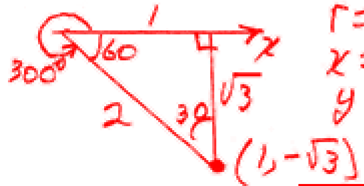


6. Draw a 45-45-90 triangle and label the standard lengths of the sides.



In problems 7-10, draw the angle in standard position. Draw in an appropriate 30-60-90 or 45-45-90 triangle. Using the triangle as a reference, label an (x, y) point on the terminal side of the angle. Using the *x*, *y*, *r* definitions, determine the sine, cosine, and tangent of the angle.

7. 300°



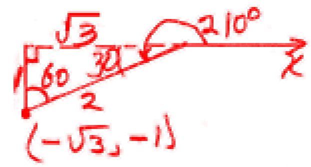
$r = 2$
$x = 1$
$y = -\sqrt{3}$

$\sin 300 = \frac{y}{r} = \boxed{\frac{-\sqrt{3}}{2}}$

$\cos 300 = \frac{x}{r} = \boxed{\frac{1}{2}}$

$\tan 300 = \frac{y}{x} = \boxed{\frac{-\sqrt{3}}{1} = -\sqrt{3}}$

8. 210°



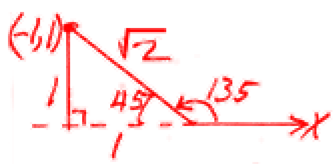
$x = -\sqrt{3}$
$y = -1$
$r = 2$

$\sin 210 = \frac{y}{r} = \boxed{\frac{-1}{2}}$

$\cos 210 = \frac{x}{r} = \boxed{\frac{-\sqrt{3}}{2}}$

$\tan 210 = \frac{y}{x} = \boxed{\frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}}$

9. 135°



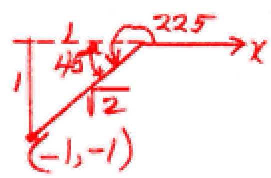
$x = -1$
$y = 1$
$r = \sqrt{2}$

$\sin 135 = \frac{y}{r} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\cos 135 = \frac{x}{r} = \frac{-1}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{2}}$

$\tan 135 = \frac{y}{x} = \frac{1}{-1} = \boxed{-1}$

10. 225°



$x = -1$
$y = -1$
$r = \sqrt{2}$

$\sin 225 = \frac{y}{r} = \frac{-1}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{2}}$

$\cos 225 = \frac{x}{r} = \frac{-1}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{2}}$

$\tan 225 = \frac{y}{x} = \frac{-1}{-1} = \boxed{1}$

In the following problems, draw the angle in standard position. Draw in an appropriate 30-60-90 or 45-45-90 triangle. Using the triangle as a reference, label an (x, y) point on the terminal side of the angle. Using the *x*, *y*, *r* definitions, determine the secant, cosecant, and cotangent of the angle.

11. 150°

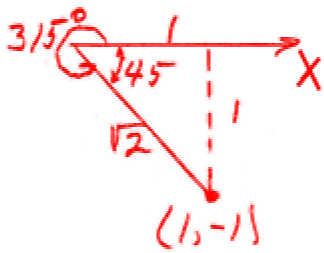
$x = -\sqrt{3}; y = 1; r = 2$

$\sec 150 = \frac{r}{x} = \frac{2}{-\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

$\csc 150 = \frac{r}{y} = \frac{2}{1} = \boxed{2}$

$\cot 150 = \frac{x}{y} = \frac{-\sqrt{3}}{1} = \boxed{-\sqrt{3}}$

12. 315^{o}



$r = \sqrt{2}$; $x = 1$, $y = -1$

$$\sec 315 = \frac{r}{x} = \frac{\sqrt{2}}{1} = \boxed{\sqrt{2}}$$

$$\csc 315 = \frac{r}{y} = \frac{\sqrt{2}}{-1} = \boxed{-\sqrt{2}}$$

$$\cot 315 = \frac{x}{y} = \frac{1}{-1} = \boxed{-1}$$

Unit 2: Lesson 05 Evaluating trig functions on the graphing calculator

See **Calculator Appendix T** (and an associated video) for how to evaluate any of the six trig functions on a graphing calculator.

Note that the calculator will only directly calculate sine, cosine, and tangent. Recall that the other three trig functions are simply reciprocals of these.

For example, to find cot 28°, enter 1/tan(28) into the calculator. (Make sure the **mode** is set to degrees for this problem.)

In the following examples, use a calculator to find the trig expression. Assume all angles are in radians unless the degree symbol (°) is used. Be sure to switch to the appropriate mode before entering each problem.

Example 1: cos(2π/7)

mode → rad

$\cos(2\pi/7) = \boxed{.6234898}$

Example 2: 1 – sec(238°)

mode → deg

$1 - 1/\cos(238)$

$= \boxed{2.8870799}$

Example 3: tan(–127°)

mode → deg

$\tan(-127)$

$= \boxed{1.3270448}$

Example 4: (14 + csc(π/5))/sin (π/7)

Mode → rad

$(14 + 1/\sin(\pi/5))/\sin(\pi/7)$

$= \boxed{36.18780839}$

Calculator Appendix S (and associated video) shows how to convert between decimal degree and degrees, minutes, & seconds.

In the following examples, use the graphing calculator to produce the desired conversion.

Example 5: Convert 128.784° to deg-min-sec form.

mode → deg ✓

128.784 ▸DMS

$= \boxed{128° 47' 2.4''}$

Example 6: Convert 45° 8' 22'' to decimal degrees form.

mode → deg

45° 8' 22'' Enter

$= \boxed{45.139\overline{4}}$

Teachers: Since this lesson is typically done very quickly, it is suggested that the Unit 2 Review be started and possibly assigned to be finished overnight.

Assignment: In problems 1-6, use a calculator to find the trig expression. Assume all angles are in radians unless the degree symbol (°) is used. Be sure to switch to the appropriate mode before entering each problem.

1. sin (48°)

Mode → deg

sin(48) = $\boxed{.7431448255}$

2. cos(11π/3)

Mode → rad

cos(11π/3)

= $\boxed{.5}$

3. 5 + cot(2π/10)

Mode → rad

5 + 1/tan(2π/10)

= $\boxed{6.37638192}$

4. tan(–11.23°) + 22

Mode → deg

tan(-11.23) + 22

= $\boxed{21.80145048}$

5. $\sqrt{\cos(11°) + \sec(11°)}$

Mode → deg

√(cos(11) + 1/cos(11))

= $\boxed{1.414335137}$

6. 56 – csc(π–π/6)

Mode → rad

56 - 1/sin(π - π/6)

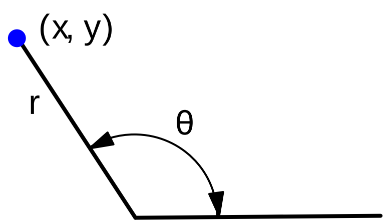
= $\boxed{54}$

In the following problems, use the graphing calculator to produce the desired conversion.

7. Convert $137^{\circ}\ 18'\ 29''$ to decimal degrees form.	8. Convert 128.784° to deg-min-sec form.
Mode → deg 137°18'29" Enter $= \boxed{137.3080556}$	128.784 ▸DMS $= \boxed{128^{\circ}\ 47'\ 2.4''}$

Unit 2: Review

1. Define the six trig functions in terms of *x*, *y*, and *r*.

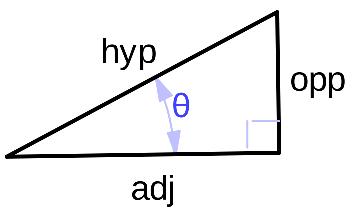


$\sin\theta = \frac{y}{r}$ $\quad$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

2. Define the six trig functions of θ in terms of *opp*, *adj*, & *hyp*.

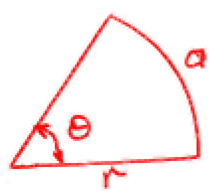


$\sin\theta = \frac{opp}{hyp}$ $\quad$ $\csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad$ $\sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\quad$ $\cot\theta = \frac{adj}{opp}$

3. Define the angle θ in radians with a drawing and accompanying equation.



$\theta = \frac{a}{r}$

4. The sine of an angle drawn inside a unit circle (radius 1) is the projection of the radius on which axis?

vertical axis (y)

5. The cosine of an angle drawn inside a unit circle (radius 1) is the projection of the radius on which axis?

horizontal axis (x)

6. Convert 272° into radians.

$\frac{deg}{rad} = \frac{deg}{rad}$; $\frac{180}{\pi} = \frac{272}{\theta}$; $180\theta = 272\pi$

$\theta = \frac{272\pi}{180} = \boxed{4.74729 \text{ rad}}$

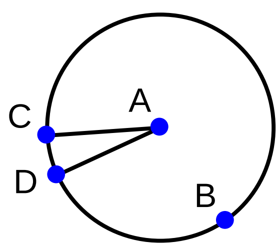
7. Convert 7π/17 into degrees.

$\frac{deg}{rad} = \frac{deg}{rad}$ $\theta = 180(7\pi/17)/\pi$

$\frac{180}{\pi} = \frac{\theta}{7\pi/17}$ $\theta = \boxed{74.11764706°}$

8. How many radians are there in a full circle? 2π radians	9. Since cot is not a function provided on a graphing calculator, how can it be done on the calculator? $\cot x \rightarrow \boxed{1/\tan(x)}$

10. Angle DAC is 32° and the radius of the circle is 4 ft. How long is arc DC?



$\frac{180}{\pi} = \frac{32}{\theta}$

$\theta = 32\pi/180$

$\theta = .5585 \text{ rad}$

$\theta = \frac{a}{r}$

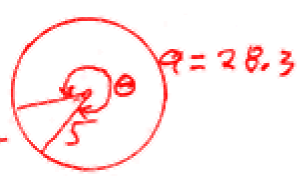
$\frac{.5585}{1} = \frac{a}{4}$

$a = 4(.5585)$

$a = \boxed{2.234}$

11. Using the drawing in problem 10, and assuming the length of arc DBC is 28.3 ft. and the radius of the circle is 5 ft., what is the measure (in radians) of the angle that arc subtends?

$\theta = \frac{a}{r}$

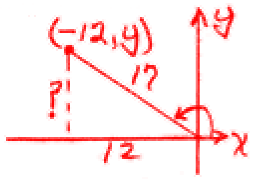


$\theta = \frac{28.3}{5}$

$\theta = \boxed{5.66 \text{ rad}}$

In problems 12-13, point P (x, y) is on the terminal side of an angle in standard position. The distance from the vertex to P is *r*. Draw and label the angle and then find the one of *x*, *y*, and *r* that is missing.

12. x = –12, r= 17, P(–12, y), is in the 2nd quadrant.



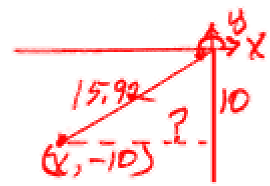
$x^2+y^2=r^2$

$(-12)^2+y^2=(17)^2$

$y=\pm\sqrt{17^2-(-12)^2}$

$=\boxed{+12.04159}$ pos because of 2nd Quad

13. y = –10, r= 15.92, P(x, –10), is in the 3rd quadrant.



$x^2+y^2=r^2$

$x^2+(-10)^2=(15.92)^2$

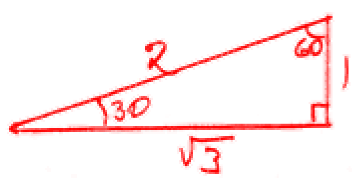
$x=\pm\sqrt{(15.92)^2-(-10)^2}$

$x=\boxed{-12.387348}$

14. List the sine and cosine of the quadrantal angles (90°, 180°, 270°, 360°).

sin 90 = 1	cos 90 = 0
sin 180 = 0	cos 180 = –1
sin 270 = –1	cos 270 = 0
sin 360 = 0	cos 360 = 1

15. Draw a 30-60-90 triangle and label the lengths of the sides. Using these sides, give sin 30°, cos 60°, sec 30°, cot 60°.



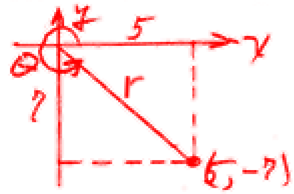
$\sin 30 = \frac{opp}{hyp} = \frac{1}{2}$

$\cos 60 = \frac{adj}{hyp} = \frac{1}{2}$

$\sec 30 = \frac{hyp}{adj} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot 60 = \frac{adj}{opp} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

16. Given point P (5, –7) on the terminal side of an angle θ in standard position, determine the cotangent and cosecant of θ.



$r^2 = x^2 + y^2$

$r = \sqrt{5^2 + (-7)^2}$

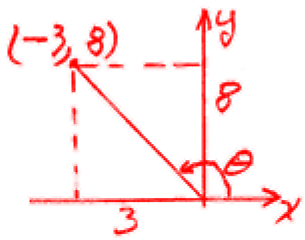
$r = \sqrt{74}$

$\cot\theta = \frac{x}{y} = \boxed{\frac{5}{-7}}$

$\csc\theta = \frac{r}{y} = \boxed{\frac{\sqrt{74}}{-7}}$

---

17. Given point P (–3, 8) on the terminal side of an angle θ in standard position, determine the sine and cosine of θ.



$r^2 = x^2 + y^2$

$r = \sqrt{(-3)^2 + 8^2}$

$r = \sqrt{73}$

$\sin\theta = \frac{y}{r} = \boxed{\frac{8}{\sqrt{73}}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{-3}{\sqrt{73}}}$

---

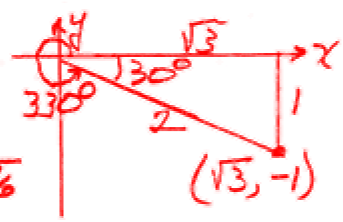
18. Determine the sine and cosine of 11π/6 radians.

$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{\theta}{11\pi/6}$

$\vdots$

$\theta = 330^\circ$

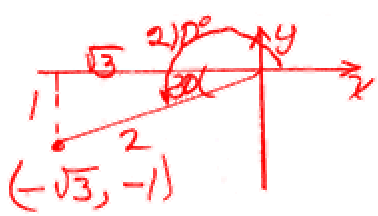


$\sin 330 = \frac{y}{r} = \boxed{-\frac{1}{2}}$

$\cos 330 = \frac{x}{r} = \boxed{\frac{\sqrt{3}}{2}}$

---

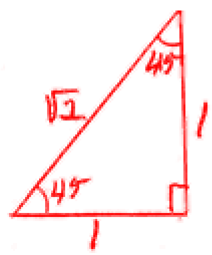
19. Determine the secant and tangent of 210°.



$\sec 210 = \frac{r}{x} = \frac{2}{-\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

$\tan 210 = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \boxed{\sqrt{3}/3}$

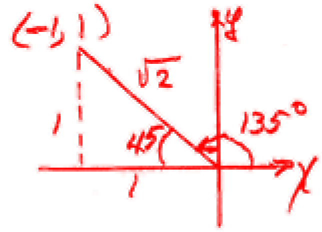
20. Draw a 45-45-90 triangle and label the lengths of the sides. Using these sides give the sine, cosine, and tangent of 3π/4 radians.



$$\frac{deg}{rad} = \frac{deg}{rad}$$

$$\frac{180}{\pi} = \frac{\theta}{3\pi/4}$$

$$\therefore \; \theta = 135^\circ$$



$$\sin 3\pi/4 = \frac{y}{r} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\cos 3\pi/4 = \frac{x}{r} = \frac{-1}{\sqrt{2}} = \boxed{-\frac{\sqrt{2}}{2}}$$

$$\tan 3\pi/4 = \frac{y}{x} = \frac{1}{-1} = \boxed{-1}$$

---

21. Use a calculator to evaluate the following expression. The angles are in radians.

147.2(sin(5π/3) – csc(14.3π))

$$147.2(\sin(5\pi/3) - 1/\sin(14.3\pi))$$

$$= \boxed{-309.4281457}$$

Pre Calculus, Unit 3

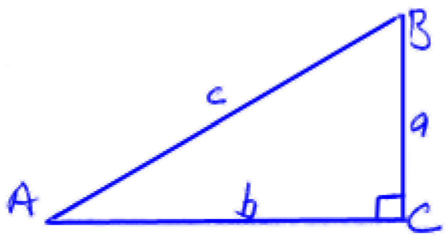
Triangle Solutions

Unit 3: Lesson 01 Abstract solutions of right triangles

There are exactly six basic things to know about a triangle: **3 sides and 3 angles**.

In triangle problems, typically, three things will be given. **"Solving"** the triangle **means determining the other three things**.

Conventional way to name the six parts of a right triangle:



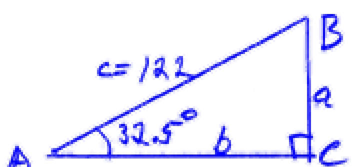
- *a* is opposite angle A
- *b* is opposite angle B
- *c* is opposite angle C

To solve right triangles, make use of the following facts (θ is one of the non-right angles):

- sin(θ) = opp/hyp
- cos(θ) = adj/hyp
- tan(θ) = opp/adj
- Sum of the interior angles is 180^o (π radians)

In the following examples, draw and label the right triangle and then solve it using the given information. Notice that when a triangle is a right triangle, this automatically gives one piece of information (one of the angles is 90^o).

Example 1: Triangle ABC is a right triangle, A = $32^o\,30'$, c = 122.



$B + 32.5 + 90 = 180$

$B = \boxed{52.5^o}$

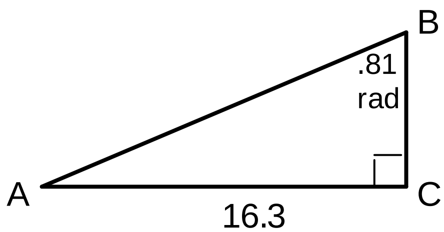
$\sin 32.5 = \frac{a}{122}$

$a = 122 \sin 32.5 = \boxed{65.55}$

$\cos 32.5 = \frac{b}{122}$

$b = 122 \cos(32.5) = \boxed{102.89}$

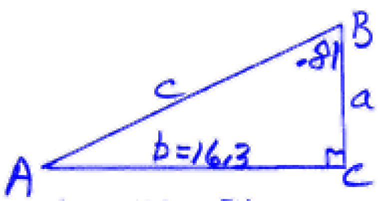
Example 2:



$\sin .81 = \frac{16.3}{c}$

$c \sin .81 = 16.3$

$c = \frac{16.3}{\sin .81} = \boxed{22.5048}$



$A + .81 = \pi/2$

$A = \boxed{.760796 \text{ rad}}$

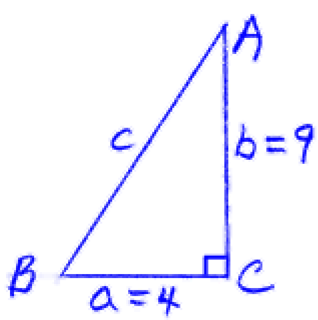
$\tan .81 = \frac{16.3}{a}$

$a \tan .81 = 16.3$

$a = \frac{16.3}{\tan .81} = \boxed{15.51708}$

Occasionally, as in the next example, we are given no angles (besides the right angle); however at least two sides are given. If, for example, we know the side opposite angle θ and the hypotenuse, then on the graphing calculator enter **2nd SIN** and the display will contain **$\sin^{-1}$(** . At this point, enter opp/hyp, close the parenthesis, press ENTER, and the value of the angle θ will appear.

Example 3: Triangle ABC is a right triangle, a = 4, b = 9.



$c^2 = a^2 + b^2$

$c = \sqrt{4^2 + 9^2}$

$c = \boxed{9.848}$

$\sin A = \frac{a}{c} = \frac{4}{9.848} = .40617$

$A = \sin^{-1}(.40617) = \boxed{23.9647^\circ}$

$\sin B = \frac{b}{c} = \frac{9}{9.848} = .91389$

$B = \sin^{-1}(.91389) = \boxed{66.0487^\circ}$

Example 3 above made use of the "inverse sine", **$\sin^{-1}$(opp/hyp)**, to find an angle. Similarly, "inverse cosine" and "inverse tangent" can be used to find angles:

- **$\cos^{-1}$(adj/hyp)**
- **$\tan^{-1}$(opp/adj)**

Admittedly, this is a very cursory introduction to inverse functions; however, for the time being, it's a quick way to calculate angles. See **Calculator Appendix U** and a related video for more on inverse trig functions. (In later lessons there will be an in-depth study of inverse trig functions.)

In all of the above examples we have produced **approximate answers** since sine, cosine, and tangent were evaluated on a calculator. These calculations generally produce irrational numbers (decimal places go on forever).

It is possible to produce **exact answers** if the angles are **special angles** (30°, 60°, 90°, 45°, or their radian equivalents).

Example 4: Right triangle ABC, $B = 45^\circ$, $c = 17$

$A + B = 90$

$A + 45 = 90$

$A = 90 - 45$

$A = \boxed{45^\circ}$

$\cos 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{a}{17}$

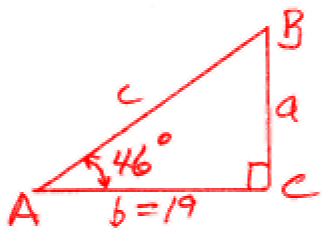
$a = 17 \cos 45 = \boxed{17\frac{\sqrt{2}}{2}}$ ← exact

$\sin 45^\circ = \frac{b}{17}$

$b = 17 \sin 45 = \boxed{17\frac{\sqrt{2}}{2}}$

Assignment: In the following problems, assume all triangles are right triangles. Draw and label the right triangle and then solve it using the given information.

1. A = 46°, b = 19



$\cos 46° = \frac{19}{c}$

$c \cos 46 = 19$

$c = 19/\cos 46 = \boxed{27.35157}$

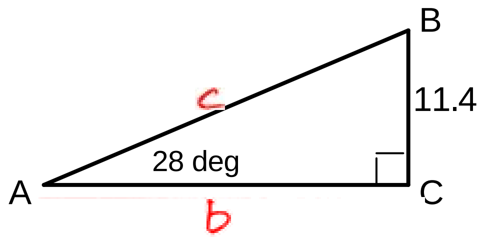
$\tan 46 = \frac{a}{19}$

$a = 19 \tan 46 = \boxed{19.67507}$

$B + 46 = 90$

$B = 90 - 46 = \boxed{44°}$

2.



$\sin 28 = \frac{11.4}{c}$

$c \sin 28 = 11.4$

$c = 11.4/\sin 28 = \boxed{24.2826}$

$28 + B = 90$

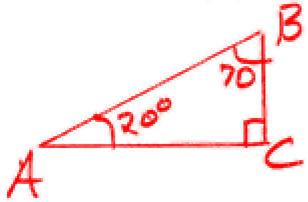
$B = 90 - 28 = \boxed{62°}$

$\tan 28 = \frac{11.4}{b}$

$b \tan 28 = 11.4$

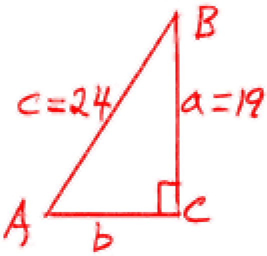
$b = 11.4/\tan 28 = \boxed{21.44028}$

3. A = 20°, B = 70°



Impossible to find sides. We must be given at least 1 side.

4. a = 19, c = 24



$a^2+b^2=c^2$

$b=\sqrt{c^2-a^2}$

$b=\sqrt{24^2-19^2}=\boxed{14.66}$

$\sin A=\frac{19}{24}=.791\overline{6}$

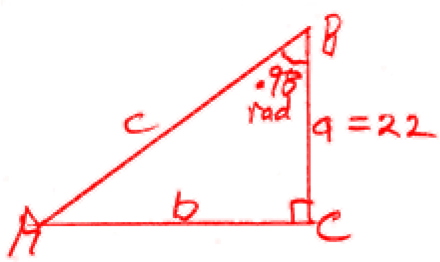
$A=\sin^{-1}(.791\overline{6})=\boxed{52.3415^\circ}$

$\cos B=\frac{19}{24}=.791\overline{6}$

$B=\cos^{-1}(.791\overline{6})=\boxed{37.65846^\circ}$

---

5. B = .98 radians, a = 22



$A+.98=\pi/2$

$A=\frac{\pi}{2}-.98=\boxed{.59079\text{rad}}$

$\cos(.98)=\frac{22}{c}$

$c\cos(.98)=22$

$c=22/\cos(.98)=\boxed{39.4957}$

$\tan .98=\frac{b}{22}$

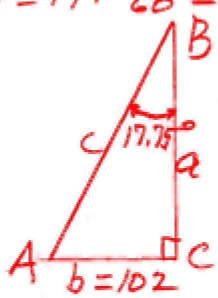
$b=22\tan(.98)$

$=\boxed{32.801}$

---

6. B = 17° 45', b = 102

$B=17+\frac{45}{60}=17.75^\circ$



$A=90-17.75=\boxed{72.25^\circ}$

$\sin(17.75^\circ)=\frac{102}{c}$

$c\sin(17.75)=102$

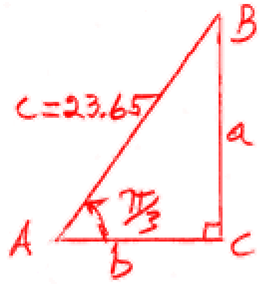
$c=102/\sin(17.75)=\boxed{334.525}$

$\tan(17.75^\circ)=\frac{102}{a}$

$a\tan(17.75)=102$

$a=102/\tan(17.75)=\boxed{318.6479}$

7. A = π/3 radians,
c = 23.65



$\sin \frac{\pi}{3} = \frac{a}{23.65}$

$a = 23.65 \sin \frac{\pi}{3}$

$a = \boxed{20.4815}$

$\cos \frac{\pi}{3} = \frac{b}{23.65}$

$b = 23.65 \cos \frac{\pi}{3}$

$b = \boxed{11.825}$

$B + \frac{\pi}{3} = \frac{\pi}{2}$

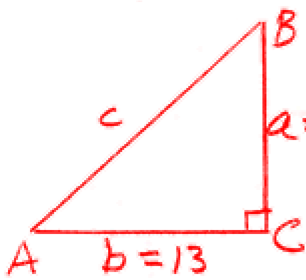
$B = \frac{\pi}{2} - \frac{\pi}{3}$

$B = \frac{\pi}{2}\frac{3}{3} - \frac{\pi}{3}\frac{2}{2}$

$B = \frac{3\pi - 2\pi}{6} = \boxed{\frac{\pi}{6} \text{ rad}}$

---

8. a = 11, b = 13



$\tan A = \frac{11}{13}$

$= .84615$

$A = \tan^{-1}(.84615)$

$= \boxed{40.23635^\circ}$

$\tan B = \frac{13}{11}$

$= 1.\overline{18}$

$B = \tan^{-1}(1.\overline{18})$

$= \boxed{49.7636^\circ}$

$c^2 = a^2 + b^2$

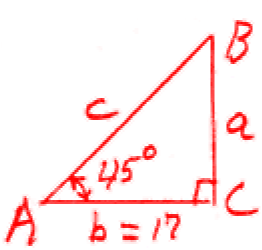
$c = \sqrt{a^2+b^2}$

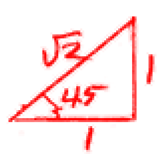
$c = \sqrt{11^2+13^2}$

$c = \boxed{17.02938}$

In the following two problems, produce **exact** results. All angles are special.

9. A = 45°, b = 17





$\cos 45 = 1/\sqrt{2}$

$\tan 45 = 1/1 = 1$

$\cos 45 = \frac{17}{c}$

exact $\frac{1}{\sqrt{2}} = \frac{17}{c}$; $c = \boxed{17\sqrt{2}}$

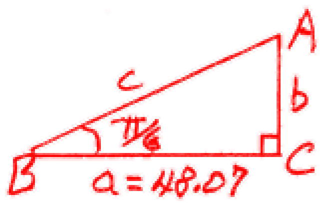
$\tan A = \frac{a}{17}$

$1 = \frac{a}{17}$; $a = \boxed{17}$

$45 + B = 90$

$B = 90 - 45 = \boxed{45^\circ}$

10. B = π/6 radians, a = 48.07



$A + \frac{\pi}{6} = \frac{\pi}{2}$

$A = \frac{\pi}{2}\cdot\frac{3}{3} - \frac{\pi}{6}$

$A = \frac{2\pi}{6} = \boxed{\frac{\pi}{3}\text{ rad.}}$

$\tan\frac{\pi}{6} = \frac{b}{48.07}$

exact

$\frac{1}{\sqrt{3}} = \frac{b}{48.07}$

$\sqrt{3}\,b = 48.07$

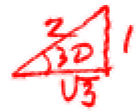
$b = \boxed{48.07/\sqrt{3}}$

exact

$\cos\frac{\pi}{6} = 48.07/c$

$\sqrt{3}/2 = 48.07/c$

$c = \boxed{96.14/\sqrt{3}}$



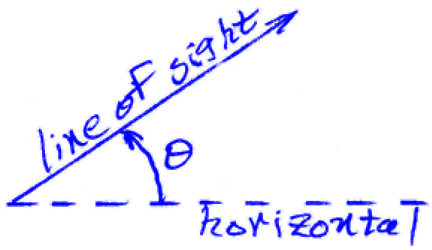
$\tan 30 = \frac{1}{\sqrt{3}}$

$\cos 30 = \frac{\sqrt{3}}{2}$

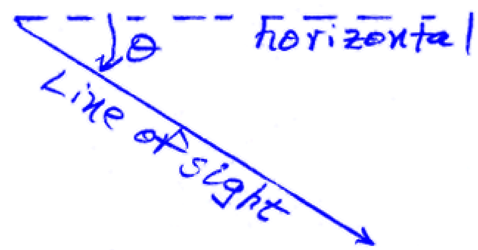
Unit 3: Lesson 02 **Right triangle word problems, triangle area**

Quite often in solving word problems we are confronted with angles that are not in "standard position."

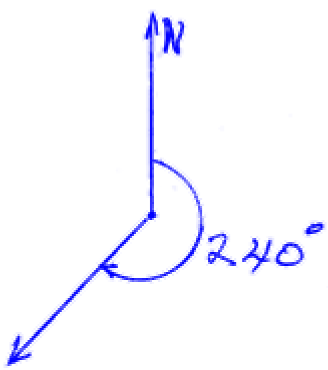
Angle of elevation (measure with respect to a horizontal line):



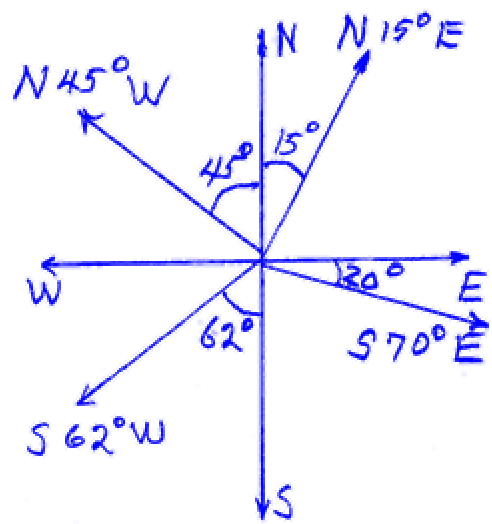
Angle of depression (measure with respect to a horizontal line):



Navigational angle (measure with respect to north, positive direction is clockwise):

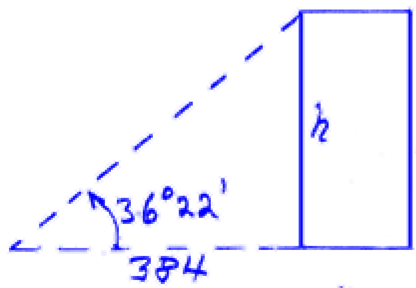


Surveying, bearing angle (the acute angle at which the direction varies to the east or west from the north-south line):



The **area** of a right triangle is given by A = (1/2)(base)(height) which is equivalent to A = .5ab.

Example 1: From a point 384 ft in a horizontal line from the base of a building, the angle of elevation to the top of the building is 36° 22'. How tall is the building?



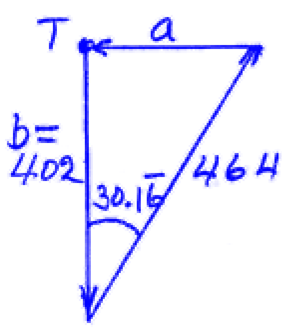
$\theta = 36°22' = 36 + \frac{22}{60}$
$= 36.3\overline{6}°$

$\tan 36.3\overline{6} = \frac{h}{384}$

$h = 384 \tan(36.3\overline{6})$

$h = \boxed{282.7645304}$

Example 2: Find the area of a parcel of land whose boundaries are marked as follows: beginning at the old oak tree, thence 402 ft south, thence 464 ft N 30.1666° E, and thence due west back to the old oak tree.



$\sin 30.1\overline{6} = \frac{a}{464}$

$a = 464 \sin(30.1\overline{6})$

$a = \boxed{233.1679}$

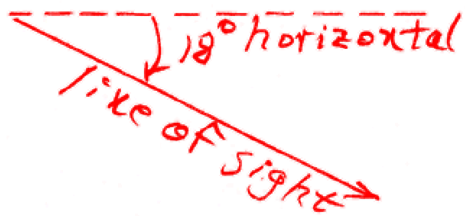
$\text{Area} = \frac{1}{2} a b$

$= \frac{1}{2}(233.1679)(402)$

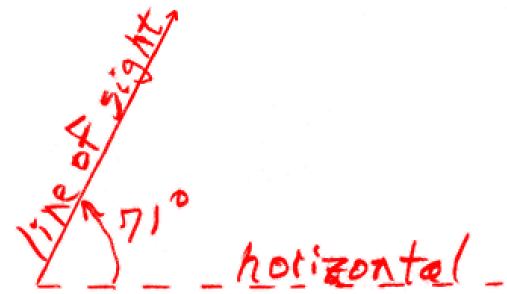
$= \boxed{46{,}866.74792 \text{ ft}^2}$

Assignment:

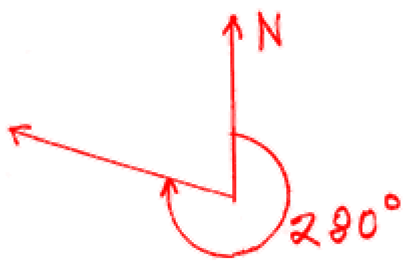
1. Draw an angle of depression of 18^o.



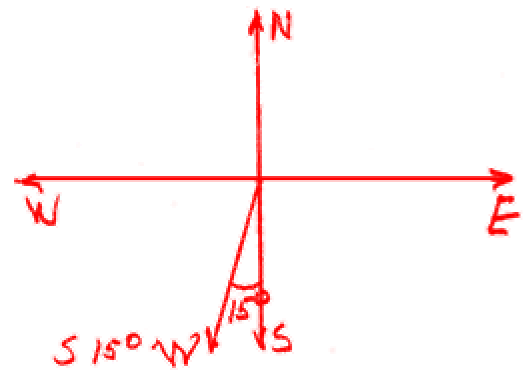
2. Draw an angle of elevation of 71^o.



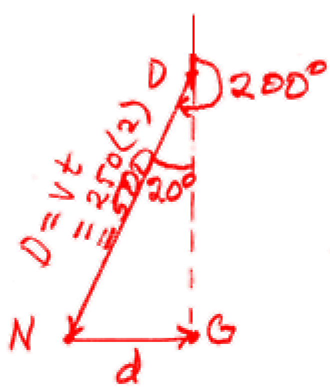
3. Draw a navigational angle of 280^o.



4. Draw a heading of S 15^o W.



5. An airplane flew from Dweeb City flying at 200^o at a speed of 250 mph for 2 hours and reached Nerdtown. From that point the plane turned and flew due east and landed at Geekville which just happened to be due south of Dweeb City. What is the distance from Nerdtown to Geekville?

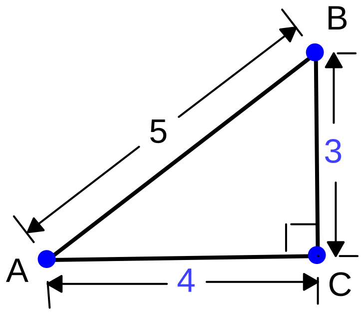


$\sin 20^\circ = \dfrac{d}{500}$

$d = 500 \sin 20^\circ$

$d = \boxed{171.01 \text{ mi}}$

6. Find the area of this triangle.



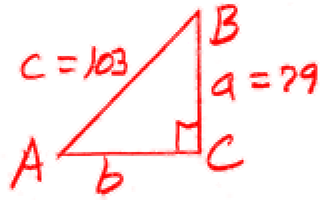
$a = 3$

$b = 4$

$\text{Area} = \frac{1}{2} a b$

$= \frac{1}{2}(3) \cdot 4 = \boxed{6}$

7. Find the area of right triangle ABC where a = 79 and c = 103.



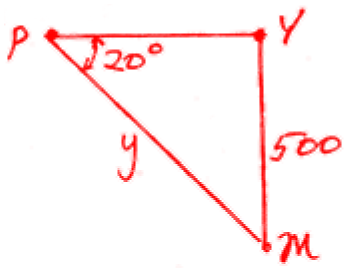
$b = \sqrt{c^2 - a^2} = \sqrt{103^2 - 79^2}$

$= 66.0908$

$\text{Area} = \frac{1}{2} a b$

$= \frac{1}{2}(79)(66.0908) = \boxed{2610.5}$

8. My house is 500 yards south of your house, and this distance subtends an angle of 20° at a point P that is due west of your house. How far is point P from my house?



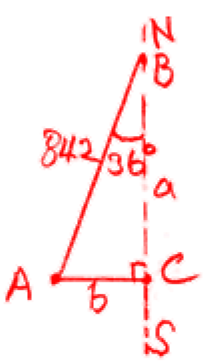
$\sin 20^\circ = \frac{opp}{hyp} = \frac{500}{y}$

$y \sin 20 = 500$

$y = 500/\sin 20$

$y = \boxed{1461.9022 \text{ yd.}}$

9. A certain piece of land is in the shape of a right triangle. The longest side is 842 meters and bears S 36° W. How many meters of fence would enclose this tract if one of the sides runs east-west?



$\cos 36^\circ = \frac{a}{842}$

$a = 842 \cos 36^\circ = 681.1923$

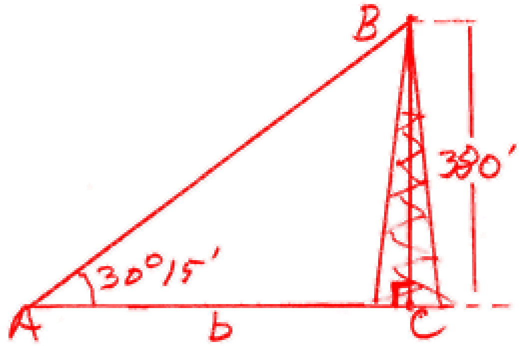
$\sin 36 = \frac{b}{842}$

$b = 842 \sin 36 = 494.9151$

$\text{Perimeter} = 842 + a + b$

$= \boxed{2018.1074 \text{ m}}$

10. A cell-phone tower is 380 ft tall. An observer is how far from the base of this tower if he and the tower are on level ground and the angle of elevation from the observer to the top of the tower is 30° 15'?



$30°15' = 30.25°$

$\tan 30.25 = \frac{380}{b}$

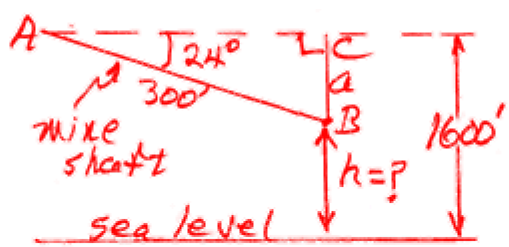
$b \tan 30.25 = 380$

$b = 380/\tan 30.25$

$b = \boxed{651.5967\ ft}$

---

11. The elevation above sea level at the entrance to a mine is 1600 ft. The mine shaft descends in a straight line for 300 ft at an angle of depression of 24°. Find the elevation of the bottom of the mine shaft above sea level.



$\sin 24 = \frac{a}{300}$

$a = 300 \sin 24 = 122.02'$

$a + h = 1600$

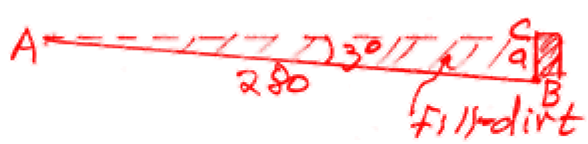
$h = 1600 - a$

$h = 1600 - 122.02$

$h = \boxed{1477.98'}$

---

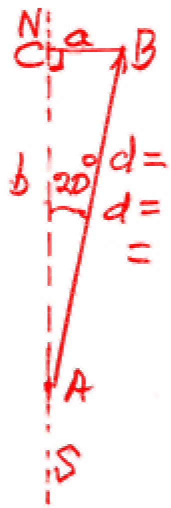
12. A piece of land slopes at an angle of 3° and runs for 280 ft in the direction of the slope. In order to level the land, a retaining wall is to be built at the lower end of the property so that fill-dirt can level the property. How high must the wall be?



$\sin 3 = \frac{a}{280}$

$a = 280 \sin 3° = \boxed{14.654\ ft.}$

13. An airplane flew at 20° for 5 hrs at 180 mph. It then turned and headed due west and landed at a point directly north of the starting point. After refueling, the plane then flew due south back to the starting point. What is the area in sq miles of the triangular pattern flown by the airplane?



$d = vt$

$d = 180(5)$

$= 900\text{ mi}$

$\sin 20 = \frac{a}{900}$

$a = 900 \sin 20 = 307.81\text{ mi}$

$\cos 20 = \frac{b}{900}$

$b = 900 \cos 20$

$= 845.723\text{ mi}$

$\text{Area} = \frac{1}{2} a\, b$

$= \frac{1}{2}(307.81)(845.723)$

$= \boxed{130{,}160.9983\text{ mi}^2}$

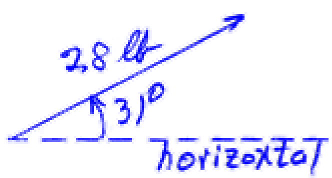
Unit 3: Lesson 03 Vectors

Quantities that have both **magnitude**(size) and **direction** are known as **vectors**.

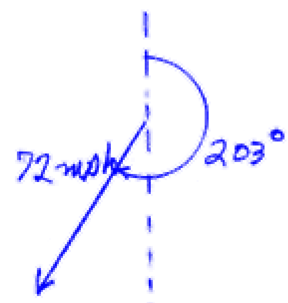
Examples of vector quantities are forces, velocities, accelerations, and displacements.

Vectors are **represented graphically with an arrow** (a directed line-segment) where the length of the arrow is proportional to the magnitude of the vector. Of course, the direction of the arrow represents the direction of the vector.

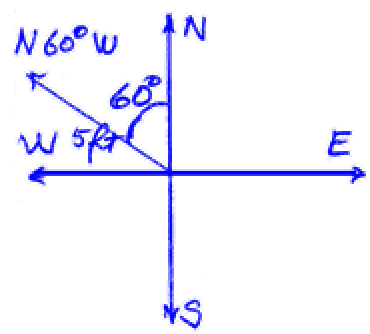
Example 1: Force



Example 2: Velocity

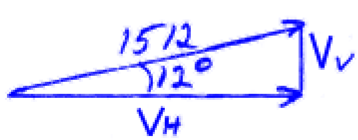


Example 3: Displacement



Vectors can be resolved into two mutually perpendicular components: usually, horizontal and vertical components. If our vector is a force represented by the symbol **F**, then the horizontal and vertical components are called F_H and F_V (or equivalently F_X and F_Y).

Example 4: The velocity of a bullet is 1512 ft/sec and inclines upward at 12°. Resolve this velocity (call it **V**) into its horizontal and vertical components.



$\sin 12 = V_V/1512$; $V_V = 1512 \sin 12$

$V_V = \boxed{314.36 \text{ ft/sec}}$

$\cos 12 = V_H/1512$; $V_H = 1512 \cos 12$

$V_H = \boxed{1478.96 \text{ ft/sec}}$

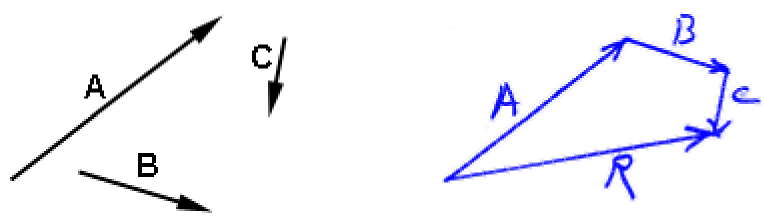
Addition of vectors:
The sum of vectors is called the **resultant**. Vectors can be added by one of three methods:

Parallelogram method (for adding two vectors): Move the two vectors (keeping them parallel to their original directions) together tail-to-tail and complete the parallelogram. The **diagonal** from the two joined tails to the opposite corner **is the resultant** (the sum).

Example 5: Using the parallelogram method, graphically show the resultant **R** when adding vectors **A** and **B**.

Head-to-tail method (for adding any number of vectors): Move the vectors (keeping them parallel to their original directions) so that they daisy-chain together in a head-to-tail fashion. The resultant (the sum) is drawn from the tail of the first vector in the chain to the head of the last.

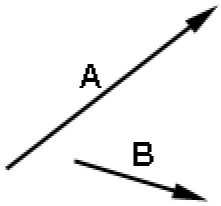
Example 6: Using the head-to-tail method, graphically show the resultant **R** when adding vectors **A**, **B**, and **C**.



Component method: Break each vector up into its two perpendicular components. Add all the horizontal components and call the sum R_H (or R_X). Add all the vertical components and call the sum R_V (or R_Y).

Create the final resultant by adding these two vector components.

Example 7: Using the component method, add vectors **A** (angle of elevation 40°) and **B** (angle of depression 19°). The magnitude of **A** is 107 and the magnitude of **B** is 51.

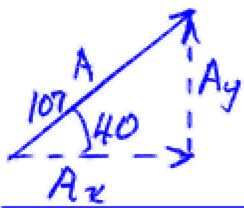


$\sin 19 = B_y/51$ minus because down

$\boxed{B_y = -16.604}$

$\cos 19 = B_x/51$

$\boxed{B_x = 48.22}$



$\sin 40 = A_y/107$; $\boxed{A_y = 68.78}$

$\cos 40 = A_x/107$; $\boxed{A_x = 81.97}$

Resultant

$R_x = A_x + B_x = 81.97 + 48.22$

$= 130.19$

$R_y = A_y + B_y = 68.78 - 16.604$

$= 52.176$

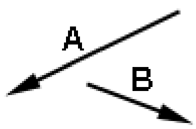
$R = \sqrt{130.19^2 + 52.176^2}$

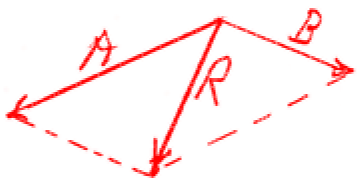
$R = \boxed{140.26}$

$\theta = \tan^{-1}\left(\frac{52.176}{130.19}\right) = \boxed{21.84°}$

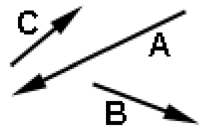
Assignment:

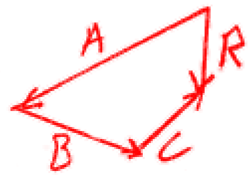
1. Add these vectors graphically using the parallelogram method.



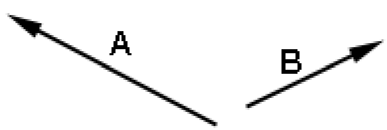


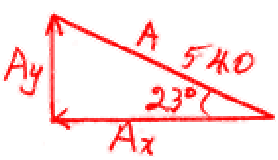
2. Add these vectors graphically using the head-to-tail method.





3. Add vectors **A** and **B** using the component method. Vector **A** has a magnitude of 540 and an angle of elevation of 23°. Vector **B** has a magnitude of 312 and an angle of elevation of 20°.



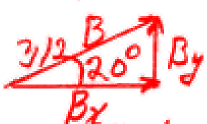


$\sin 23 = A_y/540$

$A_y = 210.99$

$\cos 23 = A_x/540$

$A_x = -497.07$ neg because to the left



$\sin 20 = B_y/312;\ B_y = 106.71$

$\cos 20 = B_x/312;\ B_x = 293.18$

$R_x = A_x + B_x$

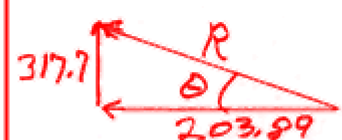
$= -497.07 + 293.18$

$= -203.89$

$R_y = A_y + B_y$

$= 210.99 + 106.71$

$= 317.7$



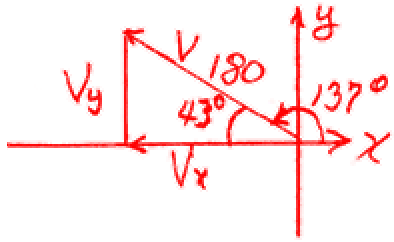
$R = \sqrt{317.7^2 + 203.89^2}$

$= \boxed{377.50}$

$\theta = \tan^{-1}(317.7/203.89)$

$= \boxed{57.31°}$

4. An object is moving with a speed of 180 m/sec in an x-y coordinate system. Its direction is that described by a 137° angle in standard position. What are the *X* and *Y* components of the velocity of the object?



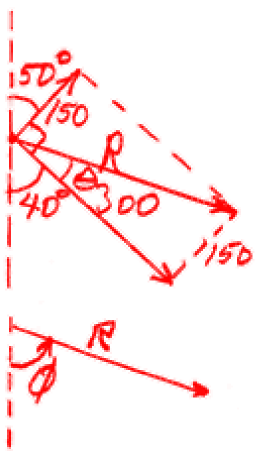
$\sin 43 = V_y/180$

$V_y = \boxed{122.76 \text{ m/sec}}$

$\cos 43 = V_x/180$

$V_x = \boxed{-131.64 \text{ m/sec}}$

5. Two forces, **F₁** and **F₂**, are pulling on the same object. **F₁** is a force of 300 lb with a bearing of S 40° E. **F₂** is a force of 150 lb with a bearing of N 50° E. Use the parallelogram method to find the single force (the resultant) that equivalently replaces these two forces. Give both the magnitude and bearing of the resultant.



$R = \sqrt{300^2 + 150^2}$

$R = 335.41$

$\theta = \tan^{-1}(150/300)$

$\theta = 26.57°$

$\phi = 40 + \theta$

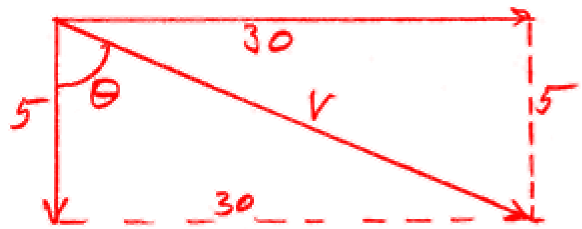
$= 66.57°$

Resultant is a vector of magnitude 335.41 and with a bearing of S 66.57° E

6. Draw the displacement vector **D** that has a magnitude of 136 and bearing N 18° W.

7. Draw the velocity vector **V** of a boat with a heading of 350° and speed 37 mph.

8. A football is thrown due south at 5 m/sec from a car that is traveling east at 30 m/sec. Find the speed of the ball relative to the ground and its direction of travel.



$$V = \sqrt{30^2 + 5^2} = \boxed{30.4138 \text{ m/sec}}$$

$$\theta = \tan^{-1}(30/5) = \boxed{\begin{array}{l} 80.538^\circ \\ S\ 80.538^\circ\ E \end{array}}$$

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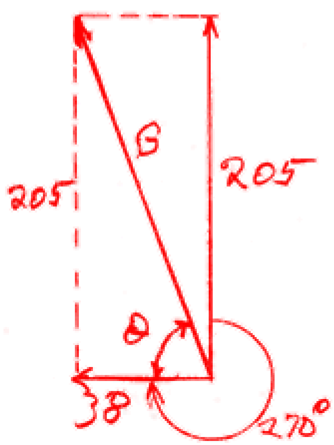
9. An airplane is headed due north with a speed of 205 mph. The wind is blowing from the east at a speed of 38 mph. What is the navigational heading and speed of the plane's shadow on the ground?

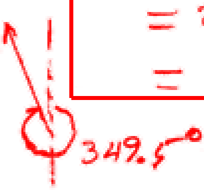


$$G = \sqrt{205^2 + 38^2} = \boxed{\begin{array}{l} 208.49 \text{ mph} \\ \text{speed} \end{array}}$$

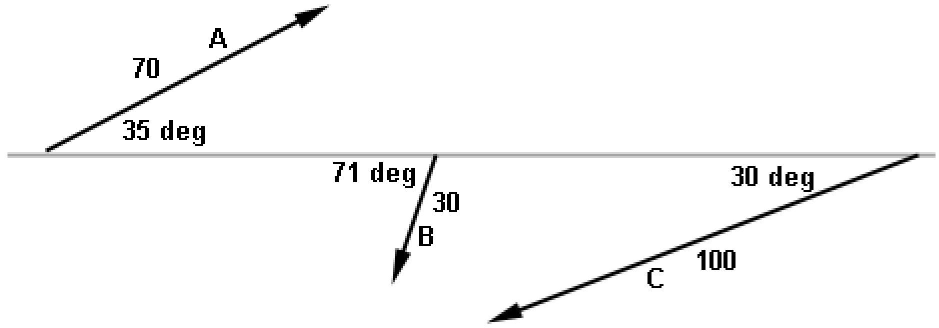
$$\theta = \tan^{-1}(205/38)$$

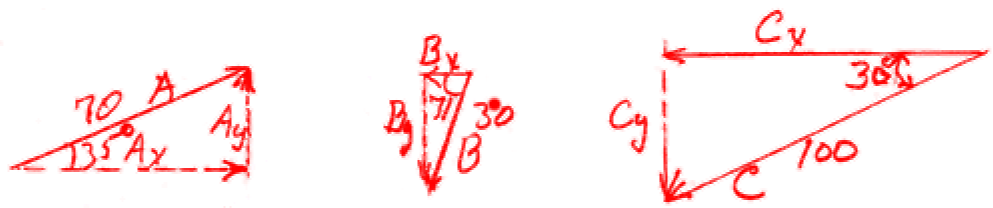
$$= 79.5^\circ$$

$$\boxed{\begin{array}{l} \text{Nav heading} \\ = 270 + \theta \\ = 270 + 79.5 \\ = 349.5^\circ \end{array}}$$



10. Using the component method, find the magnitude and direction(as an angle in standard position) of the resultant after adding these three displacement vectors.





$A_y = 70 \sin 35$
$= 40.15$

$A_x = 70 \cos 35$
$= 57.34$

$B_y = 30 \sin 71$
$= -28.37$

$B_x = 30 \cos 71$
$= -9.767$

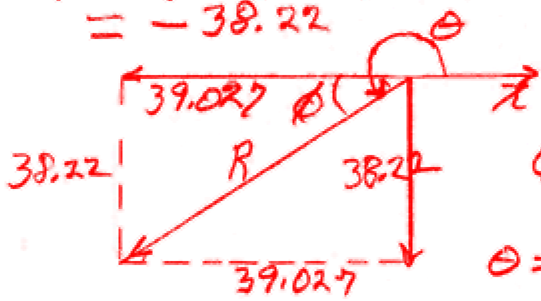
$C_y = 100 \sin 30$
$= -50$

$C_x = 100 \cos 30$
$= -86.60$

Neg here because both vector are down + to the left.

$R_x = A_x + B_x + C_x = 57.34 - 9.767 - 86.60$
$= -39.027$

$R_y = A_y + B_y + C_y = 40.15 - 28.37 - 50$
$= -38.22$



$R = \sqrt{38.22^2 + 39.027^2}$
$= \boxed{54.625}$

$\phi = \tan^{-1}(38.22/39.027)$
$= 44.401^\circ$

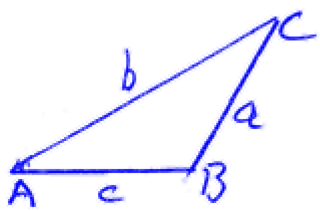
$\theta = 180^\circ + 44.401^\circ = \boxed{224.401^\circ}$

Unit 3: Lesson 04 Sine Law; more triangle area formulas

Until this point in our studies we have been restricted to solutions of only right triangles. No longer.

The **sine law** applies to **any type triangle**, not just right triangles.

Each of the following proportions is an expression of the sine law. Notice that each ratio is made of a pair of items that are **opposite each other** in the triangle.



$$\frac{\sin C}{c} = \frac{\sin B}{b} = \frac{\sin A}{a}$$

For a derivation of the sine law see **Enrichment Topic K**.

By dropping a perpendicular from a vertex to the line described by the opposite side, and then using right triangle identities and the sine law, it is possible to produce the following two important triangle area formulas. It is conventional to use **K** for the area of a triangle.

$$K = \frac{1}{2}\frac{b^2 \sin A \ \sin C}{\sin B}$$

Since the names of sides and angles within an oblique triangle are interchangeable, this area formula should be learned as follows:

The area of a triangle is equal to one-half the product of the square of any side and the sines of the adjacent angles divided by the sine of the opposite angle.

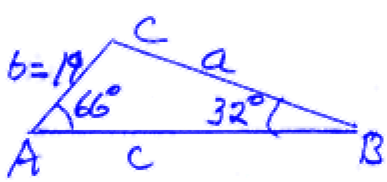
Another area formula:

K = $\frac{1}{2} a\, b \sin(C)$

Since the names of sides and angles within an oblique triangle are interchangeable, this area formula should be learned as follows:

The area of a triangle is equal to one-half the product of any two sides times the sine of their included angle.

Example 1: A = 66°, B = 32°, b = 19: Solve this triangle and find its area.



$C + 66 + 32 = 180 \rightarrow C = \boxed{82°}$

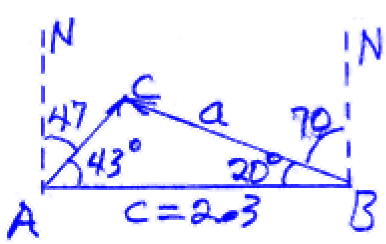
$\frac{\sin 32}{19} = \frac{\sin 66}{a} \rightarrow a = \boxed{32.75}$

$\frac{\sin 32}{19} = \frac{\sin 82}{c} \rightarrow c = \boxed{35.51}$

$K = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(32.75)(19)\sin 82° = \boxed{308.097}$

Example 2: Two observers are stationed on an east-west line and are 2.3 miles apart. Observer A sees the steeple of the old church with a bearing of N 47° E. Observer B reports a bearing of N 70° W. How far is the church from B?



$C + 20 + 43 = 180$

$C = 117$

$\frac{\sin 43}{a} = \frac{\sin 117}{2.3}$

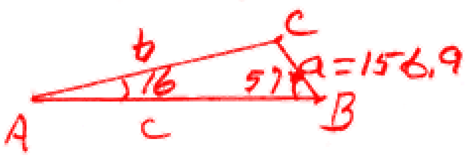
$a \sin 117 = 2.3 \sin 43$

$a = \frac{2.3 \sin 43}{\sin 117}$

$a = \boxed{1.76 \text{ mi}}$

Assignment: In problems 1-3, solve the triangle and then find its area. Draw and fully label the triangle.

1. A = 16°, B = 57°, a = 156.9



$16 + 57 + C = 180$

$C = \boxed{107}$

$\frac{\sin 16}{156.9} = \frac{\sin 57}{b}$; $b = \boxed{477.39}$

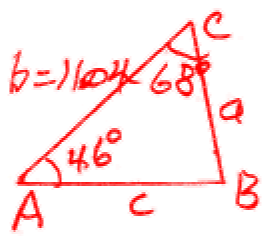
$\frac{\sin 107}{c} = \frac{\sin 16}{156.9}$; $c = \boxed{544.35}$

$K = \frac{1}{2} ab \sin C$

$= \frac{1}{2} 156.9(477.39) \sin 107$

$= \boxed{35,814.8}$

2. A = 46°, C = 68°, b = 11.04



$46 + 68 + B = 180$

$B = \boxed{66°}$

$\frac{\sin 66}{11.04} = \frac{\sin 68}{c}$; $c = \boxed{11.205}$

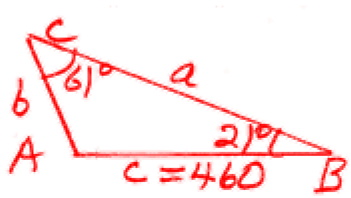
$\frac{\sin 66}{11.04} = \frac{\sin 46}{a}$; $a = \boxed{8.69}$

$K = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(8.69)(11.04) \sin 68$

$= \boxed{44.476}$

3. B = 21°, C = 61°, c = 460



$61 + 21 + A = 180$

$A = \boxed{98°}$

$\frac{\sin 61}{460} = \frac{\sin 21}{b}$; $b = \boxed{188.48}$

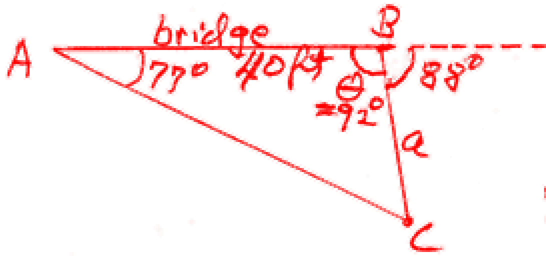
$\frac{\sin 61}{460} = \frac{\sin 98}{a}$; $a = \boxed{520.82}$

$K = \frac{1}{2} ca \sin 21$

$= \frac{1}{2}(460)(520.82) \sin 21$

$= \boxed{43,928.39}$

4. From a bridge above a river an observer looks along an angle of depression of 77° to a rock on the bank (directly under the bridge). Walking forward 40 ft, she determines a new angle of depression of 88° to the same rock. How far is the rock from her second position?



$\theta + 88 = 180$

$\theta = \angle B = 92^{\circ}$

$77 + 92 + C = 180$

$C = 11^{\circ}$

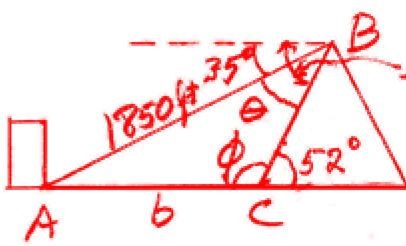
$$\frac{\sin 11}{40} = \frac{\sin 77}{a}$$

$a \sin(11) = 40 \sin(77)$

$a = \boxed{204.26 \text{ ft}}$

---

5. A surveyor at the top of a pyramid in Egypt, whose surface makes a 52° angle with respect to the horizontal, finds the angle of depression to the bottom of a building on the horizontal plain below is 35°. A laser range finder shows the distance from his position to the bottom of the building is 1850 ft. How far is the building from the base of the pyramid?



52° (alt interior angles)

$\theta + 35 = 52$

$\theta = 52 - 35 = 17^{\circ}$

$52 + \phi = 180$

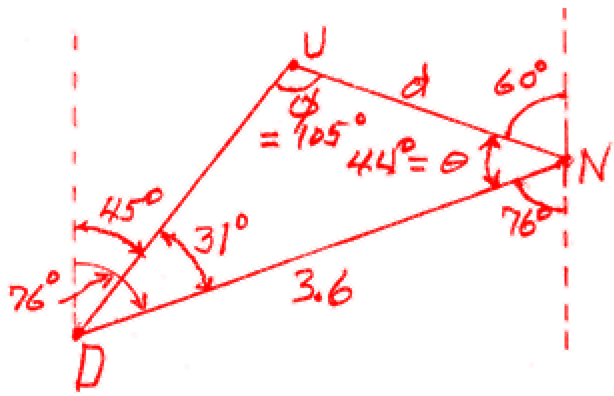
$\phi = 128^{\circ}$

$$\frac{\sin 128}{1850} = \frac{\sin 17}{b}$$

$b \sin 128 = 1850 \sin 17$

$b = \boxed{686.40 \text{ ft}}$

6. An observer in Dweeb City sights a UFO at a bearing of N 45° E. Simultaneously, an observer in Nerdville sights the same UFO with a bearing of N 60° W. How far is the UFO from Nerdville if Nerdville is 3.6 miles N 76° E of Dweeb City?



$$60+\theta+76=180$$
$$\theta = 44^\circ$$
$$31+44+\phi=180$$
$$\phi = 105$$

$$\frac{\sin 31}{d} = \frac{\sin 105}{3.6}$$

$$d \sin 105 = 3.6 \sin 31$$

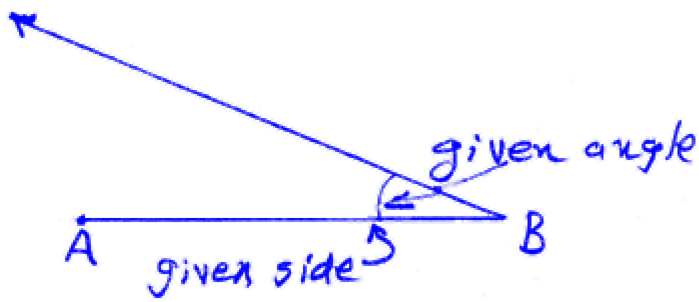
$$d = \frac{3.6 \sin 31}{\sin 105} = \boxed{1.9195 \text{ mi}}$$

Unit 3: Lesson 05 Ambiguous case of the sine law

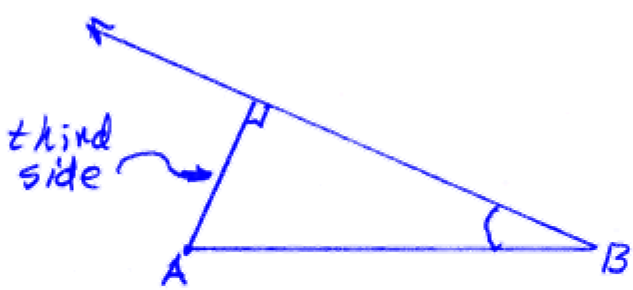
In the previous lesson the astute observer might have noticed that the three pieces of information initially given about a triangle never consisted of **two sides and a non-included angle**.

This is known as the **ambiguous case** since there are three possibilities for the solution.

First, consider being given a side (a line segment) and an angle at one end of that segment as shown here:

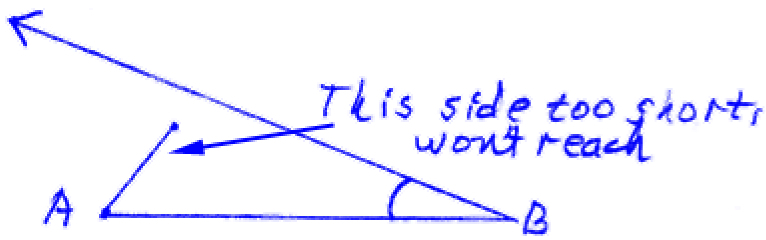


Case 1: Consider a third side that is opposite the angle (**now we have two sides and a non-included angle**) that is just barely long enough to reach the other side of the angle in a **perpendicular** fashion.



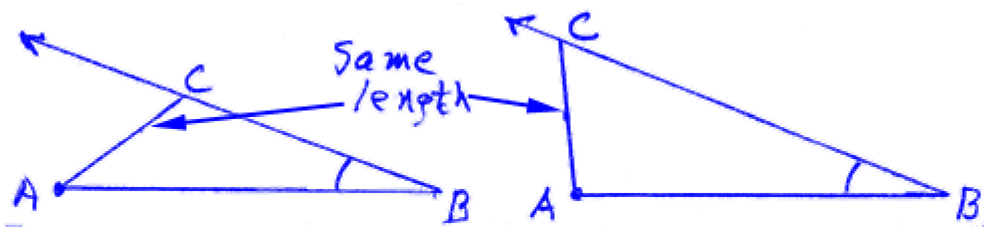
Mathematically, you will be alerted to this when the sine calculation of an angle yields a value of 1 (remember, sin $90^o = 1$).

Case 2: Consider a third side that is opposite the angle and is **too short** to reach the other side of the angle.



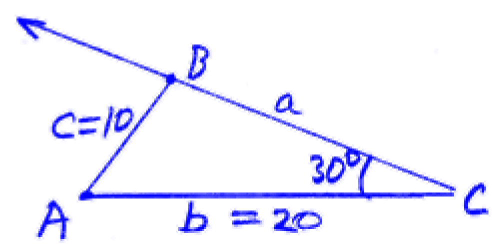
Mathematically, you will be alerted to this unpleasant possibility when the sine or cosine calculation of an angle yields an impossible value (outside the acceptable range, $-1 \le \text{value} \le 1$).

Case 3: Consider a third side (that is opposite the angle) that can actually touch the other side of the angle in **two places**.



Two different triangles are possible, and it must be decided from the physical situation represented by the problem if both solutions are acceptable or if one must be rejected.

Example 1 (representing case 1): Solve triangle ABC where b = 20, c = 10, C = 30°.



$90 + 30 + A = 180$

$A = \boxed{60^\circ}$

$\frac{\sin 30}{10} = \frac{\sin 60}{a}$

$a \sin 30 = 10 \sin 60$

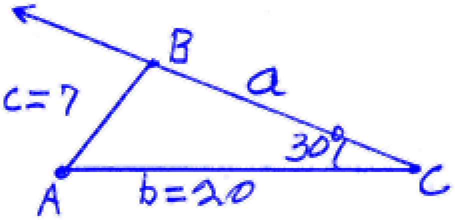
$a = \boxed{17.3205}$

$\frac{\sin 30^\circ}{10} = \frac{\sin B}{20}$

$\sin B = 1$

$B = \sin^{-1}(1) = \boxed{90^\circ}$

Example 2 (representing case 2): Solve triangle ABC where b = 20, c = 7, C = 30°.



$$\frac{\sin 30}{7} = \frac{\sin B}{20}$$

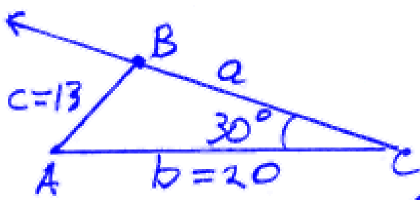
$$\sin B = (20 \sin 30)/7$$

$$\sin B = 1.428$$

Illegal, outside the range of −1 ⟷ 1.

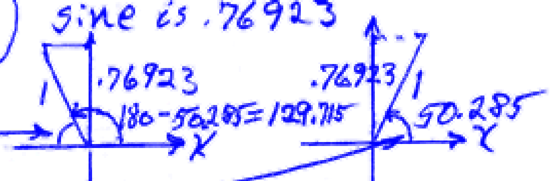
$$\boxed{\text{Triangle is not possible side c is too short!}}$$

Example 3 (representing case 3): Solve triangle ABC where b = 20, c = 13, C = 30°.

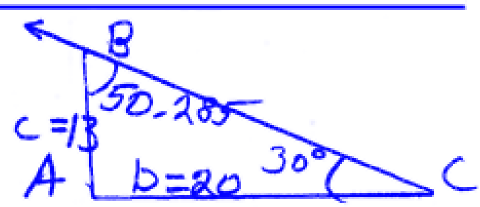


$$\frac{\sin 30}{13} = \frac{\sin B}{20}\ ;\ \sin B = .76923$$

There are 2 angles whose sine is .76923



$$B = \sin^{-1}(.76923) = 50.285°$$



B can be either of two values

$B = \boxed{129.715}$

$A + 30 + 129.715 = 180$

$A = \boxed{20.285°}$

$$\frac{\sin 30}{13} = \frac{\sin 20.285}{a}$$

$a = \boxed{9.014}$

$B = \boxed{50.285}$

$A + 30 + 50.285 = 180$

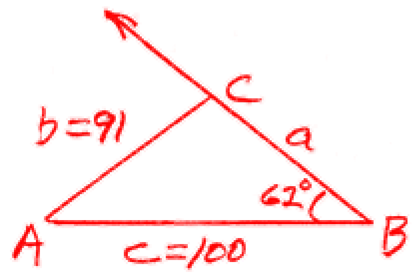
$A = \boxed{99.715°}$

$$\frac{\sin 30}{13} = \frac{\sin 99.715}{a}$$

$a = \boxed{25.627}$

Assignment:

1. Solve triangle ABC where c = 100, B = 62°, b = 91



$$\frac{\sin 62}{91} = \frac{\sin C}{100}$$

$\sin C = .97027$

$C = \sin^{-1}(.97027) = 75.994$

Two solutions: 75.994° and $180 - 75.994 = 104.006°$

Using $C = \boxed{75.994}$

$A + 75.994 + 62 = 180$

$A = \boxed{42.006}$

$$\frac{\sin 62}{91} = \frac{\sin 42.006}{a}$$

$a = \boxed{68.971}$

Using $C = \boxed{104.006}$

$A + 104.006 + 62 = 180$

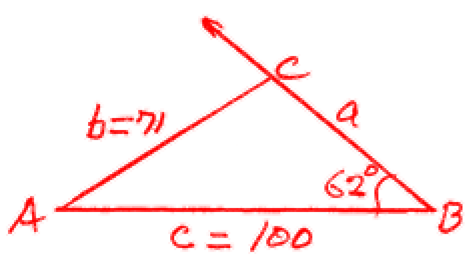
$A = \boxed{13.994}$

$$\frac{\sin 62}{91} = \frac{\sin 13.994}{a}$$

$a = \boxed{24.9229}$

---

2. Solve triangle ABC where c = 100, B = 62°, b = 71



$$\frac{\sin 62}{71} = \frac{\sin C}{100}$$

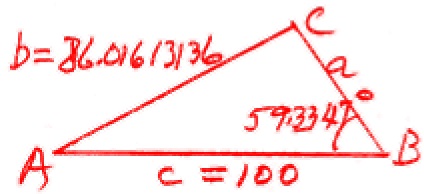
$\sin C = 1.2435$

impossible!

outside the permitted range of $-1 \longleftrightarrow 1$

$\boxed{\text{Side b is too short. Can't complete the triangle}}$

3. Solve triangle ABC where $c = 100$, $B = 59.3347^o$, $b = 86.01613136$



$$\frac{\sin(59.3347)}{86.01613136} = \frac{\sin C}{100}$$

$$\sin C = 1$$

$$C = \sin^{-1}(1) = \boxed{90^\circ}$$

$$A + 90 + 59.3347 = 180$$

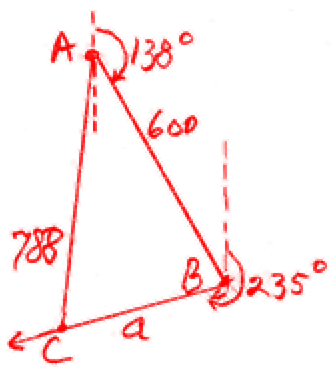
$$A = \boxed{30.6653^\circ}$$

$$\frac{\sin 59.3347}{86.01613136} = \frac{\sin 30.6653}{a}$$

$$a = \boxed{51.0022}$$

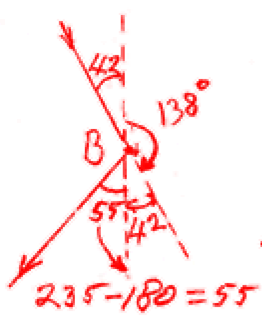
---

4. A pilot flies at a heading of 138^o from A to B and then at 235^o from B to C. If A is 600 miles from B and A is 788 miles from C, how far is it from C to B?



$$\frac{\sin 83}{788} = \frac{\sin C}{600};$$ Two solutions $C = 49.09$ & 130.91

$A + 49.09 + 83 = 180$	$A + 130.91 + 83 = 180$
$A = 47.91$	$A = -33.91$
$\frac{\sin 83}{788} = \frac{\sin 47.91}{a}$	impossible!
$a = \boxed{589.16 \text{ mi}}$	If the flight from B to C was reversed it would be possible to create this triangle

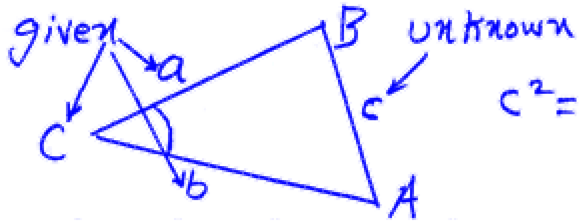


$$42 + B + 55 = 180$$

$$B = 83^\circ$$

Unit 3: Lesson 06 **Cosine Law**

In the previous lessons a problem was never given in which **two sides and their included angle** was the initial information given. Application of the sine law always results in a single equation in two variables: all progress stops. **To solve such a problem the cosine law is needed**:



$$c^2 = a^2 + b^2 - 2ab\cos C$$

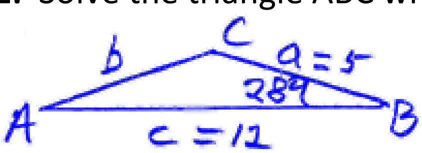
cosine law

Since the angles and corresponding opposite sides could have been labeled differently, the **cosine law** should be learned as follows:

> ***The square of a side equals the sum of the squares of the other two sides minus twice the product of the others sides and the cosine of the angle between them.***

See **Enrichment Topic L** for a derivation of the cosine law.

Example 1: Solve the triangle ABC where a = 5, c = 12, B = 28°.



$$b = \sqrt{12^2 + 5^2 - 2(12)5\cos 28}$$

$$b = \boxed{7.94}$$

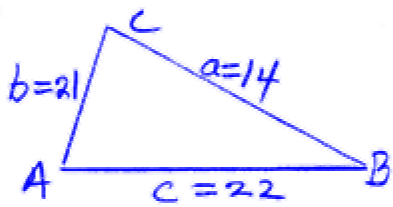
Use ambig. case of the Sine Law with either sin28/7.94=sinA/5 or sin28/7.94=sinC/12. Always choose the version having smaller of 5 & 12 and then use the resulting acute angle.

$$\frac{\sin 28}{7.94} = \frac{\sin A}{5} \quad \text{or} \quad \cancel{\frac{\sin 28}{7.94} = \frac{\sin C}{12}}$$

$$A = \boxed{17.20^\circ} \qquad A + B + C = 180$$

$$C = \boxed{134.8^\circ}$$

Example 2: Find the angles of the triangle whose sides are a = 14, b= 21, and c = 22.



$$a^2+c^2-2ac\cos B=b^2$$

$$\cos B=\frac{b^2-a^2-c^2}{-2ac}$$

$$\cos B=.388$$

$$B=\cos^{-1}(.388)=\boxed{67.17^\circ}$$

$$\frac{\sin 67.17}{21}=\frac{\sin A}{14}$$

Notice we choose the smaller of 14 and 22

$$\sin A=.614$$

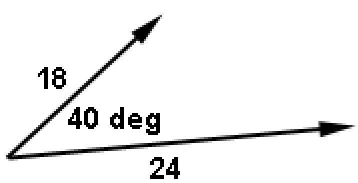
$$A=\sin^{-1}(.614)=\boxed{37.91^\circ}$$

$$A+B+C=180$$

$$37.91+67.17+C=180$$

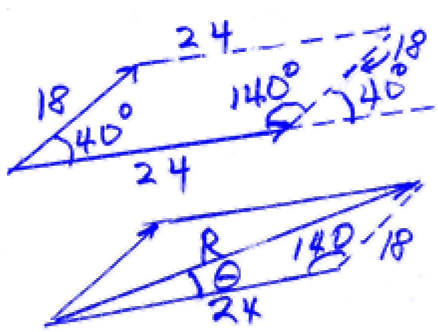
$$C=\boxed{74.92^\circ}$$

Example 3: Use the parallelogram method to add these vectors. Find both magnitude and direction of the resultant.



$$R=\sqrt{24^2+18^2-2(24)(18)\cos 140}$$

$$R=\boxed{39.52}$$



Direction is given by θ, angle with side 24.

$$\frac{\sin 140}{39.52}=\frac{\sin\theta}{18}$$

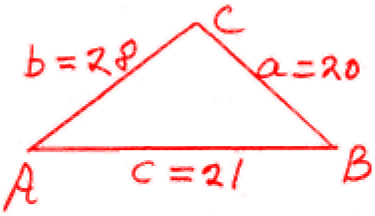
Notice we choose the smaller of 18 and 24.

$$\sin\theta=.292767$$

$$\theta=\sin^{-1}(.292767)=\boxed{17.02^\circ}$$

Assignment:

1. Find the angles of a triangle whose sides are: a = 20, b= 28, c = 21



$a^2+b^2-2ab\cos C=c^2$

$\cos C=\dfrac{c^2-a^2-b^2}{-2ab}$

$\cos C=\dfrac{21^2-20^2-28^2}{-2(20)(28)}$

$\cos C=.66339$

$C=\cos^{-1}(.66339)=\boxed{48.44^\circ}$

Now apply the sine law

$\dfrac{\sin 48.44}{21}=\dfrac{\sin A}{20}$

$A \doteq \boxed{45.45^\circ}$

Notice we choose the smaller of 20 and 28.

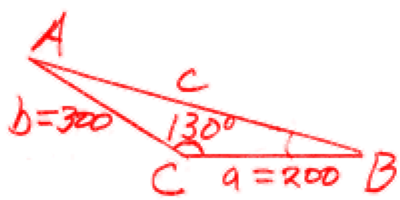
$A+B+C=180$

$45.45+B+48.44=180$

$B=\boxed{86.11}$

---

2. Solve the triangle ABC where a = 200, b = 300, C = 130°.



$c=\sqrt{300^2+200^2-2(300)(200)\cos 130}$

$c=\boxed{455.12}$

Notice we choose the smaller of 200 and 300.

$\dfrac{\sin 130}{455.12}=\dfrac{\sin A}{200}$

$\sin A=.3366$

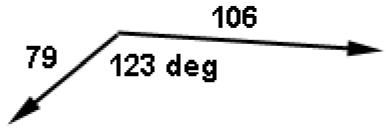
$A=\sin^{-1}(.3366)=\boxed{19.67^\circ}$

$A+B+C=180$

$19.67+B+130=180$

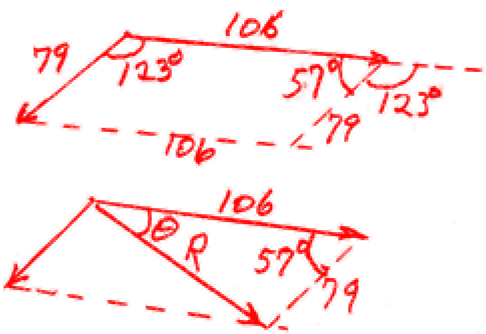
$B=\boxed{30.33^\circ}$

3. Use the parallelogram method to add these vectors. Find both magnitude and direction of the resultant.



$R = \sqrt{106^2 + 79^2 - 2 \cdot 106 \cdot 79 \cos 57}$

$R = \boxed{91.41}$



Direction is given by θ

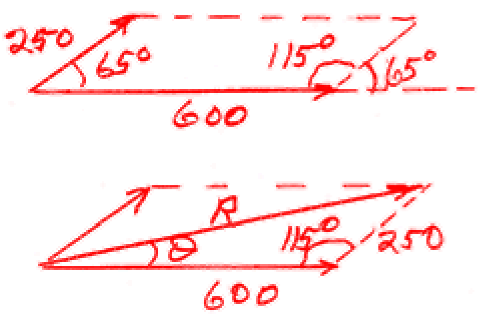
$\frac{\sin 57}{91.41} = \frac{\sin\theta}{79}$ Notice we choose the smaller of 79 and106

$\sin\theta = .7248$

$\theta = \sin^{-1}(.7248) = \boxed{46.45°}$

---

4. Two forces of 250 lbs and 600 lbs are acting at the same point and make an angle of 65° with each other. Find the magnitude of the resultant and its direction (the angle it makes with the 600 lb force).



$R = \sqrt{600^2 + 250^2 - 2(600)250 \cos 115}$

$R = \boxed{741.14}$

Direction is give by θ

$\frac{\sin 115}{741.14} = \frac{\sin\theta}{250}$

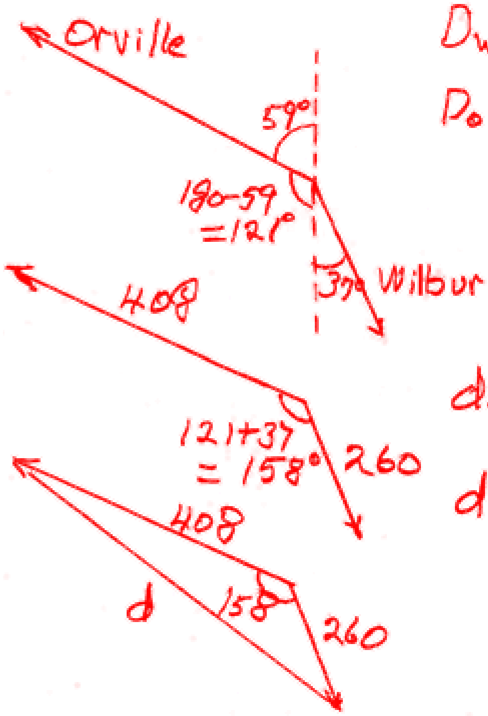
$\sin\theta = .3057$

$\theta = \sin^{-1}(.3057)$

$= \boxed{17.8°}$

Notice we choose the smaller of 250 and 600.

5. Orville and Wilbur leave the airport at the same time. Wilbur files S 37^{o} E at 65 mph while Orville flies N 59^{o} W at 102 mph. How far apart are they after 4 hours?



$D_w = vt = 65(4) = 260$ mi

$D_o = vt = 102(4) = 408$ mi

$d = \sqrt{408^2 + 260^2 - 2 \cdot 408 \cdot 260 \cos 158}$

$d = \boxed{656.33 \text{ mi}}$

Unit 3: Cumulative Review

1. Solve $4x^2 - 2x + 1 = 0$ with the quadratic formula.

$a = 4;\ b = -2;\ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4(4)1}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{-12}}{8} = \frac{2 \pm 2i\sqrt{3}}{8} = \frac{\cancel{2}(1 \pm i\sqrt{3})}{\cancel{8}\,4}$$

$$= \boxed{\frac{1 \pm i\sqrt{3}}{4}}$$

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2. Solve $3x^2 + x - 6 = 0$ by completing the square.

$\frac{3x^2}{3} + \frac{1x}{3} - \frac{6}{3} = \frac{0}{3}$ $\qquad \frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{6}$

$x^2 + \frac{1}{3}x - 2 = 0$ $\qquad \left(\frac{1}{6}\right)^2 = \frac{1}{36}$

$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{2}{1}\cdot\frac{36}{36} + \frac{1}{36}$

$\sqrt{\left(x + \frac{1}{6}\right)^2} = \pm\sqrt{\frac{73}{36}}$

$x + \frac{1}{6} = \pm\frac{\sqrt{73}}{6}$; $\boxed{x = -\frac{1}{6} \pm \frac{\sqrt{73}}{6}}$

---

3. Solve $x^2 + 2x - 35 = 0$ by factoring.

$(x + 7)(x - 5) = 0$

$x + 7 = 0 \qquad x - 5 = 0$

$\boxed{x = -7} \qquad \boxed{x = 5}$

4. Multiply $(2x^2 - 3y)(x^2 - 7y)$

F O I L

$= 2x^4 - 14x^2y - 3x^2y + 21y^2$

$= \boxed{2x^4 - 17x^2y + 21y^2}$

5. Define sin, cos, & tan in terms of opp, adj, and hyp as these terms apply to a right triangle.

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

6. Define sin, cos, & tan in terms of x, y, and r.

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

7. Simplify $\dfrac{a+\frac{3}{by}}{\frac{b}{y^2}+6}$

$$= \frac{a+\frac{3}{by}}{\frac{b}{y^2}+6}\cdot\frac{by^2}{by^2} = \boxed{\frac{aby^2+3y}{b^2+6by^2}}$$

8. Convert 3π/7 radians to degrees.

$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{\theta}{3\pi/7}$

$\theta = 3(180)/7 = \boxed{77.1429^\circ}$

9. Convert 26.12° to radians.

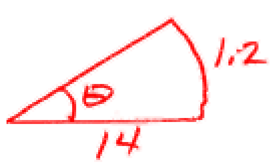
$\frac{deg}{rad} = \frac{deg}{rad}$

$\frac{180}{\pi} = \frac{26.12}{\theta}$

$\theta = 26.12\pi/180$

$\theta = \boxed{.45588 \text{ radians}}$

10. An angle, θ, subtends an arc of 1.2 meters and has a radius of 14 meters. What is the value of the θ in radians?



$\theta = \frac{1.2}{14} = \boxed{.08571 \text{ rad.}}$

11. Find the equation of the line (in slope-intercept form) that passes through (4, –1) and is perpendicular to the line given by x/3 + y/7 = 1.

$\frac{x}{3}+\frac{y}{7}=1$

$\frac{y}{7}=1-\frac{x}{3}$

$y=7-\frac{7}{3}x$

$m=-\frac{7}{3}$

$m_{\perp}=\frac{3}{7}$

$y=mx+b$

$y=\frac{3}{7}x+b$

$-1=\frac{3}{7}(4)+b$ sub in (4,-1)

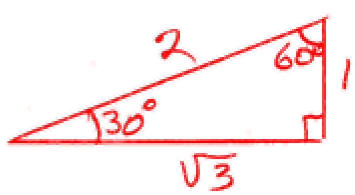
$-1-\frac{12}{7}=b$

$-19/7=b$

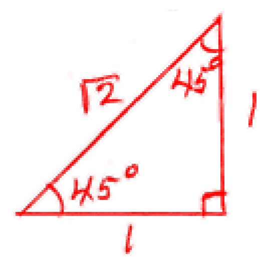
$y=mx+b$

$\boxed{y=\frac{3}{7}x-\frac{19}{7}}$

12. Draw a 30-60-90 triangle and label the standard lengths of the sides.



13. Draw a 45-45-90 triangle and label the standard lengths of the sides.



14. Solve this system of equations: 3x – 6y = 5 and x + 3y = 5

$3x-6y=5 \longrightarrow 3x-\cancel{6y}=5$

$2(x+3y)=5\cdot2 \longrightarrow 2x+\cancel{6y}=10$

$5x=15$

$x=\boxed{3}$

$3x-6y=5$

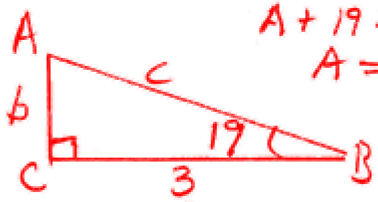
$3(3)-6y=5$

$-6y=5-9$

$y=\frac{-4}{-6}=\boxed{\frac{2}{3}}$

Unit 3: Review

1. Solve right triangle ABC where a = 3, and B= 19°.



$A + 19 = 90$

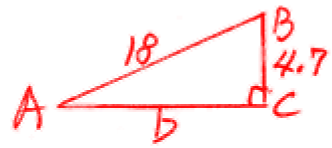
$A = \boxed{71^\circ}$

$\tan 19 = \frac{b}{3}$

$b = 3\tan 19 = \boxed{1.03}$

$c = \sqrt{3^2 + 1.03^2} = \boxed{3.171}$

2. Find the area of right triangle ABC where a = 4.7 and c = 18.



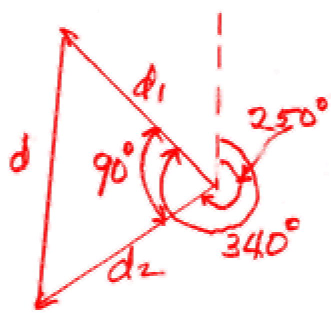
$b = \sqrt{18^2 - 4.7^2} = 17.375$

$K = \frac{1}{2}ab$

$K = \frac{1}{2}(4.7)(17.375)$

$= \boxed{40.83}$

3. Two planes leave an airport at noon. How far apart are they at 2:00 PM when one flies with a speed of 180 mph at a heading of 340° and the other with a speed of 200 mph at a heading 250°?



$d_1 = v_1 t = 180(2) = 360\text{ mi}$

$d_2 = v_2 t = 200(2) = 400\text{ mi}$

$d = \sqrt{d_1^2 + d_2^2}$

$= \sqrt{360^2 + 400^2} = \boxed{538.14\text{ mi}}$

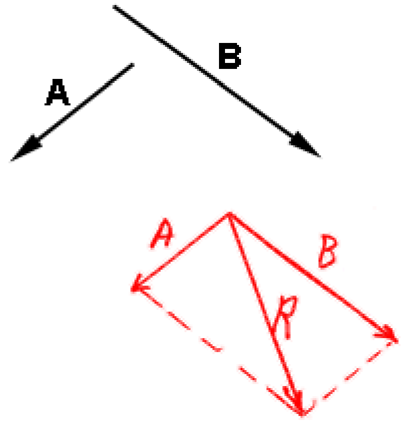
4. Write a formula for the cosine law in terms of the sides and angles of a triangle (a, b, c, A, B, C).

$c^2 = a^2 + b^2 - 2ab\cos C$

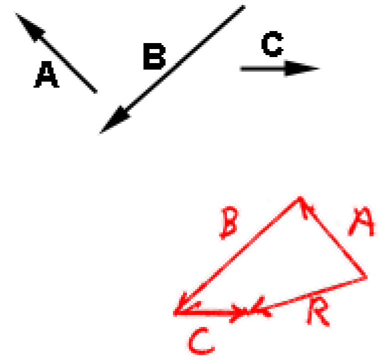
5. Write a formula for the sine law in terms of the sides and angles of a triangle (a, b, c, A, B, C).

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

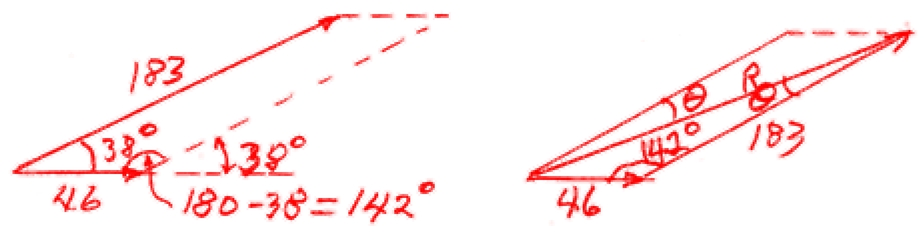
6. Graphically show how to add these two vectors with the parallelogram method.



7. Graphically show how to add these three vectors with the head-to-tail method.



8. A pilot is flying at 183 mph. The direction in which the plane is headed makes an angle of 38° with the direction of the wind which is blowing at 46 mph. Use the parallelogram method to determine the ground speed of the plane. How far off-course is the plane from the direction in which it tries to head?



$R = \sqrt{46^2 + 183^2 - 2 \cdot 46 \cdot 183 \cos 142°} = \boxed{221.07 \text{ mph}}$

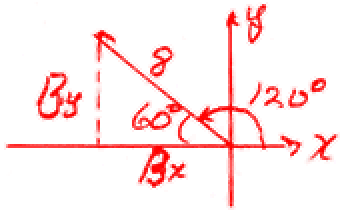
$\frac{\sin 142}{221.07} = \frac{\sin\theta}{46}$ **Choose the smaller of 46 and 183**

$\sin\theta = .128106$; $\theta = \sin^{-1}(.128106) = \boxed{7.36°}$

9. What information about a triangle constitutes the “ambiguous case” of the sine law?

2 sides + a non-included angle.

10. Vector A consists of two components, $A_X = 5$ and $A_Y = -11$. Vector B has magnitude 8 and has a direction described by an angle of 120° in standard position. Find the magnitude of the sum of vectors A and B using the component method.



$B_x = -8\cos 60 = -4$

$B_y = 8\sin 60 = 6.9282$

$R_x = A_x + B_x = 5 - 4$
$= 1$

$R_y = A_y + B_y = -11 + 6.9282$
$= -4.0718$

$R = \sqrt{1^2 + 4.0718^2}$

$R = \boxed{4.1928}$

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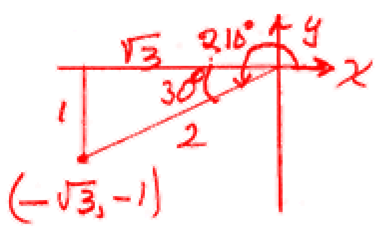
11. Find the exact values of the cosecant and tangent of 210°.



$\csc 210 = \frac{r}{y} = \frac{2}{-1} = \boxed{-2}$

$\tan 210 = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \boxed{\frac{1}{\sqrt{3}}}$

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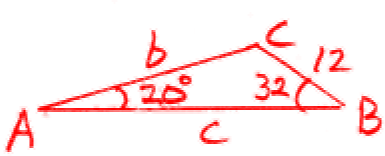
12. Solve triangle ABC where A = 20°, B = 32°, a = 12 .



$A + B + C = 180$

$20 + 32 + C = 180$

$C = \boxed{128°}$

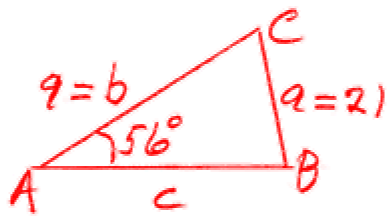
$\frac{\sin 20}{12} = \frac{\sin 32}{b}$

$b = \boxed{18.59256}$

$\frac{\sin 20}{12} = \frac{\sin 128}{c}$

$c = \boxed{27.648}$

13. Solve triangle ABC where A = 56°, b = 9, a = 21 .



$\frac{\sin 56}{21} = \frac{\sin B}{9}$

$\sin B = .3553$

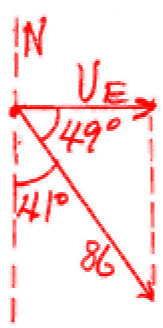
$B = \sin^{-1}(.3553) = 20.811°$

The supplement is also an answer → $B = 159.188°$

Two solutions

$B = \boxed{20.811°}$	$B = 159.188°$
$56 + 20.811 + C = 180$	$56 + 159.188 + C = 180$
$C = \boxed{103.19°}$	$C = -35.188$
$\frac{\sin 56}{21} = \frac{\sin 103.19}{c}$	impossible!
$c = \boxed{24.6623}$	No Answers for this part

14. What is the component of the vector of magnitude 86 and bearing S 41° E in the east direction?



$\cos 49 = \frac{adj}{hyp}$

$= \frac{V_E}{86}$

$V_E = 86 \cos 49$

$= \boxed{56.421}$